Asset Liability Management for Individual Households

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“Because we humanoid primates had to struggle with personal finance, we became human”

Joseph Schumpeter
Outline

• Personal finance and individual financial planning
• Asset liability management for individual households
• Dynamic stochastic model and its implementation
• iALM: Decision support tool for financial planning
• US iALM performance testing
• UK iALM example household plans

Problems of Aging and Financial Planning

Life-Cycles
- Retirement
- Earning years
- Dependent years

US
- National debt forecasts (% of GDP)
  - 2012 projection
  - 2012 (March 09)
- Population pyramid
  - Proportion by age group (%)

UK
- National debt forecasts (% of GDP)
  - 2012 projection
  - 2012 (March 09)
- Population pyramid
  - Proportion by age group (%)

Source: IMF

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Pensions and Risks

| State pensions | Governments | Reduced state social security guarantees due to high national debts
|                | DB          | Loss in value of institutional pension funds due to current crash in asset prices and low interest rates
|                | DC          | Low asset returns predicted for the next decade with the possibility of high inflation
|                | SIPP, 401K, individual savings, etc | Loss in value of savings due to low saving rates

|                | Corporate    | Reduced willingness of corporates/governments to accept funding risk of pensions and the move to 3rd pillar pension plans
|                | Corporate and Individual | Managed funds – no systematic data on their performance and risks

Should individuals rely on social security or take control of their future through individual financial planning?

Financial Planning for Individual Households

• Financial planners have traditionally resisted the academic solutions based on theoretical models
  – Asset allocation puzzle of Canner et al [J. Campbell, 2002]

• Common practice is based on the qualitative assessment of risk attitude by financial advisers
  – Rule of thumb: equity fraction of one’s portfolio equals 100 – one’s age
  – “The myth of risk attitudes” Daniel Kahneman [JPM, Fall 2009]
Kahneman’s Summary

• Classical utility theory
  – Risk aversion is measured by the curvature of the utility function for wealth
  – Common practice is to find a portfolio that fits a single number: the investor’s attitude to risk

• Prospect theory, psychology and behavioral economics
  – People are not consistently risk adverse and more sensitive to losses than to gains
  – People are risk seeking in their attraction to long shots and their willingness to gamble when faced with a near-certain loss, and hold separate mental accounts

• To understand an individual’s complex attitudes towards risk we must know both the size of the loss that may destabilize them, as well as the amount they are willing to put in play for a chance to achieve large gains

• Temporary perspectives may be too narrow for the purpose of wealth management
  – Utility theory and its behavioral alternatives are concerned with the moment of decision not with the moment of truth when consequences are experienced

“The theories (utility theory and its behavioral alternatives) assume that individuals correctly anticipate their reaction to possible outcomes and incorporate valid emotional prediction into their investment decisions. In fact, people are poor forecasters of their future emotions and future tastes – they need help in this task – and I believe that one of the responsibilities of financial advisors should be to provide that help.”

Daniel Kahneman
Financial Planning

• “Is Personal Finance an exact science? An immediate flat no. ... It is a domain full of ordinary common sense. Alas, common sense is not the same thing as good sense. Good sense in these esoteric puzzles is hard to come by.”
  
  Paul Samuelson

• Is reconciliation of theory and practice possible?
• In the search for ‘good sense’ we can apply a modelling methodology which comes from Operations Research – decision making in the face of uncertainty
• In financial planning the principal ideas should be brought together from behavioural and classical finance using stochastic optimization theory

Framing the Financial Planning Problem?

“We do not prosper by income or happiness alone”

  Samuel Brittan

“Is wealth the long-term spending that our portfolio can sustain? This definition is close to the truth, but it ignores purchasing power. Is wealth, then, the inflation-indexed real income that our assets could sustain over time? For most investors, this is probably the most useful definition of wealth.”

  Robert Arnott
**Asset Liability Management for individual investors: iALM**

- The *iALM* system is a decision support tool based on the theory of stochastic optimization.

- *iALM* generates life-cycle recommendations for managing wealth and other selected (by user) critical decisions along his/her life span such as level of saving or spending at retirement, borrowing, sending children to private schools, buying real estate, and so on.

- It allows interactive re-solving to obtain long-term financial plans with modified data inputs in order to compare the consequences of the changes in individual preference.

- Principal ideas are brought together from behavioural and classical finance and decision theory.

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**iALM Implementation**

- **Dynamic multi-stage optimization problem with stochastic data:** simulated cashflows (inflows and outflows) of incomes, liabilities, investment returns, etc.

- What-if scenario analysis

- **Implementable decisions** correspond to the root node of the scenario tree

- **Periodic recalibration** of the model parameters to market and personal data – ability to modify inputs periodically or at times of significant changes in life.

- Uses *STOCHASTICS*™ with special attention to graphics and computational speed for interactive use.
Stochastic Programming Techniques for iALM

I. Simulation
Generation of stochastic data with a discrete number of annual observations of a continuous time vector data process branching at specified times (decision times) in the future.

