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Profiling High Frequency Equity Price Movements in Directional Changes

Edward P K Tsang*, Ran Tao*, Antoaneta Serguieva** and Shuai Ma*

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*Centre for Computational Finance and Economic Agents (CCFEA), University of Essex
**Financial Computing and Analytics Group, University College London
**Bank of England

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Abstract,
Market prices are traditionally sampled in fixed time-intervals. Directional change (DC) is an alternative way to record price movements. Instead of sampling at fixed intervals, DC is data-driven: price changes dictate when one records a price. This new approach requires developing new approaches to extract information from data, particularly from data that record DCs. In this paper we introduce a set of indicators capturing such information, and demonstrate that the indicators help construct DC profiles of markets. We further demonstrate how such profiles capture and distinguish characteristics of equity.

1 Introduction

Market dynamics are traditionally captured by a time series. The observer decides how often the data is sampled. High frequency data arrive in regularly times. To summarize them in time series, some would record tick-by-tick data in minutely summaries. Guillaume et al. (1997) introduced the concept of "directional changes" as an alternative way to sample data. In this approach, sample points are data-driven: that means the observer lets the data determine when to have a sample of the market. The observer decides a threshold which he/she consider significant in price changes: for example, a price change of 5% may be considered significant. The market is seen to be in alternating uptrends and downtrends. It is considered to have changed from a downtrend to an uptrend if the price data points are sampled when the market changes direction by the predefined threshold.

In further research on DC, Glattfelder et al. (2011) presented twelve new empirical scaling laws related to foreign exchange data series across thirteen currency exchange rates and based on the directional change theory. Kablan and Ng (2011) developed a new method of capturing volatility using the directional-change event approach. Aloud et al (2012) pointed out that the length of the price-curve coastline defined by directional change events shows a longer coastline of price changes than if based on time series. Bisig et al (2012) proposed useful measures of price movements under DC. Qi (2012) studied value-at-risk under the DC framework. Masry et al (2013) studied trading patterns in FX markets using DC.

In time series analysis, researchers have developed useful indicators, such as return and volatility, to summarize market price changes. However DC is a relatively new concept. In this paper we propose potentially useful indicators for profiling markets under DC. We also assess these profiles for summarizing real market data. We believe summarizing markets under DC will contribute different information about the market, compared to time series analysis.

The remainder of the paper is organised as follows. Section 2 describes the concept of directional change and its component events. Section 3 explains how markets can be summarized into trends under the DC framework. Section 4 introduces useful indicators for extracting information from data under the DC framework. Section 5 offers an example of profiling high frequency equity price movements under DC; it explains how DC-based profiles could help us understand the markets. The paper is concluded in Section 6.

2 Directional changes

2.1 Directional Change (DC) events

Directional change (DC) is an alternative way to summarize price changes (Guillaume et al 1997). The basic idea is to divide the market into alternating *Uptrends* and *Downtrends*. An uptrend terminates when a *Downturn DC Event* takes place. Similarly, a downtrend terminates an *Upturn DC Event* takes place. A Downturn (Upturn) DC Event is an event at which price drops (rises) by a *threshold* from its highest (lowest) price in the previous trend. Here the threshold is a percentage that the observer considers significant. One observer may consider 0.05% a significant change, while another observer may consider 5% is significant. Observers who use different thresholds will observe different DC events and trends.

As a Downturn DC Event defines the beginning of a new downtrend, at the end of the Downturn DC Event, price would have dropped by the specified threshold from the highest price in the last (as well as current) trend. That highest point (or the lowest point in the case of upturn directional change) is called an *Extreme Point*. It must be emphasized that the extreme point was only confirmed to be the extreme point in hindsight, when DC is confirmed (i.e. when price has changed by the threshold or more from the extreme point).

A downtrend continues until the next upturn DC event is observed, which defines the lowest price in the current downtrend and start the next uptrend. We refer to the price change from the end of the Downturn DC Event to the lowest price in the current trend an *Overshoot Event*. In other words, each trend comprises a DC Event and an Overshoot Event, as shown in Figure 1. A formal definition of DC Events and Overshoot Events can be found in (Tsang 2010).

During a downtrend, the lowest price within the current trend, P_l, is continuously updated to the minimum of P_t (the current market price) and P_l (the last low price). Similarly, during an uptrend, the highest price within the trend, P_h, is continuously updated to the maximum of P_t (the current market price) and P_h (the Last High price) (Tsang 2010). At the beginning of the sequence, when we do not know whether we are in an uptrend or downtrend, the last high price P_h and last low price P_l are set to the initial market price at the beginning of the summarized period (Tsang 2010).

2.2 A more formal definition

In this subsection, we shall provide a formal definition of the above. A downturn DC event is an event when the absolute price change between the current market price P_t and the last high price P_h is lower than a fixed threshold (a percentage) θ:

$$P_t \leq P_h \times (1- \theta) \tag{1}$$

In a downward run, an upturn DC event is an event when the absolute price change between the current market price P_t and the last low price P_l is higher than a fixed threshold θ:

$$P_t \geq P_l \times (1+ \theta) \tag{2}$$

The starting point of an upturn DC event is an upturn point where the price last troughed at P_l. The end of an upturn DC event is an upturn confirmation DC point where the price has risen from the last upturn point by the threshold θ.

