The Event Calculus on High Frequency Finance

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Ignoring the "Obvious"

- Where can we place a piece without collapsing this pile?
- We should be able to evaluate risks given our physics knowledge
- We ignore the physics in financial markets!

The Physics in Markets

- We don't know many things in this pile
  - Pieces could be wet
  - Pieces might stick together
- But clearing rules in a market are designed!
- So we must be able to study its physics!

Event Calculus

- Attempt to define market dynamics formally
  - to avoid ambiguity in verbal descriptions
- Markets can be described by states
- Events change the state of the market
- We want to study consequences of events
  - Maintain *consequential closure* if possible
- We want to know exactly what are included in our analysis

An Event Calculus

State + Orders → State

- How should a state be modelled?

State = (Bid_Q, Offer_Q)

Bid_Q = $((P_1, V_1), (P_2, V_2), \ldots, (P_{bq}, V_{bq}))$

- Where $P_1 > P_2 > \ldots P_{bq}$

Offer_Q = $((P_1, V_1), (P_2, V_2), \ldots, (P_{oq}, V_{oq}))$

- Where $P_1 < P_2 < \ldots P_{oq}$

i.e. Order book + New orders → New State

How Orders Are Processed

- We assume that orders are in a queue
- One order is processed at a time

Orders = $(\text{Order}_1, \text{Order}_2, \ldots, \text{Order}_n)$

State + $(\text{Order}_1, \text{Order}_2, \ldots, \text{Order}_n)$ →

$(\text{State} + \text{Order}_1) + (\text{Order}_2, \ldots, \text{Order}_n)$

Order = (Order_Type, Price, Volume)

Order_Type = bid | offer

Clearing of a sell order

- Let
  - Offer_Queue1 = ((P1, V1), (P2, V2), ...)
  - Order1 = (sell, P, V)
  - If P1 < P then ((P1, V1), (P2, V2), ...) + (sell, P, V) → ((P1, V1), (P2, V2), ...) ⊕ (sell, P, V)
  - If P1 ≥ P then ((P1, V1), (P2, V2), ...) + (sell, P, V) → ((P1, V1- (min(V1, V)), (P2, V2), ...) + (sell, P, V-(min(V1, V)))
- Here ⊕ is the queue joining operator which simply puts the orders in ascending order
  - ((P1, 0), (P2, V2), (P3, V3), ...) → ((P2, V2), (P3, V3), ...)

Example snapshot of the market

	Price	Volume
offer 5	1.65	1,000
offer 4	1.64	2,000
offer 3	1.63	1,500
offer 2	1.62	2,000
offer 1	1.61	3,000
bid 1	1.60	2,500
bid 2	1.59	2,000
bid 3	1.58	2,500
bid 4	1.57	1,500
bid 5	1.56	4,000

- What is the consequence of a market order to sell 5,000 units?
- 2,500 sold at 1.60
- 2,000 sold at 1.59
- 500 sold at 1.58
- Price dropped by 1.25%*

* Assume that last transaction price was 1.60

Trader positions matter

	Position	Price	Volume	Margin	Triggered below
Trader 1	long	1.65	4,000	4.00%	1.584
Trader 2	long	1.64	2,000	4.00%	1.574
Trader 3	long	1.64	2,000	5.00%	1.558

- Snapshot of trader positions
- Trader 1 will have to sell should price drop below 1.584, for example
- Different traders have different margins

Effect of Margin Call

Market after clearing sell order of 5,000

	Price	Volume
offer 5	1.65	1,000
offer 4	1.64	2,000
offer 3	1.63	1,500
offer 2	1.62	2,000
offer 1	1.61	3,000
bid 1	1.58	2,000
bid 2	1.57	1,500
bid 3	1.56	4,000

- Price at 1.58
- Trader 1's margin exceeded
- Trader 1 has to sell its 4,000 units now*
- This will push the price down to 1.56
- From 1.60, price dropped by 2.5%

* Assume market mechanism with complete automation

Cascaded Effects

	Price	Volume
offer 5	1.65	1,000
offer 4	1.64	2,000
offer 3	1.63	1,500
offer 2	1.62	2,000
offer 1	1.61	3,000
bid 1	1.56	3,500

- Price at 1.56
- Trader 2's margins exceeded now
- Trader 2 has to sell its 2,000 units (at 1.56)
- Lesson:
  - Price drop depends on trader positions
- What happens if there are no buyers left?

Consequences of margin calls

- A trader is in long position because he/she thinks that price will go up
- Margin calls cause selling, which help to push the price down
  - Exactly the opposite of what he/she expects!
- Cascaded margin calls could lead to substantial falls in price

Remarks

- We are not trying to predict
  - We are just studying the consequences of events under our calculus
- Consequences of an order is non-trivial!
- Given a snapshot of the market, including the trader positions, we can ask:
  - ? How big an order would cause the price to go down (up) by, say, 2.5%?
- Useful for assessing Value-at-risk
- Related work: Liquidity Risk

Future Work

- Real markets are far more complex
- What about market making?
  - How should the market maker review its prices in response to a big purchase?
- We can define new events
  - Directional changes
- Wiki-style repository of programs
  - To enable global collaborative research in finance

Conclusions

- Top-down classical economics challenged
- Use of physical time questionable
- Intrinsic time is more meaningful
- An event calculus helps to identify relevant components and study the physics of markets
  - Cascading effects can be analysed
  - New perspective to Value-at-risk