Constraint Satisfaction for Decision Making

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http://csp.bracil.net/ConstraintSatisfactionTeaching

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CE884: Constraint Satisfaction Contents

1. Overview
2. Problem Formulation (Z, D, C)
3. Problem Reduction – NC, AC, PC, DAC
4. Lookahead Search algorithms
5. Gather Information While Searching
6. Search Ordering
7. Stochastic Search – Hill Climbing, GSAT
8. Meta-heuristic Search – GENET, GLS, GA

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Constraint Satisfaction

A non-technical introduction

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Constraint Satisfaction Quick Summary

- It's about decision-making
  - Large number of decisions to make
  - Complex constraints limited your options
- Ubiquitous (you find it everywhere)
  - Though you may not realize it
- Why difficult? "*Combinatorial Explosion*"
  - There are too many combinations to consider
- Challenge: to solve bigger problems faster

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Constraint Satisfaction Applications

- Resources Allocation
  - Staff rostering, timetabling
  - BT workforce scheduling
  - BA flight scheduling
- Transportation
  - *ILOG Dispatcher* for vehicle routing
  - Train and bus scheduling
  - Collective transportation
- Microsoft AutoCollage
- Gnu C Compiler
- Database queries

- Fun: Sudoku
- Logistics
  - Satellite scheduling
  - Radio links assignment (military & mobiles)
  - Phone calls routing
  - Airport / container port
- Industrial Scheduling
  - Just-in-time scheduling
  - Hardware configuration
  - Car sequencing

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The Sudoku Problem
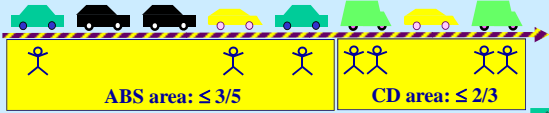
	2	6				8	1	
3			7		8			6
4				5				7
	5		1		7		9	
		3	9		5	1		
	4		3		2		5	
1				3				2
5			2		4			9
	3	8				4	6	

- Given: a 9 × 9 grid, with some entries blank, some containing a digit.
- Find: a complete grid.

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Car Sequencing Problem

Options				
ABS	×	√	√	×
CD	×	×	√	√
...				Total:
Production:	30	30	20	40	120

ABS area: ≤ 3/5

CD area: ≤ 2/3

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Airline Applications

Constraint satisfaction and optimization are behind many applications in airline operations; e.g. British Airways uses ECLiPSe (IC-Parc) for scheduling aircraft to serve their flights and ILOG Solver for aircraft stand allocation at airports.

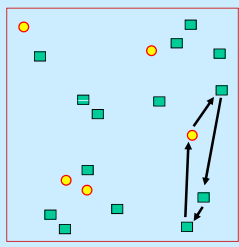Edward Tsang (Copyright) 10 Ap PRCSLS F U Tuesday, 17 January 2012

Transportation

Constraint satisfaction is one of the core technologies in transportation optimization. For example Guided Local Search was used in ILOG Solver's vehicle routing package, Dispatcher. Cairo/Line schedule for collective transportation.

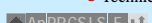Edward Tsang (Copyright) 11 Ap PRCSLS F U Tuesday, 17 January 2012

BT's Workforce Scheduling

BT has many jobs to be done in UK every day. It has to schedule a large number of teams to serve these jobs, subject to time, skill and other constraints. Saving of 0.5% could mean Millions of Pounds per year. Guided Local Search achieved the best results in one of BT's challenge problems.
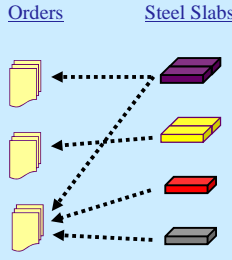
○ Technicians   ■ Jobs

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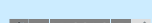
CP in Steel Industry

IBM helped Korea's biggest steel manufacturer to apply constraint technology to schedule its production to meet orders. This include the allocation of existing stocks to orders, subject to various constraints, such as size, quality and colour, in order to minimize waste and cost.