A downturn DC event is followed by a downward overshoot event that is ended by the next upturn DC event, which is itself followed by an upward overshoot event that is ended by the next DC downturn event (Tsang 2010) (see Figure 1). The overshoot event (OS) represents the time interval of price movement beyond the DC event.

Under the DC framework, price movement is summarized in a four-event cycle:

… → Downturn DC Event→

Downward Overshoot Event →

Upturn DC Event →

Upward Overshoot Event →

Downturn DC Event→ …

A *total price movement* (TM) is constituted by a downturn event and a subsequent downward overshoot event, or an upturn event and a subsequent upward overshoot event (Glattfelder et al, 2011), as illustrated in Figure 1.

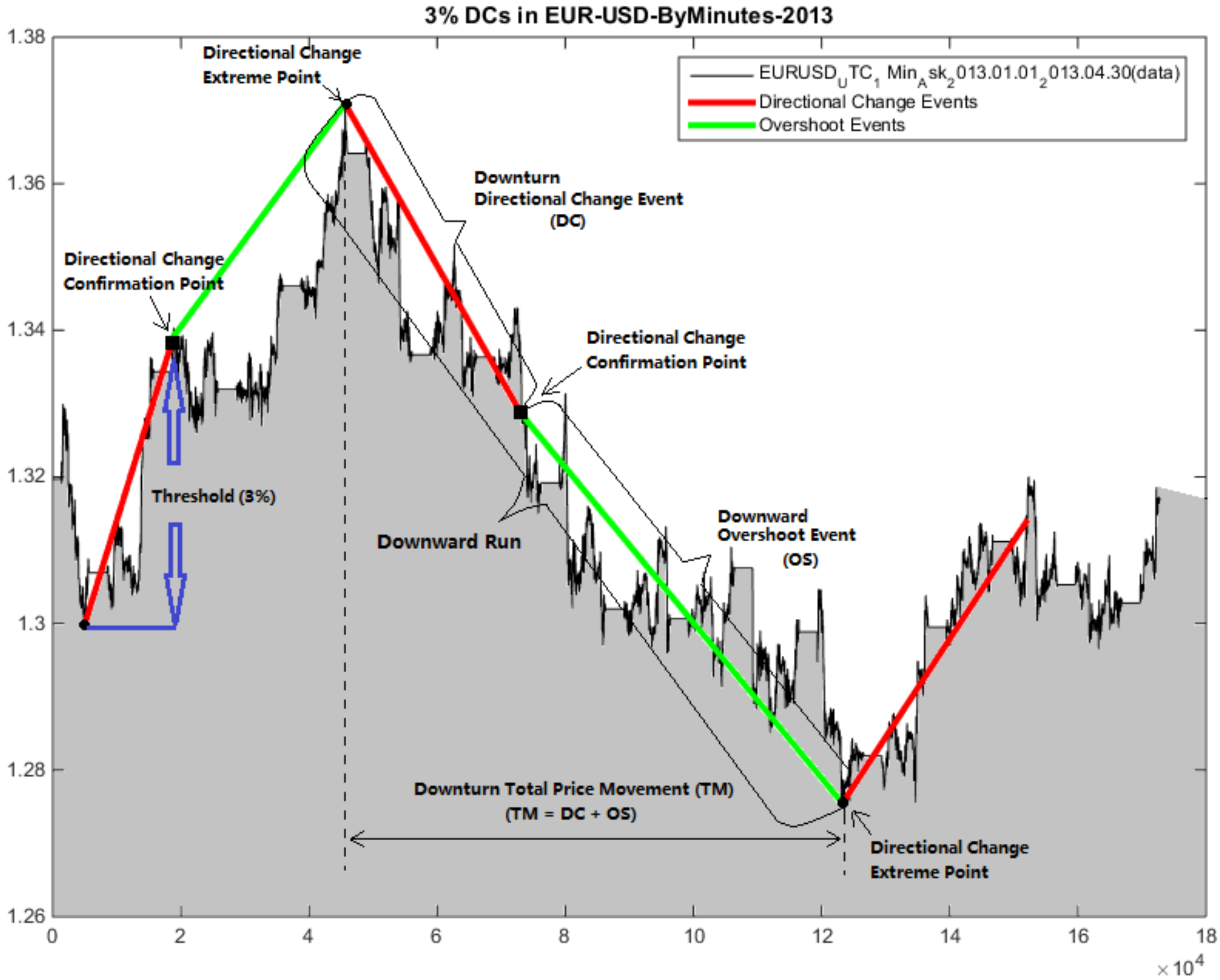

Figure 1: Directional Changes in EUR/USD (threshold = 3%) (While the emphasis of this paper is on high frequency data, DC can be applied to time series, as shown here.)

3 Summarizing time series under DCs

In this section, we propose a procedure for summarizing price movements through DC. The first step for summarizing time series applying the DC theory is to locate the significant points of each event: Directional Change Extreme Point (EXT), Directional Change Confirmation Point (DCC) and Theoretical Directional Change Confirmation Point (DCC*).

Directional Change Extreme Point (EXT) is the starting point that is an Upturn Point or a Downturn Point. It can be also seen as the end of one TM event (Figure 2). Directional Change Confirmation Point (DCC) is the point at which to confirm one DC event. For an Upturn Event, it is the first point that rises past $P_{EXT} \times (1+ \theta)$. And for a Downturn Event, it is the first point that drops past $P_{EXT} \times (1- \theta)$ (Figure 2).