Scenario tree is a schema for forward simulation – along each branch a multiple number of stochastic processes are simulated. Some are independent, other may be correlated.

Simulation discrete time steps correspond to the data sampling frequency of the process of interest.

iALM involves simulation of asset returns and liabilities punctuated by life events.

II. Optimization
Discrete time and state optimization giving a different optimization problem (given by its objective and constraints) at each node of the scenario tree dependent on both its predecessors and successors.

Major decision time points are stages of the tree.

Implementable decisions are at the root node which are the most constrained decisions robust against all alternative scenarios generated while the remainder allow what-if prospective analysis.

iALM solves a large scale linear optimization problem.

Consumption (goal) maximization at each decision time subject to constraints such as risk, budget, cash flow balance and so on annually.
Sustainable wealth maximization across all years and generated scenarios simultaneously.

Overview of individual ALM

- Gather Individual and Market Data
- Econometric and Actuarial Modelling
- Scenario Tree Simulation
- Optimization Model: Tailored Portfolios, Goals, Spending, Cashflow balances, etc

- Dynamic optimization model for assets-liabilities
  - Objective
  - Various Constraints
- Visualization of decisions

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Modelling Changing US Markets

Investment securities
- Domestic and International Equities
- Government Bonds
- Corporate Bonds
- Alternatives
- T-bills and all bond coupons
- Treasury Inflation Protected Securities (TIPS)
- Cash
- CPI
- Other fixed assets

Fundamental financial models
- Multi-dimensional GBM process
  \[ d \ln X_{i,t} = \mu_i dt + \sigma_i dW_{i,t} \]
- Geometric Ornstein Uhlenbeck (OU) process
  \[ d \ln r_i = (\alpha - \beta \ln r) dt + \sigma dW_i \]
- OU process
  \[ dr_i = (\alpha - \beta r_i) dt + \sigma dW_i \]

Annual Returns of the S&P 500 Index
Modelling Events

• Random events
  – death (D) with probability of dying at age t
  • Simulation of length of life scenarios
  – long-term care (LTC): a single event drawn from an historical frequency distribution in an interval beginning at age 65 and ending at the realization of the last of two independent deaths at T
  – Terminal healthcare is currently incurred for exactly two years prior to death by all persons with the out-of-pocket costs paid by the terminally ill of age 63 and older having a rate of increase above inflation
• Maximum horizon T equals 115 years minus the starting age of the youngest head of household

Length of Life

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Modelling Life Style

- Construction of a problem suitable for a general household from different age and wealth groups which must reflect individual circumstances
  - Planning horizon for each problem depends on the age of individuals
  - Major impacts of uncertain events: Long Term Care and Death
  - Medical expenses depend on the state of health and insurance
- Forecasting of earned income
- Client’s defined specific goals and spending on these within a range of desirable, acceptable and minimum levels

Framing of the Problem

- **Broad Framing:** overall objective is to provide ‘sustainable spending’ over a household’s lifetime in terms of desired multiple life goals specified by preferences on goal choice and their priorities

- **Narrow Framing:** maximization of goal consumption
  - each single goal utility function is defined with respect to reference points chosen by household specifying its individual consumption preferences
  - example of a goal with high preference – private education of a child
The Value Function

- Recall the value function of prospect theory

\[ v(x) = \begin{cases} 
  x^2 & \text{if } x > 0 \\
  -\lambda(-x^2) & \text{if } x < 0 
\end{cases} \]

(with a typical \( \alpha = 0.88 \) and \( \lambda = 2.25 \))

Reference point

Individual Goal Utility

- Individual goal utility function is given by three reference points
- For each single goal the level of spending \( y \) is in the range between acceptable (s) and desirable (g) subject to existing and foreseen liabilities, i.e. minimum (h) spending. These values specify the shape of the utility function for each goal
- Objective to maximize goal spending with piecewise linear utility functions for goal spending with priorities
Overall Objective

• The objective is to maximize the expected present value (over all scenarios) of life time consumption, i.e. spending on all selected goals

\[
E \left[ \sum_{t=1}^{\infty} u_{t} \right]
\]

where \( u_{t} = \sum_{g \in G} u'_{g,t} = \frac{1}{\varphi_{t}} \left( \pi^{g} z_{t}^{g} + \pi^{g} l_{t}^{g} \right) \)

Here \( z_{t}^{g} \) is excess borrowing, \( l_{t}^{g} \) is total tax payment and \( \varphi_{t} \) is the inflation index at \( t \)

• Consumption refers to all “elective” spending on chosen goals

Key Modelling Features

• Portfolio return and risk are driven by desirable consumption subject to existing and future liabilities

• Risk management of portfolio by
  – Constraining the portfolio drawdown in each scenario
  – Constraining the proportion of assets in the portfolio

• Length of each individual scenario represents a possible duration of life, i.e. we solve a problem with a random time horizon
Wealth Generation Through Optimum Resource Allocation

- iALM objectives are achieved through optimum resource allocation over a network of cashflows
  - cash flows of liabilities
  - cash flows of different incomes and portfolio returns
  - income from portfolio returns provides optimal consumption

