A Directional Changes based study on stock market

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A thesis submitted for the degree of Doctor of Philosophy in Computational Finance

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June 2018

Contents

Abstract

This thesis is based on the concept of directional change (DC). To put simply, a directional change is a price change event that confirmed by a pre-determined parameter – theta (θ) along with an extremum price. When the price changes, from a extremum by θ, then there is a directional change. If the extremum is a local minimum, then this is an upward directional change; likewise if the extremum is a local maximum, then this is a downward directional change. And the price where the directional change is confirmed is called the directional change confirmation point (DCC). Following the directional change event, there is the overshoot event, which is the price change between the DCC, and the next extremum price.

Unlike the DCs, the overshoots are not pre-determined, and as a result, the lengths of the overshoots vary. However, the Average Overshoot Length scaling law (AOL scaling law), which made its debut in Glattfelder et al. (2010a), states that on average, the mean absolute length of the overshoots is approximately equal to the chosen threshold θ. Yet, this AOL scaling law is not confirmed in the stock markets.

This thesis is going to examine the Average Overshoot length scaling law (AOL scaling law) in the stock markets. As the AOL scaling law is a functional relationship between the threshold and the average length of the overshoots, the exponent in the power law could be used to judge whether the two variables approximately equal, as it is also referred as the characteristic exponent in Müller et al. (1990).

Following this, two trading strategies built based on the AOL scaling law are going to be introduced. In order to prove that the trading strategies based on the AOL scaling law are profitable, this thesis is going to propose two strategies as proof of concept. This could be used as an attempt of more sophisticated ones.

As they are proof of concept, they only take long positions, and rules are going to be introduced.

Second, trading strategies are made based on the concept of Directional Changes. Every trade in the market is recorded. To study market movements, information of trading (such as price) is summarised. Most analysts summarise market movements with periodic sampling of data, such as daily closing prices. Instead of using regular sampled methods, two trading strategies based on Directional Changes are introduced: Trading Strategy 1 & 2 (TS1&2). These trading strategies is later evaluated with stock markets' data.

Lastly, we attempt to examine the behaviour of sub Directional Change when the price get closer to the EXTs.

Acknowledgements

I would like to sincerely thank Professor Edward Tsang who provides me enormous support during the whole course.

I would also like to thank my parents for their great support.

Chapter 1

Introduction

1.1 Background

This thesis is heavily based on the concept of Directional Change (DC), therefore, Directional Changes is going to be introduced first. A Directional Change throughout this thesis is an event when the price changes, starting from a previous extremum, by a certain percentage in a different direction other than its previous trend. A Directional Change could be a upturn or a downturn Directional Change event. For instance, if its previous extremum is a maximum, then the previous trend is considered a upward trend, when the price changes (decreases in this case) by a certain percentage, then there is a downturn Directional Change, and vice versa if the extremum is a minimum. And this *certain percentage* is an observer-determined parameter called the threshold or denoted as θ, which is used to determine the number of Directional Changes in a time series.

A Directional Change is a certain percentage of price change from an extremum determined by θ. When a Directional Change is confirmed, then the price point where the it is confirmed is a Directional Change confirmation point (DCC). between this DCC, and next extremum, there is an connecting event called an Overshoot event. To be more precise, an Overshoot is the price change between a DCC and an extremum that starts next Directional Change.

The scaling laws are reported widely in the foreign exchange markets, of which we believe could be utilised as a tool to build trading strategies in stock markets if it could be also confirmed with stock data. Scaling laws are also widely considered as the tool to understand complex systems. The scaling law is formally called the Average Overshoot Length Scaling Law (AOL Scaling Law). The term overshoot is part the concept of the Directional Changes.

Trading strategies are widely used among the financial markets. The Directional Changes are believed to provide new insights towards the financial markets. Therefore, we would like to apply the insights that provided by the Directional Changes to the trading strategies, in order to explore what could be achieved.

The Average Overshoot Scaling law has been around for quite some time, and it suggests that on average the price changes in an overshoot should be approximately equal to the threshold, on which the overshoot are generated. Therefore, with this property, it would be interesting to see if trading strategies could be built with it. And could it potentially make profits.

And the AOL Scaling Law is originally discovered in the FX markets, however it is not really tested with stock markets in the existing literature, therefore, it is also of interest to see whether the explaining exponents are exact the same as in the FX markets.

Some research suggests that Directional Changes could potentially makes useful indicators of the markets, therefore, we would also attempt to make indicators with Directional Changes.

1.2 Motivation

As mentioned previously, this thesis is motivated the potential of Directional Changes. As no trading strategies built on Directional Changes has been existing in the literature, we aim to attempt making trading strategies using the property discovered by Glattfelder et al. (2010a) and test it with stock markets' data and see whether it could generate positive returns. Furthermore, we would like to take a closer look at the AOL scaling law, to find out if it is the same in the stock markets as in the FX markets. Lastly, it would be very nice if any indicators could be built that has potential predicting power to the EXTs.

1.3 Objectives

There objectives are as following:

First, test the AOL scaling law with stock markets data to see if it shares the same explaining exponents as in the FX markets. Second, make trading strategies using Directional Changes, and test them using stock markets' date to see if they produce profits. Third, examine how the sub Directional Changes behave as the price get close to EXTs.