The Theoretical Directional Change Confirmation Point (DCC*) is the minimal or maximum directional change confirmation price for an upturn or downturn directional change event. It may not exist in the real market. We use DCC* rather than DCC as the EXT point and DCC point can be the same point, which could distort the summary. The price of DCC* is defined as follows:

In an uptrend: $P_{DCC\uparrow*} = P_{EXT} \times (1+\theta) \leq P_{DCC\uparrow}$;

In a downtrend: $P_{DCC\downarrow*} = P_{EXT} \times (1+\theta) \geq P_{DCC\downarrow}$,

Here P_{EXT} is the price at the directional change extreme point (EXT). P_{DCC} is the price at the directional change confirmation point (DCC), θ is the fixed threshold. ↑and ↓ here represents Upturn and Downturn event. Therefore $P_{DCC\uparrow*}$ is the DCC* price of an upturn directional change event and $P_{DCC\downarrow*}$ is the DCC* price of a downturn directional change event.

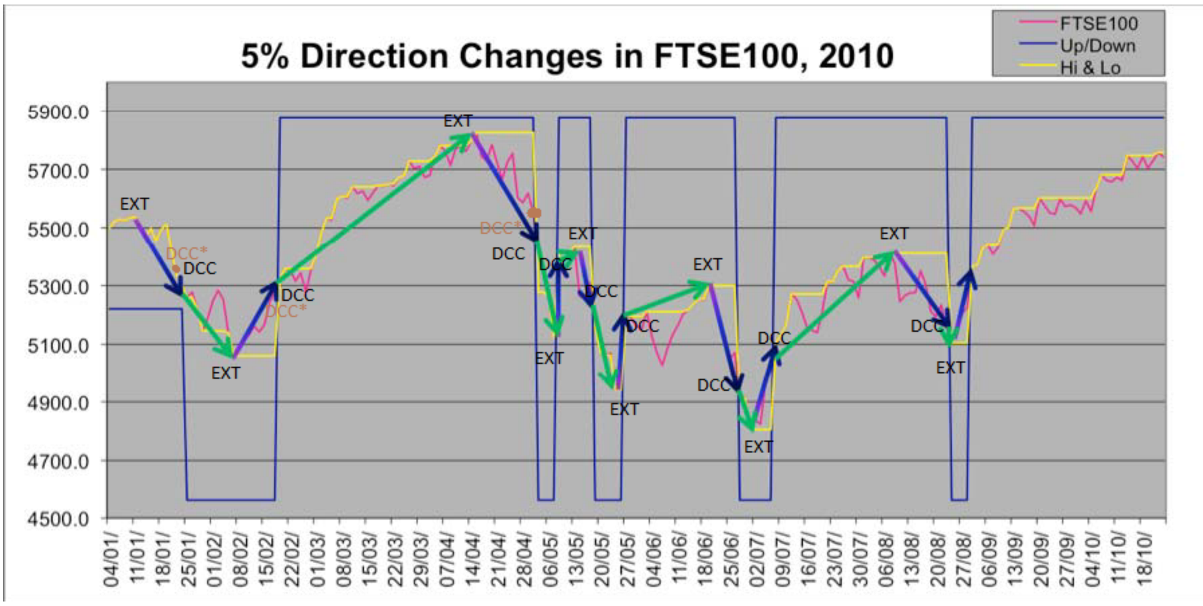

Figure 2: EXT, DCC and DCC* in DC summarizing the FTSE100 in daily closing prices (Source: Tsang 2010)

Having located the DCC, DCC* and EXT points, the second step is to define indicators for directional change profiling. Some indicators define the market trend, others measure the directional change volatility or the risk. The indicators are described in Section 4.

We implement a programme (in MatLab) to generate the indicators. The programme produces two files: a DC-Data file and a Profile Summary File. The DC-Data file includes the details of every DC point and its indicators. The Profile Summary File is generated from the DC-Data file. It attempts to provide information for studying the price movements in the whole period.

4 Useful indicators in directional change

DC is a new way of summarizing price changes. In order to analyse price dynamics, we need to extract useful information from DC summaries. In this section, we propose indicators which could be useful for extracting information. With these indicators, we aim to construct profiles for price changes summarized under the DC framework.

4.1 Number of directional change events (NDC)

NDC measures the frequency of DC events over the profiling period, which is the time period summarizing the price movements under DC. By recording the NDC within the profiling period, the DC method provides unique information about market price movements.

4.2 Overshoot Values at Extreme Points (OSVEXT)

The current magnitude of an overshoot is the price change from the last directional change confirmation price (DCC) to the current price. We define Overshoot Value (OSV) for measuring the current magnitude of an overshoot. Instead of using the absolute value of the price change, we introduce this measure as relative to the threshold, θ. Therefore:

$$OSV = ((P_c - P_{DCC*}) \div P_{DCC*}) \div \theta \quad (3)$$

Here P_c is the current price, P_{DCC*} is the last theoretical directional change confirmation price, θ is the threshold. At DC confirmation, P_c is approximately P_{DCC*} and OSV is approximately 0.

