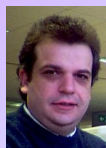


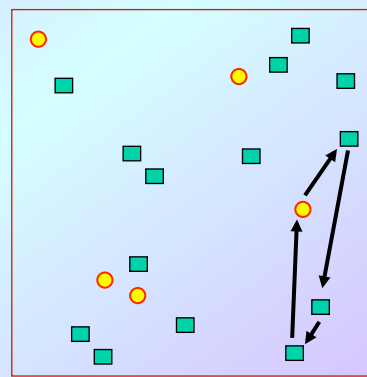
Constraint Satisfaction

Professor Edward Tsang
University of Essex
URL: <http://www.bracil.net/CSP/>



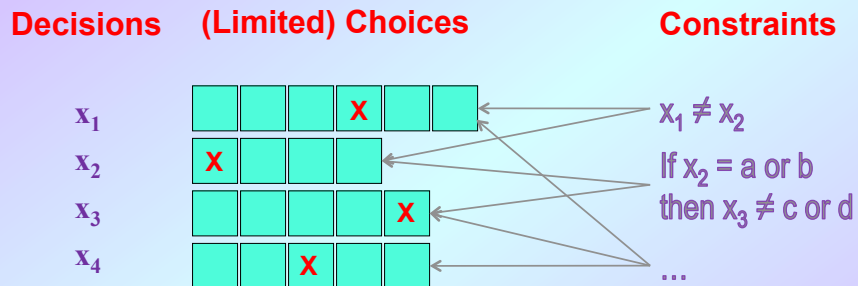
BT's Workforce Scheduling

BT has many jobs to be done in UK every day. It has to schedule a large number of teams to serve these jobs, subject to time, skill and other constraints. A saving of 0.5% could mean £Millions per year.



● Technicians ■ Jobs

What is Constraint Satisfaction?



- Constraint satisfaction is a **decision problem**
- Task: make decisions without violating constraints
 - Sometimes you want the “best” solution

Airline Applications

*British Airways uses constraint satisfaction to **schedule aircraft** to serve their flights, satisfying various constraints.*



Transportation



*Constraint satisfaction is one of the core technologies in transportation optimization. For example **Guided Local Search** was used in ILOG Solver's vehicle routing package, Dispatcher. Cairo/Line schedule for **collective transportation**.*

Image Processing

Microsoft AutoCollage uses constraint satisfaction (among other techniques) to blend photos together.




The Unsung Heroes

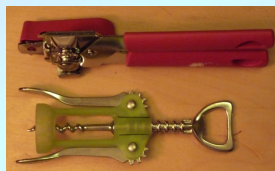
- People recognize brand names / end products
- When looking at a car, few would ask:
 - Who built the engines?
 - Who made the nuts and bolts?
 - What metal did they use?
- Constraint satisfaction is ubiquitous



Why Constraint Satisfaction?

- It is **ubiquitous** (seen everywhere)
 - Logistics, scheduling, resources allocation, ...
- **Specialized methods** available 
 - Special tools will solve larger problems quicker

Special Tools



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General Tools



16/12/2016

Scientific Challenges

- Take BT's job scheduling
- **Modelling** (Engineering Problem)
 - Defining the decisions & constraints
- **Combinatorial Explosion**
 - How to decide who to do which job, when?
 - There many decisions, many choices
 - Fundamental problem in computer science

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Summary, Constraint Satisfaction

- Decision problem
 - Limited conflicting choices
- Constraint satisfaction is everywhere
- Clever algorithms have been developed
 - One tries to exploit features of the problems
- This is a successful multi-£M business
 - Though most people don't recognize it

Remarks

- Constraints are the very reason why we have a hard problem to solve
- But to the trained eyes, constraints tell us how to solve the problem!