Orders   Steel Slabs

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Microsoft AutoCollage 2008

What is Constraint Satisfaction?

- Constraint satisfaction is a decision problem
  - You are given a number of decisions to make
  - For each decision, you are given all the choices
  - Decisions constrain each other
- Your task is to make those decisions without violating any of the constraints
- Sometimes you want the "best" solution
  - If so, you have a (constrained) *optimisation* problem

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The Constraint Satisfaction Problem

Variables (Decisions)   Domains (values available)   Constraints On assignments
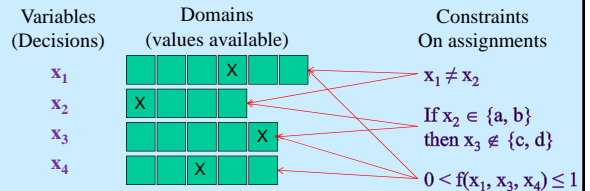
x_1
x_2
x_3
x_4

$x_1 \neq x_2$

If $x_2 \in \{a, b\}$ then $x_3 \notin \{c, d\}$

$0 < f(x_1, x_3, x_4) \leq 1$

- Constraint satisfaction is a decision problem
- Task: make decisions without violating constraints
- Sometimes you want the "best" solution

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Why constraint satisfaction?

- It is a very general problem (seen everywhere)
  - Mainly logistics, scheduling, resources allocation
- Specialized methods available
- Now multi-million Pounds/Dollars business
- Scientific challenges:
  - Combinatorial explosion (fundamental problem)
  - Modelling (engineering problem)

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Combinatorial Explosion of the Car Sequencing Problem

- Schedule 30 cars:
  - Search space: 30 factorial $\cong 10^{32}$ leaf nodes
- Generously allow:
  - Explore one in every 10^{10} leaf nodes!
  - Examine 10^{10} nodes per second!
- Problem takes over **32 thousand years** to solve!!!
  - $10^{32} \div 10^{10} \div 10^{10} \div 60 \div 60 \div 24 \div 365 \cong 31,710$
- How to contain *combinatorial explosion*?

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Use The Force

- Constraints brought the problem here
- Constraints guide us to solutions
- Experts will follow the lead by constraints
- See *Guided Local Search* later
- The *Incentive Method* was used in financial forecasting and bargaining
- Reference to Daoism (Taoism)

http://www.bracil.net/edward/library/Zhuangzi-cook

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Constraint Satisfaction

A more technical account

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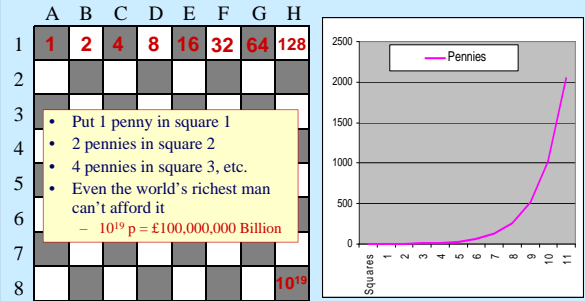
Constraint Satisfaction: What is it all about?

- Many problems involve finding combinations of values to variables, e.g.
  - Telecom services scheduling
  - Industrial scheduling
- Combinatorial problems are hard in nature
- How big a problem can one solve?
- Constraint research aims to extend our boundaries in these problems
  - It's all about algorithms, efficiency, problem solving …

Combinatorial Explosion

	A	B	C	D	E	F	G	H
1	1	2	4	8	16	32	64	128
2								

- Put 1 penny in square 1
- 2 pennies in square 2
- 4 pennies in square 3, etc.
- Even the world's richest man can't afford it
  - 10^{19} p = £100,000,000 Billion

10^{19}

Problem Formulation (Modelling)

Is my problem a constraint satisfaction problem?

If so, I can apply established techniques

What is constraint satisfaction?