Overshoot values at extreme points (OSV_{EXT}) is an indicator for measuring the magnitude of an overshoot based on the price distance between fixed points. It measures how far the overshoot goes from the theoretical directional change confirmation point (DCC*) to the next extreme point (EXT). We define OSV_{EXT} as follows:

$$OSV_{EXT} = ((P_{EXT} - P_{DCC*}) \div P_{DCC*}) \div \theta \quad (4)$$

Here P_{EXT} is the price at the extreme point that ends the current trend, P_{DCC*} is the price of the theoretical directional change confirmation point of the current trend, and θ is the threshold.

4.3 Time for completion of a trend (TT)

The time that a trend takes to complete contributes important information. We create an indicator TT for measuring the time from the beginning Extreme Point to the ending Extreme Point of a trend (Figure 3).

4.4 Directional Change volatility (VT)

Directional Change Volatility (VT) is defined to describe a TM event's volatility over the defined trends. Unlike volatility in time series analysis, here it represents the number of directional change events that happened in every time unit in the defined trend. VT is therefore a way of looking at volatility in interval-based summaries from a different angle. DC Volatility is defined as follows:

$$\text{VT}\ (\theta, \text{n}) = \frac{n}{T(\theta,n)} \tag{5}$$

where T (θ, n) represents the physical time to complete the last *n* TM events. Throughout the paper, *n* equals 1 and θ is the fixed threshold. Thus VT (θ, n) = 1 ÷ TT. We simplify the notation by using VT to refer to the DC volatility of the last DC trend.

A DC summary is an event-based summary focused on significant price changes. The VT indicator brings physical time into the DC framework. In the standard time series, volatility is measured by price changes in fixed time intervals. In the DC framework, one way to measure volatility is by measuring the physical time that it takes to complete a trend. Thus is VT is defined to capture and contribute additional information to market analysis. VT has the designed property that the higher the VT value, the more volatile the market is.

It may help to relate the volatility measures in time series and DC-based summaries as follows. In time series, volatility is calculated by returns, which are measured by changes in the price (the y-axis) over a fixed amount of time (x-axis). In DC, the threshold determines the minimum amount of change in price (y-axis). The variance that we measure is time (the x-axis). Therefore, volatility in time series and VT in DC are two complementary angles of looking at volatility, with the former measuring the changes in the y-axis and the latter in the x-axis.

4.5 Total Price Movements Value at Extreme Points (TMV_{EXT}) and Price-Curve Coastline (C)

A total price movement value (TMV) is an indicator for estimating the size of total price movement based on the price distance between current price and the last directional change extreme prices. It measures the length of price-curve coastline over the defined time period. Throughout the paper, TMV is defined by:

$$TMV = ((P_c - P_{EXT}) \div P_{EXT}) \div \theta \quad (6)$$

Here θ is a fixed threshold, P_c is the current price, P_{EXT} is the price of the last directional change extreme point.

Total price movements value at extreme points (TMV_{EXT}) measures the price distance between two adjacent directional change extreme points. It measures the potential maximum profit for each TM event. TMV_{EXT} is defined by:

$$TMV_{EXT} = ((P_{EXT\_i+1} - P_{EXT\_i}) \div P_{EXT\_i}) \div \theta \quad (7)$$

Here P_{EXT_i} represents the price at the i-th directional change extreme point, P_{EXT_i+1} represents the price at the (i+1)-th directional change extreme point, and θ is the fixed threshold.

Since TMV_{EXT} represents the potential maximum profit of each TM event, we define the length of the price-curve coastline under DC (LenC) as the sum of all TMV_{EXT} over the profiling period:

$$LenC = \sum_{i=1}^{N(\theta)} TMV_{EXT_i}$$

(8)

Here θ is the threshold (in %), N (θ) is the total number of DC events over the profiling period under θ and TMV_{EXT_i} is the Total Price Movements Value at each directional change extreme point.

LenC is defined and calculated in dynamic time intervals confirmed by price changes. The calculation only pays attention to price changes; time is ignored. It sheds different light from the price-curve coastline in time series analysis, which is calculated within a fixed time interval (e.g. the sum of each ten days price change).

4.6 Speed of TM events (Sigma)

Speed of TM events (Sigma) is the speed of an upturn or downturn event, i.e. the ratio between each TM event and time interval (TT) (see Figure 3). Sigma therefore measures the angle of price movements and the efficiency for making a profit in each TM event. A high Sigma means the profit can be earned in a short time period. Since TMV_{EXT} measures the number of thresholds in up/downtrend. We define Sigma as:

$$\text{Sigma} = \frac{TMV_{EXT} \times \theta}{TT} \tag{9}$$

Here TMV_{EXT} is total price movement value at extreme points and TT is the time interval between each EXT, θ is a fixed threshold. Here Sigma measures the percentage of price rising/dropping per time unit.