- Use the force!
莊子: [庖丁解牛](#)



Technical Introduction

A Glimpse of Constraint Satisfaction

Maintaining Arc-Consistency

- Variables: x, y, z
- Domains: $\{1, 2, 3, 4\}$
- Constraints:
 $x < y; y < z$

x	1	2	3	4
y	1	2	3	4
z	1	2	3	4

- $x < y$ means $\langle x, 4 \rangle$ is not supported by y and $\langle y, 1 \rangle$ is not supported by x
- $y < z$ means $\langle y, 4 \rangle$ is not supported by z and $\langle z, 1 \rangle$ & $\langle z, 2 \rangle$ is not supported by y

- Re-check $x < y$ would delete $\langle x, 3 \rangle$ as now (with $\langle y, 4 \rangle$ gone) it has no support from y

The 8-Queens Constraint Satisfaction Problem

- Task: to put 8 queens onto the board such that no queen attacks others
- No two queens appear in the same row, column and diagonal

	A	B	C	D	E	F	G	H
1	█	□	█	□	█	□	█	□
2	□	█	□	█	□	█	□	█
3	█	□	█	□	█	□	█	□
4	□	█	□	█	□	█	□	█
5	█	□	█	□	█	□	█	□
6	□	█	□	█	□	█	□	█
7	█	□	█	□	█	□	█	□
8	□	█	□	█	□	█	□	█

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Friday, 16 December 16

Backtracking Search In The 8-Queens Problem

Complete search, till solution found, or “no solution” is concluded

- Place one queen per row

Strategies at dead-ends:

- Learning “no goods”
- dependency directed backtracking

column

Backtrack at dead-ends

	A	B	C	D	E	F	G	H
1	♛							
2	♛	♛	♛					
3	♛	♛	♛	♛	♛			
4	♛	♛						
5	♛	♛	♛	♛	♛	♛	♛	♛
6	♛	♛	♛	♛	♛	♛	♛	♛
7								
8								



Forward Checking Search

- Problem reduction – a major technique
- Combined with search methods
- Reduce domain of future variables
- Detect dead-ends – To backtrack early

Dead-end detected after Queen 4 – no legal space for row 6, backtrack...

	A	B	C	D	E	F	G	H
1	♛							
2	X	X	♛					
3	X	X	X	X	♛			
4	X	♛	X	X	X	X		
5	X	X	X		X	X	X	
6	X	X	X	X	X	X	X	X
7	X	X	X		X		X	X
8	X	X	X		X	X		X



Arc-Consistency Lookahead


- 8-Queens Problem
- Three queens have been placed
- Maintaining AC
- Result: dead-end found in row 7
- Backtrack required – typically remove Queen 3

	A	B	C	D	E	F	G	H
1	♔							
2			♔					
3					♔			
4	X	1 ⁶	X	X	X	X		
5	X		X	1 ⁶	X	X	X	3 ⁴
6	X	X	X		X	X	X	X
7	X	4 ⁵	X	2 ⁶	X	5 ⁸	X	X
8	X	2 ⁶	X	2 ⁶	X	2 ⁶		X

Back Jumping

- Jump to the latest culprit
- Recorded earliest conflict
- Identify the latest culprit, which is 4
- Undo queens 5 and 4, continue

	A	B	C	D	E	F	G	H
1	♔							
2			♔					
3					♔			
4		♔						
5				♔				
6	1	3	2	4	3	1	2	3
7								
8								

 **Heuristic Repair, Example**












Solution found

- Start with random assignments
- C2 attacks G6
D8 attacks E7
- Randomly pick one, say, E7, to repair

Count number of conflicts in each square

Randomly pick a square with least attacks, say, B7

Repeat repair

	A	B	C	D	E	F	G	H
1								
2								
3								
4								
5	2			2	2	4	3	2
6								
7	1		3	3		2	2	3
8								

Constraint Handling Techniques Summary

- Problem Reduction
 - Reduce the problem to an easier problem
- Exhaustive Search Techniques, e.g.:
 - Look ahead in order to detect dead-ends
 - Backtrack intelligently at dead-ends
- Stochastic Search
 - Handle problems which are too big to solve
 - Normally through “repairing” solutions