Given:
- Z: a set of variable (*Decisions*)
- D: mapping each variable x to a domain Dx (*fixed choices*)
- C: constraints restricting the values that variables can take simultaneously

Task:
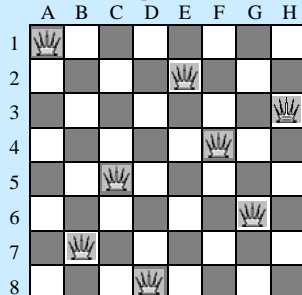- *Assign one value to each variable satisfying all the constraints*

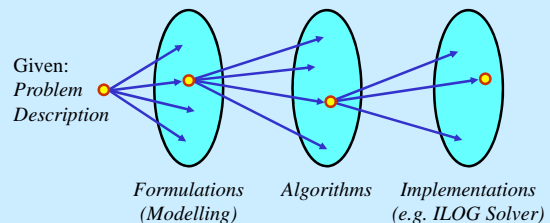## The 8-Queens Constraint Satisfaction Problem

Sample Solution

- Task: to put 8 queens onto the board such that no queen attacks others
- No two queens appear in the same row, column and diagonal

Constraint Programming

Given:
Problem Description

Formulations (Modelling)   *Algorithms*   *Implementations (e.g. ILOG Solver)*

- Areas: modelling, algorithms, algorithm mapping, monitoring, packages

Constraint Satisfaction Techniques
A Brief Overview

- Propagation constraints in order to rule out dead-ends
- Search systematically to ensure that solutions are found if they exist
  - Many heuristics regarding where to *look* next
- Heuristic search: exploring promising solutions at the risk of missing solutions
  - Many heuristic regarding where to *move to*

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Constraint Techniques Overview

| Constraint satisfaction |
| Constrained optimisation |

1. Problem Reduction
  - To reduce domain size
  - To add or tighten constraints
  - Aims:
    - To reduce a problem to easier ones
    - To detect dead-ends
    - *To achieve backtrack-free search*

2. Search
- Systematic Search
  - Lookahead
  - Learning nogoods or backtrack intelligently at dead-ends
  - Ordering heuristics
  - Aim: to exhaustively search space
- Stochastic methods
  - To *repair* solutions heuristically
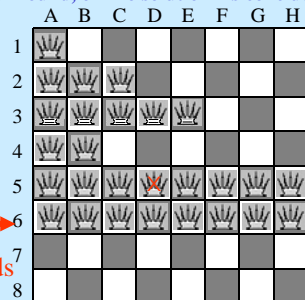 - Aim: to find solutions fast, sacrificing completeness

3. (Solution Synthesis)

Backtracking Search In The 8-Queens Problem

Complete search, till solution found, or "no solution" is concluded

- Place one queen per row
- Place one queen at a time
- Examine each column

Backtrack at dead-ends

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Forward Checking Search

- Problem reduction
  - a major technique
- Combined with search methods
- Reduce domain of future variables
- Detect dead-ends
  - To backtrack early

Dead-end detected after Queen 4 – no legal space for row 6, backtrack…

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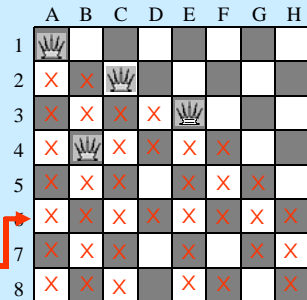
Constraint Optimization

- Constraint Satisfaction & Optimization
  - Some solutions are better than others
- Partial Constraint Satisfaction
  - Accept best solution when no solution is found
- Relevant techniques:
  - Complete: Branch & Bound
  - Incomplete: HC, SA, Tabu, NN, GA, GLS

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Commercial Constraint Business
(Multi-million Pounds business)

- ILOG Solver (SAP subsidiary, France)
  - "The ILOG-Solver inside" solutions
- Cosytec CHIP (European consortium)
  - Pioneer of constraint technology, consultancy
- ECLIPSE (IC-Parc, Imperial College, UK)
  - Supported by ICL, BA, etc
- Many more companies & products
  - many specialized in scheduling

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