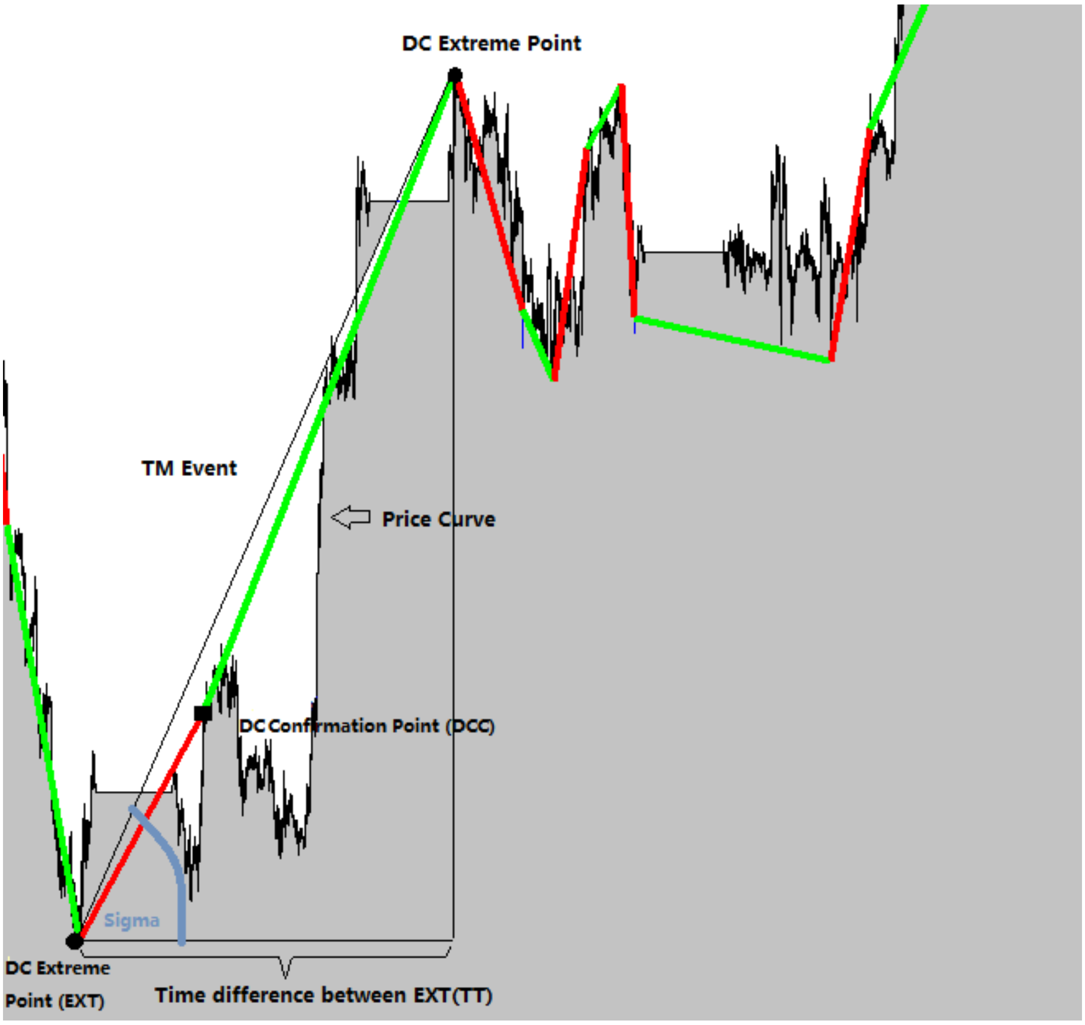

Figure 3: Illustration of TT & Sigma

Time Series Analysis Indicators		DC Events Analysis Indicators	
		NDC	The number of DC within the profiling period.
Risk, volatility(standard deviation, variance)	Variance measures variation of price of a financial instrument over time.	**VT**	VT describes DC event volatility and measures the number of DC events in given time periods.
Net Return	Relative changes of price over a given period.	**OSV_{EXT}**	Overshoot value at extreme points.
Time interval	Fixed time interval, which is of the same size.	**TT**	Time intervals are data-driven, and therefore are uneven and changeable.
		Sigma	The speed of an up/downtrend.
Inflection Point	A point in price movements at which a change in the direction of curvature is observed.	**Directional Change Extreme Point (EXT)**	The starting point of an Upturn Point or Downturn Point.
Price Curve Coastline	Defined and calculated in a fixed time interval (e. g. the sum of each ten days' price change).	**Price Curve Coastline under DC (LenC)**	Defined and calculated in dynamic time intervals that are decided by price changes.

Table 1: Contrast between time series indicators and DC-based indicators

Compared with traditional market data analysis indicators based on time series, our new indicators for directional change series provide a different angle to observe and profile the market.

5 PROFILING HIGH FREQUENCY PRICE MOVEMENTS IN EQUITY MARKETS

In this section, we shall explain how we could profile three actively traded stocks in the London Stock Exchange, namely BT, HSBC and RDS, using DC. A program called TR1 was developed for producing DC profiles. The specification of this program and its output can be found in (Tsang et al 2015). Full output of the program for the analysis below will be published online[1].

[1] All output reported in this paper will be accessible from http://www.bracil.net/finance/DirectionalChanges/

We profiled two periods, September 2014 and February 2015, for each of the three equities BT, HSBC and RDS. We fed the program with tick by tick transaction prices. Figures 4 and 5 show the directional changes found in BT in September 2014 and February 2015, respectively. Key results are shown in Table 2. Changes from September 2014 to February 2015 are shown in Table 3.

TL records the number of trades in the six profiled periods, PC records the price changes over the profiled periods. Results in Table 2 show that there were more transactions in HSBC than BT than RDS. Results in Table 3 show that there are more trades in February 2015 than in September 2014 in all the three equities. The increases shown in Table 3 for BT, HSBC and RDS are 78%, 12% and 71%, respectively.

