Exercise – Model Selection

$$\begin{array}{cccccc} & S & E & N & D \\ + & M & O & R & E \\ \hline M & O & N & E & Y \end{array}$$

a) **What is the size of the search space of each of these two formulations?**

b) **Is the size of the search space affected by the decision of maintaining 1-consistency before the search starts?**

c) **How do the constraint graph of these two formulations look like? There is no need to draw the entire constraint graphs, but describe them clearly.**

d) **If you run Forward Checking on each of the above two formulations, which run would you expect to explore more nodes? Justify your answer carefully.**

e) **Based on your answer to the above sections, explain carefully how you would choose between these two formulations.**

Formulation 1:

Variables: {S, E, N, D, M, O, R, Y}

Domains: {0, 1, …, 9}

Constraints:

C1: $\forall x,y \in \{S, E, N, D, M, O, R, Y\}, x \neq y$

C2: M = 0 or M = 1

C3: (1000×S+100×E+10×N+D) + (1000×M+100×O+10×R+E) = (10000×M+1000×O+100×N+10×E+Y)

Formulation 2:

Variables: {S, E, N, D, M, O, R, Y} + {C1, C2, C3} representing the three carries from right to left

Domains: $\forall x \in \{S,E,N,D,M,O,R,Y\}$ $D_x = \{0, \ldots,9\}$

$\forall y \in \{C1, C2, C3\}$ $D_y = \{0, 1\}$

Constraints:

C1: $\forall x, y \in \{S, E, N, D, M, O, R, Y\}, x \neq y$

C2: M = 0 or M = 1

C3: D + E = 10×C1 + Y

C4: N + R + C1 = 10×C2 + E

C5: E + O + C2 = 10×C3 + N

C6: S + M + C3 = 10×M + O

Exercise – Model Selection Answer

a) **Size of search space = Π DS_x, where DS_x is the domain size of variable x**

- **The size of the search space in Formulation 1 is 10^8**
  - **Or 8! if all-different constraint considered**
- **The size of the space in Formulation 2 is $10^8 \times 2^3$**
  - **Or $8! \times 2^3$**

b) **Small difference – after maintaining 1-C, domain of M is reduced to {0, 1}**

- **Size of space in Formulation 1 becomes $10^7 \times 2$ (or $7! \times 2$).**
- **Size of space in Formulation 2 becomes $10^7 \times 2^4$ (or $7! \times 2^4$).**

c) **In both Formulations: the "all-different" constraint ensures that every node is connected to every other node.**

d) **Formulation 1 doesn't allow much constraint propagation. FC performs will not prune any nodes. Therefore, it will explore more nodes than FC running in Formulation 2.**

e) **In general, comparing two formulations is not easy – more a craft than a science. In this particular problem,**

- **Based on (a) and (b), F.2 searches a bigger space, hence less favourable.**
- **Based on (c), no Formulation is better than the other;**
- **Based on (d), Formulation 2 is superior to Formulation 1.**
- **Formulation 2 is far more superior than Formulation 1 as the latter allows no constraint propagation apart from the all-different constraint.**