1.4 Scope

First of all, applying the methods used by Müller et al. (1990), the AOL scaling laws are tested with the stock market's data. However, the out comes could be 1) the explaining exponents are not significantly differ from 0, which means the scaling law does not hold in the stock markets. 2) the explaining exponents have the explaining power and are exact the same as in the FX markets. 3) the explaining exponents have the explaining power and are not exact the same as in the FX markets.

Second, we would like to build trading strategies that are able to make profits, as a proof of concept that the trading strategies built on Directional Changes are profitable. However, as the knowledge of this potential is not widely explored, potentially the trading strategies may not generate positive returns.

Third, the indicators are built from scratch, thus, they could potentially have no predicting power. Or the indicators together have predicting power, but it is hard to specify which one of them have the predicting power and the empirical model is going to be extremely complicated.

1.5 Overview

Apart from this introductory chapter this thesis has the following chapters. The second chapter is a literature review. From the way financial markets are studied to the concept of Directional Changes and Scaling Laws, Trading strategies. The 3rd chapter is going to examine the AOL scaling law in the stock markets and see the the explaining exponents also varies among different stock markets. The 4th chapter is going to be the one introduce the trading strategies. There are mainly two of them, which are built on Directional Changes and the 1:1 ratio of the mean price change in an overshoot and the threshold. The 5th chapter is going to examine the behaviours of sub Directional Changes. If there is any pattern appears, then this could contribute to the trading strategies built on DCs.

Chapter 2

Literature Review

2.1 Approaches to study finance in a nutshell

In general, finance is the activity of managing money. Overtime, people invent all kinds of ways to manage their money. It perhaps all originated from human trading activities, for convenience, human invent money as media of exchange instead of goods exchange Jones (1976). People grow wheat worried about their selling, people process wheat worried about their buying, then there comes derivatives. From examples as above, finance develops and evolves.

Yet, despite the success on concluding the relations between financial variables (empirical studies), it seems none of them succeeded in giving a full picture of how financial systems work or why financial systems behave the way they behave.

Talking about laws, it is natural to think of physics, which determines behaviours in the universe and tries to grab the laws of nature. However, it straggles to describe financial systems analytically (Glattfelder et al. 2010b).

Galileo made a statement that laws of nature are written in the language of mathematics properly three hundred years ago. Mathematics is what behind physics, when a physicist find a connection between two quantities that resembles a well-known mathematical connection, he jumps to the conclusion discussed in mathematics Wigner (1960). If there is not a connection in mathematics, a physicist may invent one using same language – mathematics (for example, the innovation of calculus). It seems like it is that mathematics makes physics work so well.

The question now becomes how mathematics works in finance. A prevailing method is econometrics. Although it is useful to deny that variables are not related, it straggles to truly prove they are related (although instrumental variables, or IVs, are introduced to determine causal relations, it raises other questions like how to pick IVs). Other approaches like differential equations (such as Fokker-Planck differential equation, Langevin differential equation and Gardiner (n.d.)), are truly as incredible as the creating of econometrics. However, it seems under certain circumstances, solving these equation are less efficient than running a dynamic simulation on computers (Glattfelder et al. 2010b).

With recent developments in computer science, another possibility appears. Together with the characteristics of a chaotic system – endogeneity, nonlinearity and unpredictability, a financial system may be recognised as a highly complex system (Guillaume (1995)). Since the complexity, it is hard to be analytically described, but not hard to be simulated on a computer (Glattfelder et al. 2010b). What is more, with a computer dynamic simulation, a complex system can be studied in a totally different way.

2.1.1 Market Behaviour

As Hussein (2013) mentioned is her thesis, there are three main streams examining market behaviour. They are Behavioural Approach, Empirical Microstructure Study and Agent Based Modelling.

Behavioural Approach

Traders' psychology is studied by this approach. And eventually how traders' over- and under-reaction are related to new information release and price movement is examined by this approach Hussein (2013). Cited by Hussein (2013), Frankel (1990*a*,*b*), Ito (1990), MacDonald & Marsh (n.d.), Menkhoff et al. (2009), Oberlechner (2001) study traders' heterogeneous expectations, Baber & Odean (2000), Glaser & Weber (2007), Oberlechner & Osler (2008) examine overconfidence, loss aversion and deposition effect are studied by O Connell & Teo (2009) and Shefrin & Statman (1985) respectively, Aguirre & Saidi (1999), Bjønnes & Rime (2005), Laopodis (2005) study feedback trading. Above are examples of human irrationality and empirical evidence.

Empirical Microstructure Study

Using empirical modelling, the effect of cumulated order flowBerger et al. (2006), Evans & Lyons (2002*b*, 2007), news arrival Almeida et al. (1998), Andersen et al. (2003), Chang & Taylor (2003), Evans & Lyons (2002*a*, 2004), fundamentals Evans & Lyons (2002*b*, 2004), feedback trading Engel & Kenneth (2004), Evans & Lyons (2002*b*), Froot & Ramadorai (2005) and institutional interventions Neely (2005), Payne & Vitale (2003) are analysed on a micro structure level Hussein (2013).