NDC records the number times that price has changed by 0.6% or more from "extreme points" under our definition. Results suggest that direction has changed more times in February 2015: 75%, 65% and 170% for BT, HSBC and RDS, respectively.

NDC could give some idea about the volatility of the market, but with DC profiling, one could look at volatility from another angle: TT measures the amount of time that it takes to complete a trend. One could also consider the market to be more volatile if on average a trend takes shorter time to complete. Therefore, we could also use VT (the inverse of TT) to measure volatility to measure the volatile the market of the market. Results in Table 3 show that TT in BT, HSBC and RDS dropped from September 2014 to February 2015, by 48%, 49% and 67%, respectively. From Table 2, one can work out that the corresponding VT rose by 91%, 97% and 203% respectively.

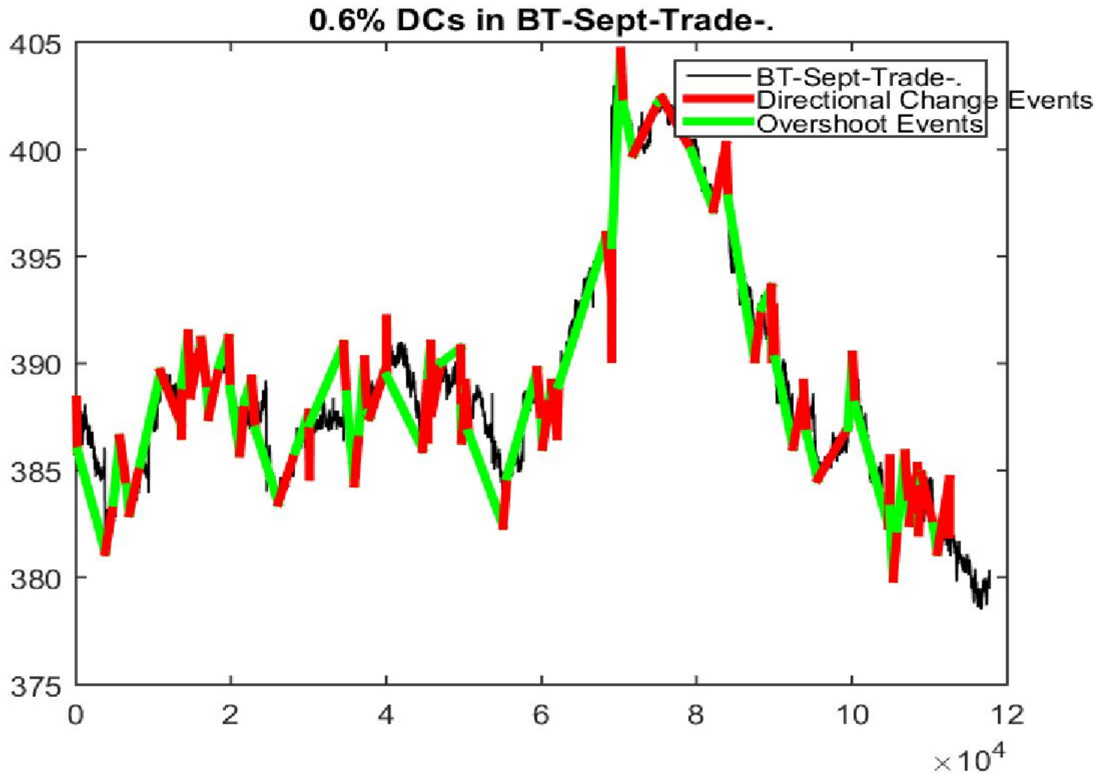

Figure 4: DC and Overshoot Events found in tick-by-tick data in BT, September 2014