Portfolio Allocation Sub-problem

- Fundamental constraints of portfolio allocation sub-problem
  - Initial holding
  - Portfolio cash flow
  - Asset inventory balance
  - Investment limits, position limits
  - Portfolio drawdown
  - etc

- Optimal allocation between different types of account
  - taxable and savings portfolios such as 401K (USA) or SIPP and ISA (UK)
Challenges Overcome in the iALM Solution

- Up to 90 annual decision periods using 4 major portfolio rebalancing (tree branching) points using novel information constraints on most decisions in between these points
- Automatic placement of major rebalancing points based on problem instance data
- Random scenario lengths due to deaths of household heads
- Occurrence of non-terminal random events such as entry and exit from long term care
- Indexing of future incomes and expenditures at appropriate rates relative to inflation
- Second order moment matching in market return scenario tree
- No solver tuning for first time solution of arbitrary client instances
**iALM Financial Plan**

- iALM provides optimum values for many decision variables – spending, amount of savings, tax-efficient allocation between multiple portfolios, etc – across time simultaneously for multiple scenarios of random processes representing market returns, foreseen liabilities and life events.
- Current iALM model includes 20 random processes that vary over the client’s lifetime and around 200 mathematically formulated conditions (constraints) per node of the scenario tree.
- Average desktop computer solving times are 1-10 minutes (Problem size over 3mln non-zero entries).

➢ An interactive process for analysing retirement and saving alternatives

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**Performance of iALM**

- **Testing** on real profiles of UK and US investors and comparison with recommendations of financial advisors.
- **Comparison** with MVO based methodology.
- **Behavioural aspects tested** using ability to analyse relationship between current wealth, earnings, savings and desirable consumption.
Comparison with MVO

Efficient Frontier

- Portfolio Return
- Portfolio Volatility

Initial allocation

Historical Backtest

1995-1999
- Asset Return / Volatility

2000-2002
- Asset Return / Volatility

2003-2004
- Asset Return / Volatility

MA-Profile
Technical Summary

• Average desktop computer solving times are **1-10 minutes**
  – **Pimlott profile: 102sec** (Dell i5)

• **iALM** provides **optimum values for multiple decision variables**
  – Recommended allocation for current year is robust with respect to the most unfavourable scenarios

• Probabilities of goals and shape of the corresponding distributions are a good **indication of uncertainty** inherited in the plan

• Many **other aspects of financial plans** are available, e.g. cash flow statements, graphs of individual cash flows for liabilities, goal spending, taxes, borrowing through life, and so on

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UK Household Data

• FT weekly ‘Money’ supplement 2005-2007

• Family member describes the household’s financial position and goals and asks expert financial advisers for recommendations on investment, savings and appropriate spending

• Quantity and quality of data provided by household may vary significantly

• Adviser’s opinions may differ significantly

• Example – **Pimlott household profile**
Model Illustration: Pimlott Household

<table>
<thead>
<tr>
<th>New</th>
<th>Expected retirement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Standing $69,401</td>
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<table>
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<tr>
<th>Client</th>
<th>Co-Client</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Assets</td>
<td>$697,100</td>
</tr>
</tbody>
</table>

- Spending $98,400
- Education - Pre-University
- Education - University

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Visual Summary of Profile

Getting an Overview

Cash Flows

Goals

Portfolio

Wealth

Getting Related Graphs

Clickable Chart

Simulation Years

Actual Values
Linking Strategic and Tactical Decisions

• **Strategic** allocation in market indices of iALM takes long-term view of individual circumstances
  – Implements dynamic allocation

• **Tactical** allocation exploits financial advisors’ knowledge at the level of individual fund characteristics
  – Adding alpha without increasing beta

• **Both levels** must consider legal and institutional framework
  – Taxation
  – Pension regulations

Helping households become involved in managing their investments

**Strategic (iALM)**
- Constructs the optimum consumption and investment policy for life-long investment
- Defines risk attitude by life-style goals
- Helps clients identify affordable goals and manage their liabilities
- Allows investigation of the benefits of insurance products relative to identified risks
- Generates client profiles useful for new product design

**Tactical (MVO)**
- Chooses efficient frontier point consistent with risk and returns of strategic portfolio recommendation
- Selects quality instruments in the market by strategic asset class
- Allows benchmarking of client portfolio performance versus indices
- Allows optimization of post tax return by separating instrument portfolio into taxable and non-taxable components consistent with strategic asset classes
Benefits Offered by iALM

• **Comprehensive, long-term solution to wealth management tailored to individual needs**
  - Free format of specification of life goals and their values
  - Construction of the utility function based on distinct client needs
  - Hedging against longevity risks by solving random horizon optimization problem
  - Combination of life insurance with retirement saving plan
  - Consideration of different options for borrowing
  - Optimum use of tax-shielded accounts

• **Interactive process** for analysing investment and savings alternatives for long term financial planning

• **New paradigm in wealth management**

References

• Dempster *et al.* (2009). Risk profiling defined benefit pension schemes. *Journal of Portfolio Management* **35.4** 76-93