Agent-Based Modelling

Thanks to the recent development in computer science, aiming to make inferences to the market behaviour and casualties of the emergence of the market anomalies, agent-based modelling (ABM) has been adopted in explaining the market behaviour.

Mimicking the real market, using artificial intelligence, the market is represented as a group of heterogenous agents who are able to adapt to their environments by learning from the information the obtain. These agents interact and interconnect to each other, and as a whole, they make the market LeBaron (2001), Samanidou et al. (n.d.).

2.1.2 Financial systems as complex systems

As mentioned previously, financial systems are highly complex systems. The basic financial rules are simple, and are examinable (for example Tsang et al. (2012)). However, knowing these rules of how investors interact with each other does not explain the emergence of financial markets.

Scaling Laws

Understanding financial markets as complex systems, methods used in examining complexity could be adopted. One of those methods is to use agent-based modelling. Alternatively, this paper focuses on discovering scaling laws which are used to find regularities in nature. Scaling laws are discovered in many areas (for example West et al. (1997*a*), Barabasi & Albert (1999), Newman (2005)). The greatness of scaling laws is that they establish invariance of scale and play an important role in describing complex systems (Glattfelder et al. 2010a).

There is a scaling law reported by Guillaume et al. (1997) and other 12 scaling laws by Glattfelder et al. (2010a). These scaling laws are crucially based on an algorithm called Directional Changes. This event-based algorithm offers a new view of financial markets.

Intrinsic Time

Earth revolves the sun and earth rotates itself. By observations, civilisations discover periodicities and invent different calendars to guide their agriculture, religions, daily lives and etc. (Richards 1998).

A interesting fact is that there is not an universal calendar that tells you every event. Although every event can be mapped into an 'universal' calendar, some are calculated by different events. For example, the calculation of Easter (Richards 1998).

Conventionally, physical time is used when analysing financial data. Could there be another event[1] that reveals periodicities?

Using intrinsic time to summarise markets is an alternative to using physical time in studying financial time series Mandelbrot & Taylor (1967) (cited Aloud et al. (2012)). Intrinsic time is defined by events, here in this thesis we focus on the events called Directional Changes.

Direcional Changes

Tsang Tsang (2011) formally defined Directional Changes (See the following section for a more detailed definition). Briefly, a Directional Change is an event, at which the current momentum (or direction) of price changes [2]. Obviously, there are two types of Directional Change events, "Upturn Event" and "Downturn Event" Tsang (2011). However, not every change in directions is called a Directional Change event. Instead, only when the price changes a certain rate in the opposite direction is called a Directional Change event. This certain rate is pre-determined, called a threshold.

A Directional Change Event is usually followed by an overshoot event. When the

[1] When physical time is used, the event is the earth revolution: around its axis and around the Suns

[2] In terms of price, there are only two directions; which are up and down.

change of price reaches the threshold, then a Directional Change Event is confirmed. However, usually, the price would not start another Directional Change Event immediately but continuously goes in the same trend until another Directional Change Event is confirmed.

2.2 Directional Changes, Definitions

In this section, we summarise the key definitions of Directional Changes since this thesis is heavily based on the concept of 'Directional Changes', following is the definitions formally presented on Tsang (2011).

A **Directional Change Event** can be a **Downturn Event** or an **Upturn Event**.

A **Downward Run** is a period between a Downturn Event and the next Upturn Event. An **Upward Run** is a period between an Upturn Event and the next Downturn Event.

In a Downward Run, a **Last Low** is constantly updated to the minimum of (a) the current price and (b) the Last Low. In an Upward Run, a **Last High** is constantly updated to the maximum of (a) the current price and (b) the Last High. Last Low and last High are called **Extremum (EXT)**.

In a Downward Run, given a **Threshold** (a percentage), an **Upturn Event** is an event when the price is higher than the Last Low by the Threshold. An Upturn Event terminates a Downward Run, and starts an Upward Run.

In an Upward Run, given a **Threshold** (a percentage), a **Downturn Directional Change Event** is an event when the price is lower than the Last High by the Threshold. A Downturn Event terminates an Upward Run, and starts a Downward Run.

The point when the price reaches the Threshold is called the point of **Directional**

Change (DC).

The above definitions are mutual recursive. Operationally, we set both the Last High and Last Low to the price at the beginning of the sequence, where neither downward run nor upward run is defined.

A Downturn Event is followed by a **Downward Overshoot Event**, which is ended by the next Upturn Event, which is itself followed by an **Upward Overshoot Event**, which is ended by the next Downturn Event. So time is defined by sequences of event cycles of four events, as shown below:

... → Downturn Event →

Downward Overshoot Event →

Upturn Event →

Upward Overshoot Event →

Downturn Event → ...

2.3 Summarising Data with Fixed Time Interval

Financial data is in high quality, as it is recorded at every trading. However, as a result, the quantity of the data tends to be large. Therefore, although information of every trading is recorded, it is not practically accessible until recent years with the developments of computer science. For example, in foreign exchange markets, not until the beginning of the 1990s, intra-daily data have been broadly studied, while daily data is very much used in the 80s. The later represents only a very small subset of information available intra-daily, and the size of the former is 100 to 1000 times larger than daily data (Guillaume et al. 1997).