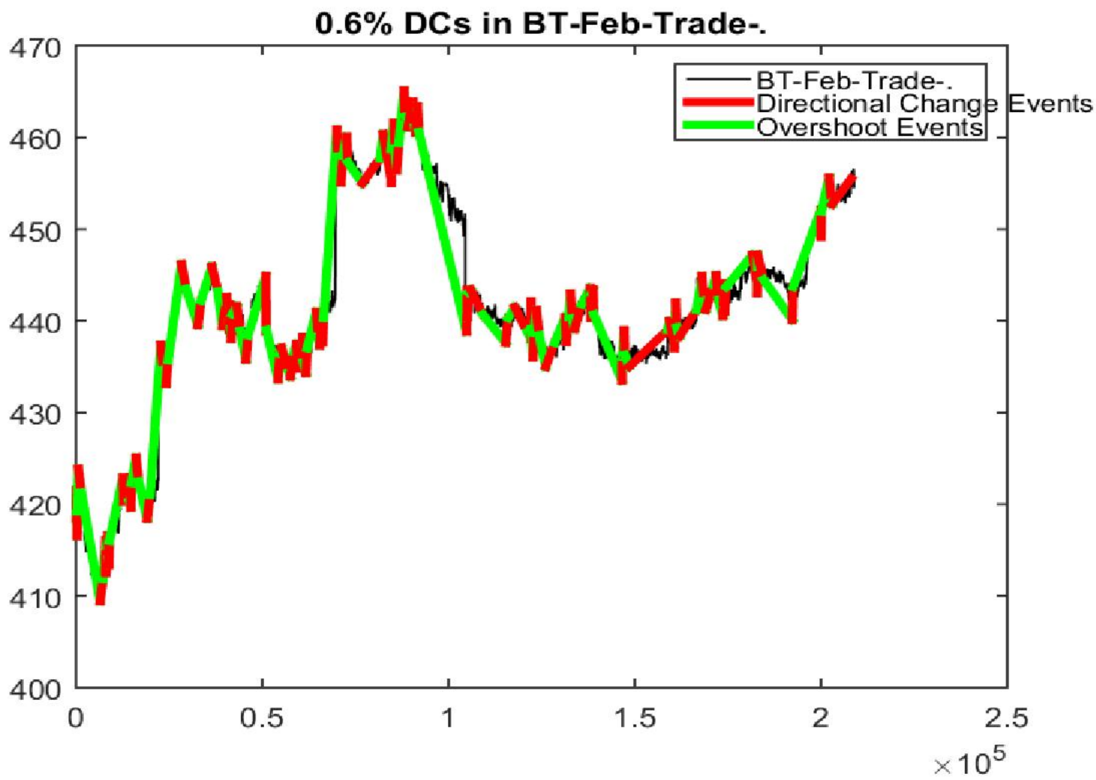

Figure 5: DC and Overshoot Events found in tick-by-tick data in BT, February 2015

	BT September 2014	BT February 2015	HSBC September 2014	HSBC February 2015	RDS September 2014	RDS February 2015
Trades (TL)	117694	208920	228248	256000	110551	188581
NDC	59	103	55	91	46	124
PC	99.04%	108.18%	96.62%	95.12%	96.73%	104.47%
Mean OSV_{EXT}	0.9918	1.0096	0.8371	0.9172	0.9330	0.9283
Mean TT (sec)	12054	6317	14045	7140	15783	5204
Mean Sigma	2.11×10^{-9}	1.07×10^{-7}	2.97×10^{-9}	4.16×10^{-8}	5.68×10^{-8}	9.42×10^{-9}
Total LenC	115.52	205.06	99.17	172.51	86.96	237.22
Mean LenC	1.9917	2.0104	1.8365	1.9167	1.9325	1.9286

Table 2: DC Profiles with a threshold of 0.6% on BT, HSBC and RDS with tick-by-tick transaction prices, September 2014 and February 2015

	BT	HSBC	RDS
Trades (TL)	78%	12%	71%
NDC	75%	65%	170%
PC	9.23%	-1.55%	8%
Mean OSV_{EXT}	1.79%	9.57%	-0.51%
Mean TT	-48%	-49%	-67%
Mean Sigma	4959%	1301%	-83%
Total LenC	77.52%	73.94%	172.79%
Mean LenC	0.94%	4.37%	-0.20%

Table 3: Changes from September 2014 and February 2015 based on DC Profiles with a threshold of 0.6% on BT, HSBC and RDS with tick-by-tick transaction prices

OSV_{EXT} is another aspect of volatility: while NDC and TT measure the frequency of directional changes, OSV_{EXT} measures the magnitude of the overshoot. With large scale experimentation, (Glattfelder et al 2011) show that overshoot approximates to the threshold in foreign exchange data. That means the value of OSV_{EXT} is approximately 1. Table 2 shows that all OSV_{EXT} values are around 1, with the exception of HSBC in September 2014, which is equal to 0.8371. OSV_{EXT} in Table 3 reveals reveal different changes in the three companies from September 2014 to February 2015. Combined with NDC, we can see increased frequent directional changes (+65%) as well as increased magnitude in overshoot (+9.57%) in BT. On the contrary, in RDS we observed substantially increased frequency in directional changes (+170%) but reduction in the magnitude of overshoot (-0.51%). BT was somewhere in between, with +75% increase in NDC and +1.79% increase in magnitude of overshoot.

Similarly, most of the MeanLenC figures are close to 2, with the exception of HSBC September 2014, which agrees with the findings in the foreign exchange market observed by (Glattfelder et al 2011).

It is important to note that Mean OSV_{EXT} and MeanLenC do not tell the full story. The value of OSV_{EXT} and LenC in each trend could vary significantly. The distribution of these values could provide useful information to researchers. Given the complication, discussion on such distributions will be left to another occasion.

Table 3 shows that Total LenC increased substantially from September 2014 to February 2015 in all the equities studied. This suggests that there is potentially more profit to be made in February 2015.

Glattfelder et al (2011) discovered patterns that persist over different thresholds in all currency pairs. If their observation applies to the equities that we observed, we would expect OSV_{EXT} and Mean LenC to be constant under different thresholds. On the other hand, as we use larger thresholds, NDC should drop; hence TT should increase. Table 4 summarizes our DC observations under different thresholds on BT prices in February 2015. DC and overshoot events found under different thresholds are shown in Figure 6, it shows how certain points are ignored as threshold is increased.

