CC484 - Constraint Satisfaction

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Outline

General Concepts (1/4)

- A graph $G = \{V, E\}$ is a structure that consists of a finite set of vertices or nodes, $V = \{v_1, \cdots, v_n\}$, and a set of edges or arcs, $E = \{e_1, \cdots, e_\ell\}$.
- Each edge e is incident to an unordered pair of vertices $\{u, v\}$ that are not necessarily distinct.
- If $e = (u, v) \in E$, we say that e connects u and v and that u and v are adjacents or neighbours.
- The degree $d(u)$ of a vertex u in a graph is the number of its adjacent vertices.
- A path is a sequence of edges $e_1, \cdots, e_k$ such that e_i and e_{i+1} share an endpoint.

General Concepts (2/4)

- It is convenient to describe a path using its vertices $v_0, \cdots, v_k$, where $e_i = (v_{i-1}, v_i)$. In this case v_0 is called the start vertix of the path, v_k is called the end vertix and the length of the path is k
- A cycle is a path whose start and end vertices are the same.
- A path is simple if no vertex appears on it more than once.
- A cycle is simple if no vertex other than the start-end vertex appears more than once and the start-end vertex does not appear elsewhere in the cycle.
- If for every two vertices u and v in the graph there exists a path from u to v, then the graph is said to be connected.
- An undirected graph with no cycles is called tree.

General Concepts (3/4)

- A directed graph (digraph) is defined similarly to an undirected graph except that the pair of endpoints of an edge is now ordered.
- The edge $e = (u, v)$, also denoted $u \rightarrow v$, is said to be directed from u to v.
- The outdegree of a vertex v is the number of edges that have v as their start vertex.
- The indegree of v is the number of edges that have v as their end vertex.
- A directed path is a directed cycle if the start vertex of the path is the same as the end vertex.
- A directed graph is strongly connected if for every vertex u and every vertex v there is a directed path from u to v.
- A directed graph is acyclic if it has not directed cycles.

General Concepts (4/4)

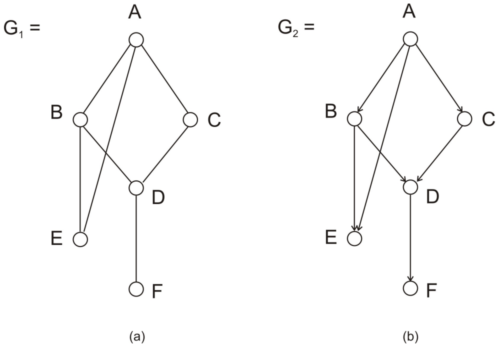

Figure: 1. (a) undirected graph and (b) directed graph.

Basic Framework (1/1)

- A constraint network R consists of a finite set of variables $X = \{X_1, \cdots, X_n\}$, with respective domains $D = \{D_1, \cdots, D_n\}$ and a set of constraints $C = \{C_1, \cdots, C_n\}$.
- A constraint network can be viewed as a triple (X, D, C).
- A constraint C_i is a relation R_i defined on a subset of variables $S_i, S_i \subset X$.
- The arity of a constraint refers to the cardinality of its scope. A unary constraint is defined on a single variable, a binary constraint, on two variables.
- A binary constraint network has only unary and binary constraints.

Formulating the n-Queens Problem (1/2)

- Think of the columns of the chessboard as the variables $x_1, \cdots, x_n$, and the possible row positions, $D_i = \{1, \cdots, n\}$, as domains of the variables.
- Assigning a value $j \in D_i$ to a variable x_i means to place a qeen in row j on column x_i of the board
- The complete definition of the 4-queens problem is $R = (X, D, C)$, where $X = \{x_1, x_2, x_3, x_4\}$, and for every i, $D_i = \{1, 2, 3, 4\}$. There are six constraints: $C_1 = R_{12}, C_2 = R_{13}, C_3 = R_{14}, C_4 = R_{23}, C_5 = R_{24}, C_6 = R_{34}$.

Formulating the n-Queens Problem (2/2)

- $R_{12} = \{(1,3),(1,4),(2,4),(3,1),(4,1),(4,2)\}$
- $R_{23} = \{(1,3),(1,4),(2,4),(3,1),(4,1),(4,2)\}$.
- $R_{24} = \{(1,2),(1,4),(2,1),(2,3),(3,2),(3,4),(4,1),(4,3)\}$.
- $R_{34} = \{(1,3),(1,4),(2,4),(3,1),(4,1),(4,2)$.

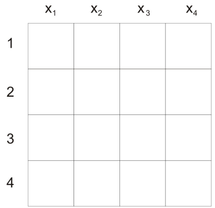

Figure: 1. The 4-queens constraint network. The network has four variables, all with domains $D_i = \{1,2,3,4\}$. The labeled chessboard.