Even though, the daily data still does not use every piece of data recorded. As a result, only tick-by-tick data contains every piece of underlying information, which we call raw data in the rest of this chapter. And a fact is that except tick-by-tick data, all other data used in our daily research is somehow summarised from tick-by-tick data. Therefore, in this sense, except using tick-by-tick data, the ways of summarising become crucial. A good summary should reflect the information of raw data as much as possible.

A common way to summarise raw data is to first choose a time interval, and then sample raw data at fixed time points with the chosen interval; for example, hourly, daily or monthly. We call data summarised this way an "interval-based summary". Naturally, an interval-based summary becomes a time series. In such a summary, the time interval is the arbitrarily chosen parameter, and the amplitude of the change of price is variable (Guillaume et al. 1997). And based on the summary, analyses can be performed, and our established knowledge is very much built on it. For example, one might describe the trend or volatility in the last n days (Hamilton 1994)[3]

A possible explanation of why interval-based summarising becomes the prevailing way would be this: Before tick-by-tick data became available, the daily or hourly even second-by-second quotations were the most accurate data available. Knowing them is seen as knowing all information available. Later, as a convention, when trading became more frequent, data are still summarised in such ways.

Although raw data is not necessary to be summarised; in finance, each trade is recorded, this includes the price, volume and the time that a trade occurs. As a result, the amount of data collected is potentially large. To prevent combinatorial explosion (Krippendorff 2010), these recorded data, or raw data, are usually not directly used, but are normally sampled into summaries. So that a comparatively smaller number of data are used for analyses.

Guillaume et al. (1997) introduced an alternative way to summarise raw data. In this approach, one summarises raw data by "Directional Change events". In this

[3]A time series is a collection of observations indexed by the date of each observation, pp.25

algorithm, compare to interval-based summarising, the change of price is fixed and time is the varying parameter. Briefly, a Directional Change is an event during which price momentum changes the direction – from upward to downward or vice versa. In this section, we recapitulate the formal definition of DC, and evaluate its appropriateness in capturing market dynamics.

2.4 Alternative Ways of Summarising Raw Data

Raw data can be summarised in many ways. However, in this section, we focus on a traditional ways – the interval-based summary and an alternative event based summary – what we called the Directional Change event-based summary or DC-based summaries in short.

2.4.1 Interval-based Summary of Transactions

As mentioned above, financial data (See figure 1, graph a for raw data) is often summarised using fixed time intervals. In other word, it is sampled with regular observation frequencies (intervals are as shown in figure 1 graph d). Samples collected this way are called interval-based summaries (see figure 1, graph b and c), i.e. time series. For example, using 400-business-day's daily[4] (figure 1 graph a) closing price data (from 06/07/2011 to 01/02/2013) of HSBC as raw data. With a monthly sampling, we can have an interval-based summary of 22 observations (21 intervals, shown in figure 1 graph d).

[4]Although a daily data is already an interval-based summary, for simplicity, we use daily data as raw data to illustrate the concept.