	0.2%	**0.4%**	**0.6%**	**0.8%**
NDC	940	264	103	54
Mean OSV_{EXT}	1.0414	0.9131	1.0096	1.0416
Mean TT (sec)	709	2532	6317	11373
Total LenC	1916.94	503.22	205.06	108.26
Mean LenC	2.0415	1.9134	2.0104	2.0426

Table 4: Selected DC indicators with varying threshold on BT February 2015 tick-by-tick transaction prices – we expect Mean OSV_{EXT} and MeanLenC values to remain similar while NDC, Total LenC and TT to vary

Table 4 shows that, as expected, NDC increases as threshold increases. Naturally, as fewer DCs were observed, Mean TT grows with threshold. A longer coastline (Total LenC) is observed when a smaller threshold is used, which is consistent with Mandelbrot (2004) suggested. Mean OSV_{EXT} and Mean LenC values remain relatively stable, around 1 and 2, respectively. Therefore, our observation supports those found by Glattfelder et al (2011).

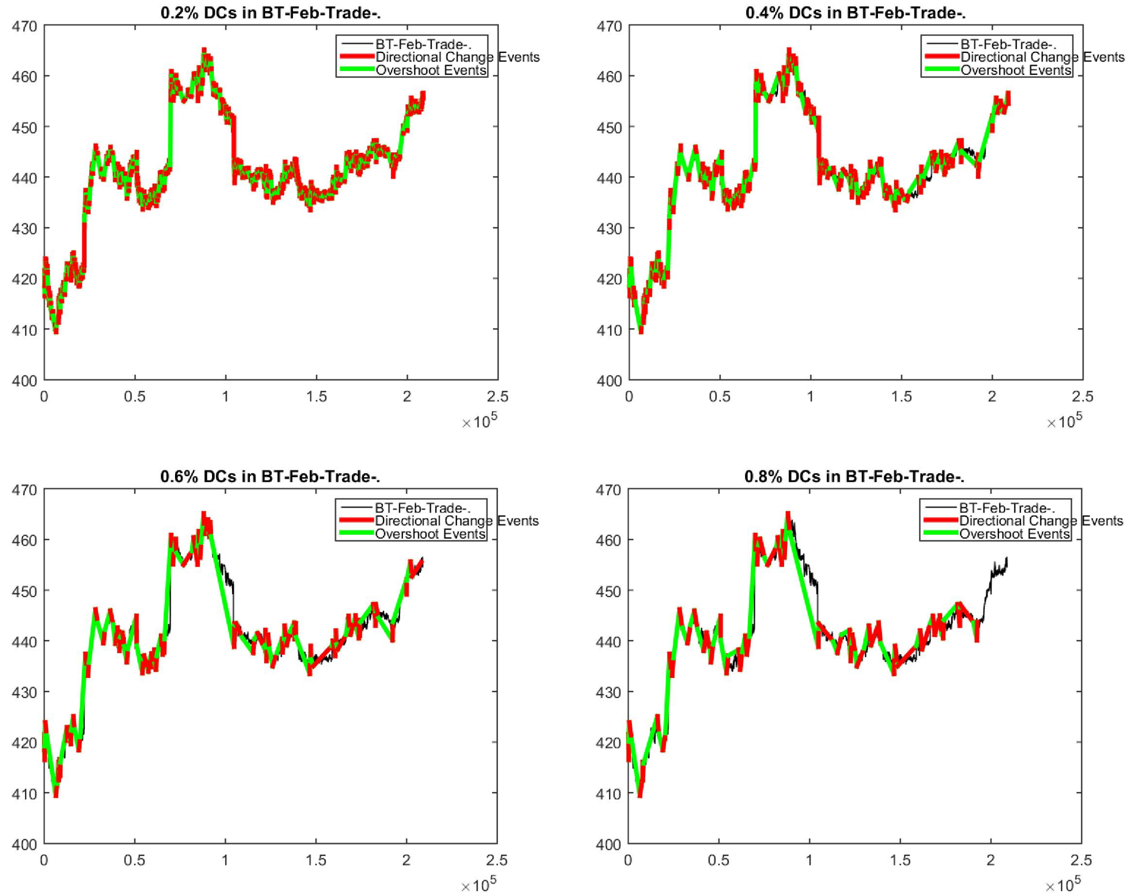

Figure 6: DC and Overshoot Events found in tick-by-tick data in BT, February 2015 using different thresholds (0.2%, 0.4%, 0.6% and 0.8%)

6 Conclusion and Further Research

In this paper, we have introduced DC as an alternative way to summarize and profile price changes. DC is different from the traditional time series, and therefore provides an additional angle for capturing and analysing market changes. We have introduced a set of indicators for capturing and extracting information from data. For example, Sigma captures the speed of a completing trend. The coastline captures the potential profit of trading within a time period under a selected threshold. This is useful information in DC events analysis that we do not find in time series analysis.

We have demonstrated how DC profiles could be used to summarize price changes in the high frequency data in equity markets. Through these indicators, we can discover useful information about the profiled period. For example, DC profiling adds new ways to look at volatility: NDC (or VT when the length of the profiled periods varies) and OSV_{EXT}. The former measures the frequency of directional changes and the latter the magnitude of overshoots. MeanLenC provides us with an idea of the potential profits to be made in a profiled period. Large scale experimentation in foreign exchange trading (Glattfelder et al 2011) provides us with reference values of 1 for OSV_{EXT}, which helps one to assess the magnitude of overshoots. Our observations in the equity market seem to support these reference values.

With the indicators presented in this paper, we aim to initiate further research in the DC framework. One direction is to define new indicators for DC profiles. Another direction is to combine directional change analysis with the traditional time series analysis, and explore synergies. Since they are two different methods to observe the same market, they may provide complementary market information.

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