

## Assignment 1, Constraint Satisfaction For Decision Making (CE884-7-SP), 2009-10 Set by Edward Tsang, University of Essex

### 1. Introduction:

This is an assignment on problem formulation. This assignment accounts for 10% of your total marks in this course. This assignment should be submitted electronically. The deadline of this assignment is **11:59:59, Friday 26 February 2010**.

### 2. Objective:

The objective of this assignment is to give you a chance to formulate a constraint satisfaction problem.

### 3. A Stable Marriage Problem:

Given are  $n$  men and  $n$  women. Each man has his own ranking of all the women and each woman has her own ranking of all the men. The task is to marry the men and the women to make  $n$  couples, ensuring that all the marriages are stable. Every man and woman is allowed to marry one person only. A marriage is *unstable* if:

- (i) a man  $X$  prefers any woman  $Y$  to his wife, and  $Y$  prefers  $X$  to her husband; or
- (ii) a woman  $X$  prefers any man  $Y$  to her husband, and  $Y$  prefers  $X$  to his wife.

More formally, given are two  $n \times n$  matrices,  $M$  and  $W$ . All entries of these matrices take values 1 to  $n$ .  $M$  stores the preferences of the men.  $W$  stores the preferences of the women (in bold italic). Each element of  $M[i]$  takes a unique value between 1 and  $n$ . If  $M[i, j]$  is less than  $M[i, j']$ , that means Man  $i$  prefers Woman  $j$  to Woman  $j'$ . Similarly, each element of  $W[i]$  takes a unique value between 1 and  $n$ . If  $W[i, j]$  is less than  $W[i, j']$ , that means Woman  $i$  prefers Man  $j$  to Man  $j'$ .

		<b><i>w[1]</i></b>	<b><i>w[2]</i></b>	<b><i>w[3]</i></b>	<b><i>w[4]</i></b>
M[1]	1	<b><i>1</i></b>	2	3	4
M[2]	4	<b><i>2</i></b>	2	3	1
M[3]	2	<b><i>3</i></b>	3	1	4
M[4]	1	<b><i>4</i></b>	3	4	2

The above table shows the preference of each person in an instance of the stable marriage problem. For example, the row M[3] shows the preferences of Man 3, which is Women 3, 1, 2 and 4 in that order. The  $W$  matrix reads column first. For example, the first row, third column (with value 3) represents  $W[3, 1]$ . Since  $W[3, 1]$  is greater than  $W[3, 2]$  (which has value 2), Woman 3 prefers Man 2 to Man 1.

To illustrate the problem further, if we use  $M_j-W_k$  to indicate the marriage between Man  $j$  and Woman  $k$ , then  $\{M1-W1, M2-W2, M3-W3, M4-W4\}$  is not a solution to the above problem. This is because  $M2-W2$  is *not* a stable marriage, as Man 2 prefers Woman 4 to his wife (Woman 2), and at the same time, Woman 4 prefers Man 2 to her husband (Man 4). This violates constraint (i) above. On the other hand,  $\{M1-W1, M2-W4, M3-W3, M4-W2\}$  is a solution to the above problem.

#### **4. Your task:**

Formulate the above problem as a constraint satisfaction problem. You must state clearly what the variables, domains and constraints are, and why they should be part of the formulation. Bonus will be given if your formulation is general for any number of couples, on condition that clarity is not compromised.

#### **5. Submission requirements:**

Write your name clearly on the first page of your submission. Underline your surname. Submit a report of strictly no more than 1,000 words, stating your formulation of this problem. State precisely the variables, domains and constraints. Clearly explain how you arrive at your formulation. State clearly where domain-specific knowledge is used. Evaluate the size of your search space. Explain how any of the constraints that you have defined might be particularly useful for solving the problem.

#### **6. Assessment criteria for this assignment:**

Correctness in formulation is the main criteria for evaluating your report. It is also important that you explain your formulation clearly. Reference should be made to constraint satisfaction techniques that you have learned in the lectures.

#### **7. Please refer to the Student's handbook on the Departmental Policy on Plagiarism and Late Submission**