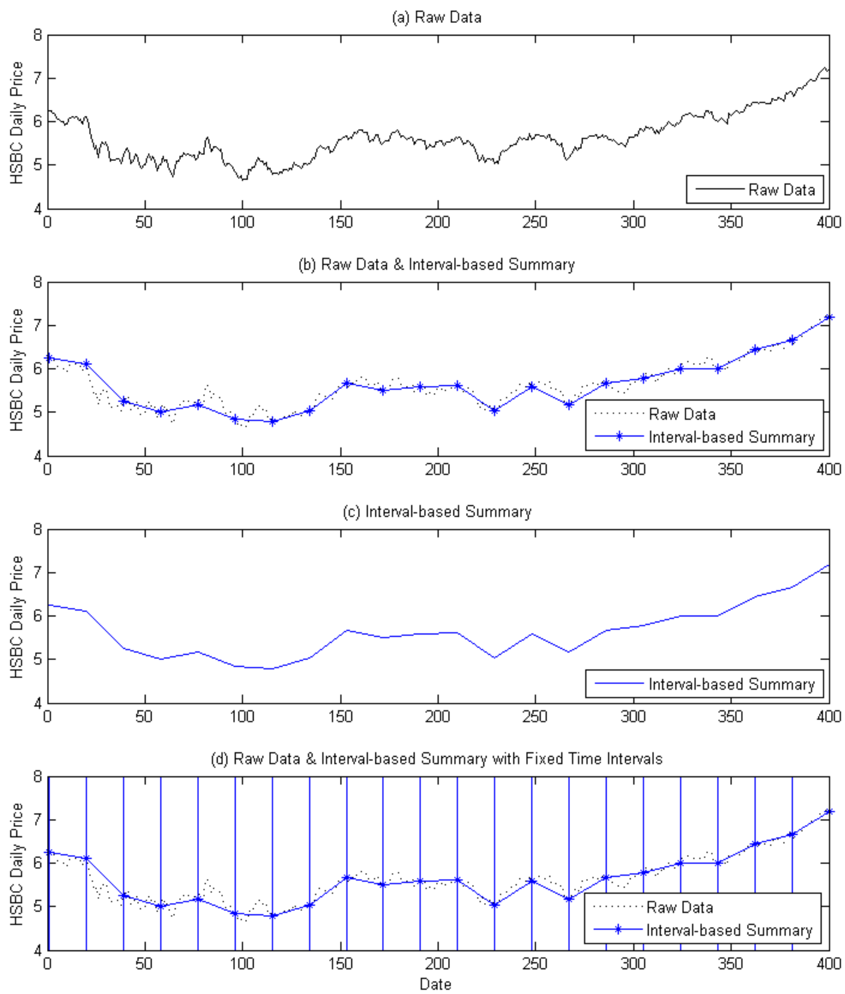

Figure 1. Summarising Raw Data
Interval-based summary of HSBC stock daily price (400 business days from 06/07/2011 to 01/02/2013). Blue curves is the interval-based summary of the original price curve (the black curve, chart a*). For simplicity, we use daily closing prices as raw data, and contrast them with monthly (20 days) interval-based summary*

2.4.2 Using Directional Change Events to Summarise Transactions

Instead of summarising transactions with a chosen time interval, we can summarise them by events. A DC-based summary is a summary of raw data sampled at each Directional Change event. Although, there are many ways of defining events; in this section, we focus on using one specific type of events defined by Guillaume et al. (1997), namely Directional Changes (or DC for short).

In order to understand what a DC-based summary is, it is necessary to first introduce the concept of "Directional Change".

Tsang Tsang (2011) formally defined Directional Changes. Briefly, a Directional Change is an event, at which the current momentum (or direction) of price changes[5]. Obviously, there are two types of Directional Change events, "Upturn Event" and "Downturn Event" Tsang (2011). However, not every change in directions is called a Directional Change event. Instead, only when the price changes a certain rate in the opposite direction is called a Directional Change event. This certain rate is pre-determined, called a threshold.

A Directional Change Event is usually followed by an overshoot event. When the change of price reaches the threshold, then a Directional Change Event is confirmed. However, usually, the price would not start another Directional Change Event immediately but continuously goes in the same trend until another Directional Change Event is confirmed.

After understanding what a Directional Change event is, a DC-based summary is possible to be explained. A DC-based summary of raw data is a summary resulted by sampling raw data at each Directional Change event with a certain threshold.

A DC-based summary is depicted in figure 2, graph *a* shows the raw data, which is

[5] In terms of price, there are only two directions; which are up and down.

the same as in figure 1. Graph b and c show the DC-based summary of the raw data, and graph d gives the intervals defined by Directional Changes. It is also shown in graph d (vertical lines) that the interval widths are not fixed.

In figure 2, from graph a to d, it shows how Directional Changes summarise raw data (graph a) into a DC-based summary (graph c to d). And in graph d, blue vertical lines represent sample points. For comparison, in figure 3, graph a and b show the Interval-based summary and DC-based summary respectively. And in graph c, the dramatic difference between them is shown. A more detailed analysis is shown in next section.

2.5 The Value of DC-based Summaries

2.5.1 DC-based Summaries Focus on Periods That Matter More

As an alternative of summarising raw data, DC-based summarising deals data in a different way. Whenever a change of price reaches the threshold, Directional Changes capture it. In contrast, interval-based summary records only at pre-determined (such as hourly, daily or second-by-second etc.) time points. Therefore, using Directional Changes means more data in periods with violently changing prices, fewer data in calm periods. For example, price P_1 starts to increase at t_1. Then at t_2, P_2 becomes $p + \Delta p$, and it starts to decrease. After, when at t_3, price P_3 becomes p again (becomes the same as at t_1, i.e. $p_1 = p_3$). Assume that t_1 and t_3 are sampling time points of an interval-based summary; the observation result would be no-change. But if the threshold T determined is smaller than p (this means that one actually cares changes that are bigger than T), then this move of price is definitely captured by Directional Changes.

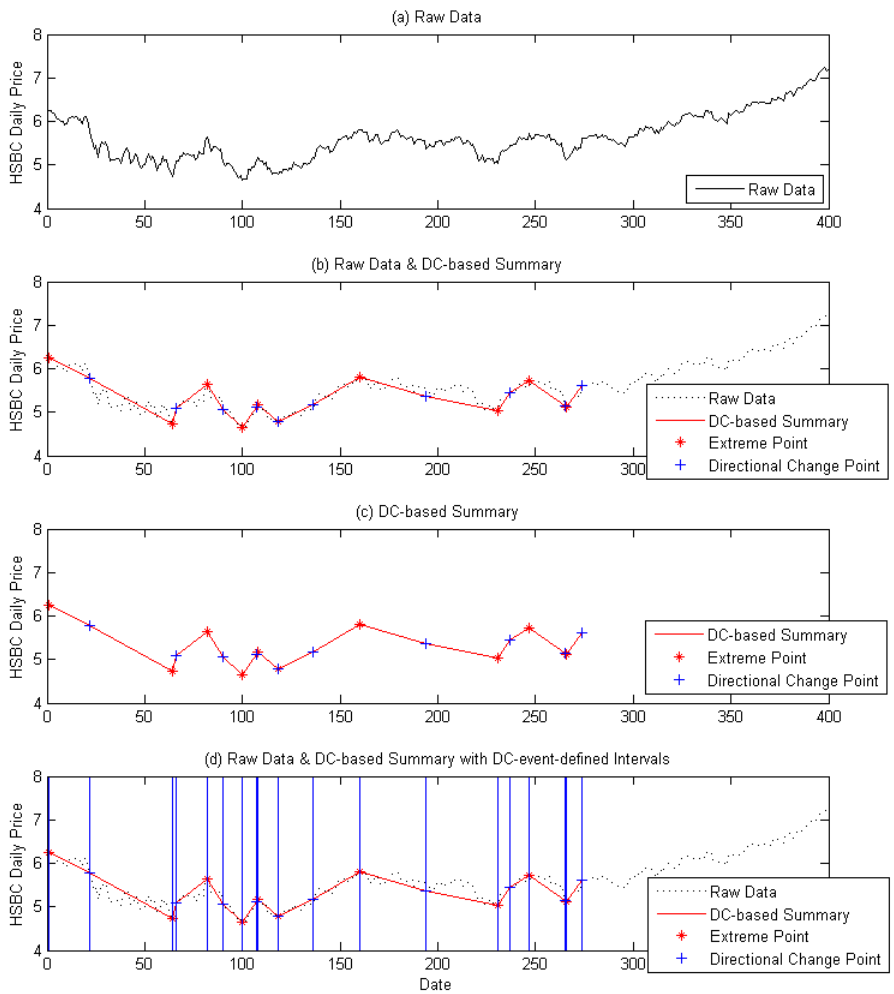

Figure 2. DC-based summary of HSBC stock daily price (400 business days from 06/07/2011 to 01/02/2013, red curve), under threshold of 0.075 (20 observations). Like figure 1, for simplicity, we use daily closing prices as raw data

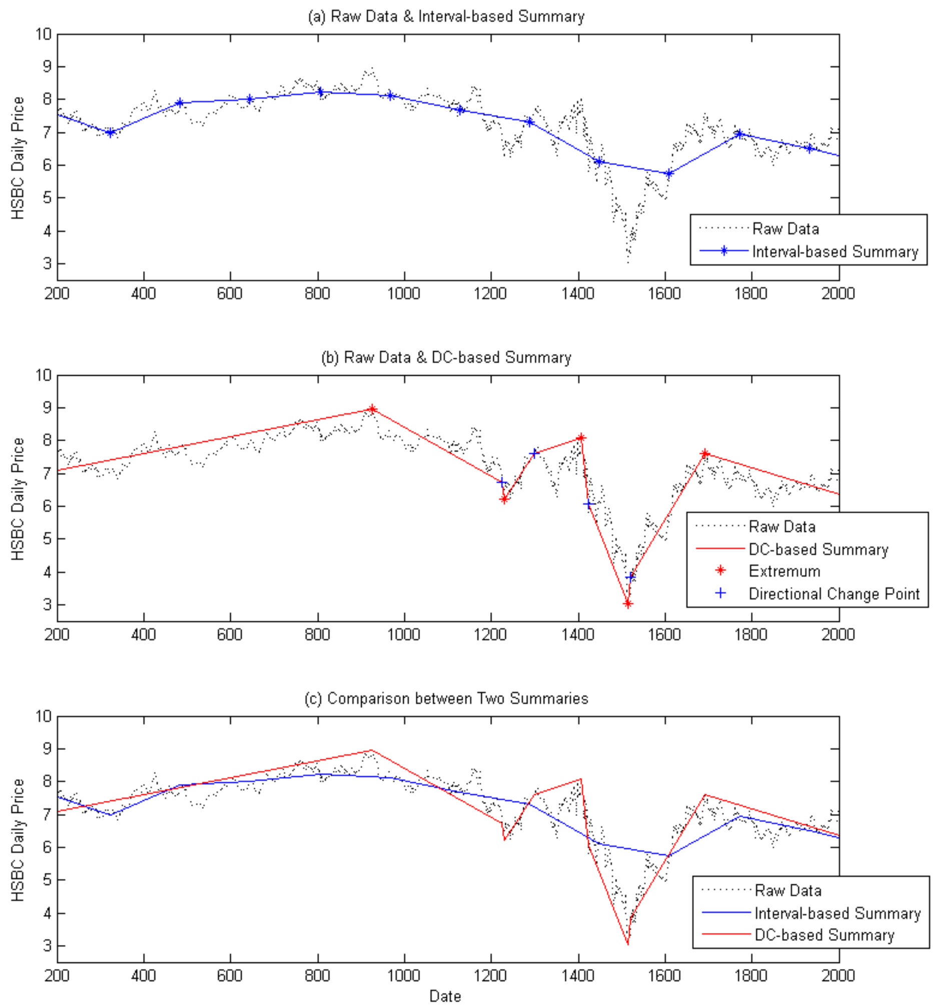

Figure 3. Comparison between interval-based and DC-based summaries (HSBC stock daily price from 24/12/2003 to 16/02/2009, under threshold 0.22)
In this figure, red curves are DC-based summaries and blue curves are interval-based summaries. As can be seen in the figure, extreme are missed by the Interval-based Summary but captured by the DC-based Summary.

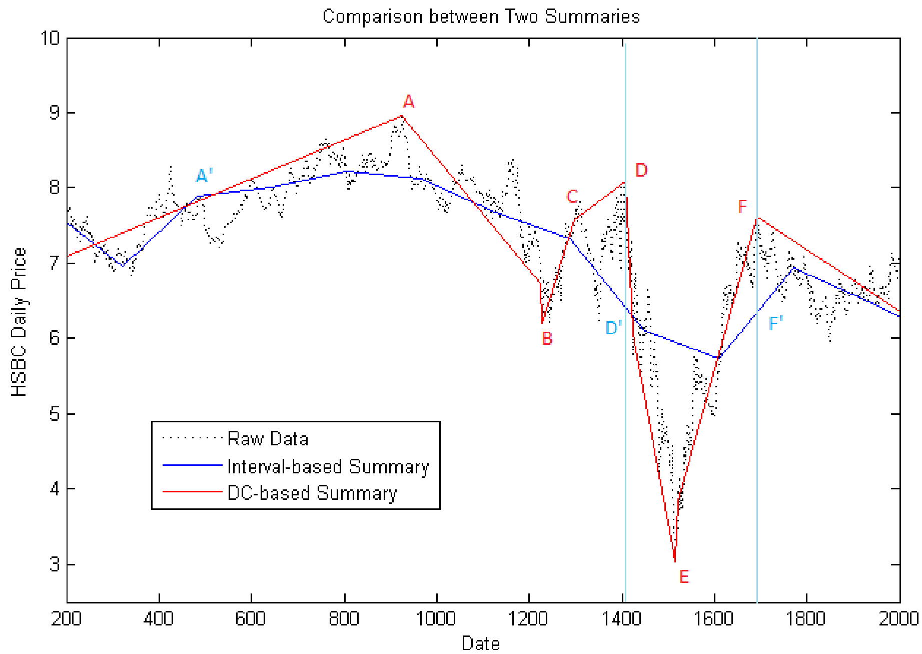

Figure 4. Comparison between interval-based and DC-based summaries (HSBC stock daily price from 24/12/2003 to 16/02/2009, under threshold 0.22)

For example, in figure 4, there are six extreme points marked as significant move — A, B, C, D, E and F. It appears that only A' is partly captured by the Interval-based Summary (not exact captures the peak point, but a sub-peak). As interval-based summaries sample at fixed points, it has a certain chance to capture peaks as A. In other word, no matter how big the change is, Interval-based Summary has a certain chance to miss it. As shown in figure 4, points B, C, D, E and F are completely missed by the Interval-based Summary, in which, a dramatic move (from D to E, and E to F) is inappropriately sampled to a much gentle decreasing trend (D' to F'). In a contrast, DC-based Summary captures all those significant moves except A'.

2.5.2 DC-based summaries Offer Longer Coastlines

Directional Changes offer a longer price coastline than Interval-based summaries. With consideration of profitability, Directional Changes capture all events that reach the threshold; of which is determined to find out changes in prise that concerning ones interests. Aloud et al (2012) show that price-curve coastlines measured by intrinsic time are longer than those measured by physical time. A longer coastline indicates higher potential of profitability. This is because longer coastline measures a bigger accumulative change (i.e. bigger $\sum_{i=1}^{n}|\frac{p_{i+1}-p_i}{p_i}|$).

A possible way of comparing coastlines summarised by intervals and DC (Directional Changes) is to calculate the cumulative changes. This is because both summaries are samples of original data, that is, the horizontal length of both summaries should be the same (see figure 1, 2)[6]. Therefore, the only matter of the length of the coastline is vertical movements, i.e. cumulative change of the price. In order to make comparison possible, we first define a threshold so that observation number (i.e. Directional Changes events) of a threshold is calculated. After, the interval of interval-based summary is defined by using raw data length divided by the observation number (DC event number). To compare the coastlines of both summaries, we test vertical coastline under thresholds of 1%, 2%, 3% and 5% on the stock daily price of HSBC. And the results show that the vertical DC-based summary coastlines are longer than interval-based ones (See table 2.1).

[6]Usually, DC-based coastlines are, horizontally, shorter than interval based ones. This is because that at the end of raw data, the remainder data do not confirm another Directional Change event (see figure 2). But this is not a problem as: First, when calculating the coastline, the interval based summary coastline is counted up to where the DC-based one ends. Second, even without adjustment, according to later experiments DC-based ones are longer than interval-based ones. If considering this issue, DC-based ones should be even longer than interval-based ones.

Table 2.1: Vertical Coastline Calculation

Threshold	1%	2%	3%	5%
Observations	2403	1602	961	601
Interval based coastline	255.53	207.64	154.62	124.42
DC-based coastline	322.67	277.89	243.54	193.70

2.5.3 DC-based Summarising Skips Data Holes

A basic fact of tick-by-tick data (raw data) is that the prices is irregularly spaced in physical time, or the transactions take place irregularly in terms of physical time (τ_j). However, most statistical analyses rely upon the use of regularly spaced data (t_i). Guillaume et al. (1997)

Consequently, interval-based summaries are often used, and the price at t_i can be defined as:

$$p(t_i, \Delta t)$$

Where t_i is a sequence of the regular spaced time data, and Δt is the time interval ($\Delta t = 1day$, $\Delta t = 1hour$, $\Delta t = 1second$, etc.)

However, when summarising raw data, there is a possibility that the sample point (t_i) is laid between ticks ($\tau_{j-1} < t_i < \tau_j$). In other word, there are data holes in interval-based summaries ($\{p(t_i)|\tau_{j-1} < t_i < \tau_j\}$ does not exist).

To fill the data holes (to obtain $\{p(t_i)|\tau_{j-1} < t_i < \tau_j\}$), linear interpolation can be adopted (Müller, 1990 Müller et al. (1990)). In this case an estimate p^* of $\{p(t_i)|\tau_{j-1} < t_i < \tau_j\}$ can be calculated as

$$p^*(t_i) = wp(\tau_{j-1}) + (1 - w)p(\tau_j)$$

Where

$$w = \frac{\tau_j - t_i}{\tau_j - \tau_{j-1}}$$

An alternative method is using $p(\tau_{j-1})$ as p^* (Wasserfallen and Zimmermann, 1985 Wasserfallen & Zimmermann (1985)).

However, if raw data is summarised by Directional Change events, the above issue no longer needs to be considered. Because Directional Change events always take place at τ_j. By replacing t_i by τ_j, traditional statistical analyses can still be employed without data holes.

2.5.4 DC-based Summaries Offer a Potential New Risk Measure

The Directional Change frequency over period S can be defined Guillaume et al. (1997):

$$d(S) \equiv d(\Delta t, n, r_c) \equiv \frac{1}{S} N(\{k | m_k \neq m_{k-1}, 1 < k \leq n\})$$

where

$$S = n\Delta t$$

and $N(\{k\})$is the counting function, $n\Delta t$ is the sampling period on which the counting is performed. m_k indicates the event type – upturn event or downturn event – of current trend. r_c is a constant threshold. $d(S)$ calculates the frequency of Directional Change events in the period.

DC-based summaries can be used as a new risk measure in two senses. First, like volatility, measuring DC frequency gives an idea that how volatile the price is in a certain period. Second, unlike volatility, the threshold is chosen by the traders, it gives the knowledge that the price is likely to move beyond the threshold. Guillaume et al. (1997)

2.6 Research using DC-based summaries

2.6.1 Regularities Based on Directional Changes Have Been Discovered

Since Guillaume *et al.* Guillaume et al. (1997) introduced Directional Changes, as regularities in a complex system, scaling laws are discovered in foreign exchange markets by Guillaume et al. (1997). A scaling law or power law is a simple polynomial function relationship: $f(x) \propto x^{-a}$. In the study of Directional Changes, Guillaume et al. (1997) presented the Directional-Change count scaling law:

$$N(\Delta x_{dc}) = \left(\frac{\Delta x_{dc}}{C}\right)^{E}$$

Where, $N(\Delta x_{dc})$ is the number of directional changes measured for the threshold Δx_{dc}. What is more, Glattfelder et al. (2010b) introduced a scaling law relates the length of the average overshoot segment to the directional change threshold:

$$\langle|\Delta x^{os}|\rangle = \left(\frac{\Delta x_{dc}}{C}\right)^{E}$$

And it turns out that the average length of overshoot Δx^{os} is about the same size as the threshold: $\langle|\Delta x^{os}|\rangle \approx \Delta x_{dc}$ Glattfelder et al. (2010b). In addition, another 12 empirical scaling laws are found in high-frequency foreign exchange data Glattfelder et al. (2010a).

These scaling laws can be seen as the law of the nature Glattfelder et al. (2010b), or regularities of science, or patterns in financial data. Because they are regularities, they happen under certain conditions. Therefore, trading strategies can be made upon these laws.

For example, according to the scaling laws and what Directional Changes reveal.

After a confirmation of a Directional Change event (a t% Directional Change), a t% overshoot will be expected (on average). A natural decision would be taking a long position when the price is expected to rise, and taking a short position when the price is expected to fall. Therefore, what the strategy suggests would be buying with all wealth at an upturn event confirmation point, as at which the lowest price is available to an investor when price is expected to rise. And selling (or short selling if possible) all assets at a down turn event confirmation point, as at which the price would be the highest to an investor when the price is expected to decline.

However, it is obvious that the strategy will not work when encounters zero overshoot[7] or when overshoot is smaller than expected. The scaling laws only apply on average, as a result the strategy may face a possibility of losing money. A possible solution will be selling a proportion of total assets, say (a_i%, where a_i decreases exponentially), whenever a small rise, say Δt% (Δt% is smaller than t%), happens after purchasing. The position will not be closed till next Directional Change event. And vice versa in short selling. Doing this, when meeting zero overshoots and overshoots that are smaller than expected, this rule will act as cut-loss strategy.

The trading strategies presented here are rather simple and more works need to be done to make it more sophisticated. However, though it is only tested in foreign exchange markets, these studies show that DC-based summaries hold the ability of revealing regularities of underlying financial data. Based on which trading rules are possible to be made.

2.6.2 Useful Market Indicators Have Been Proposed

A pioneer work for measuring the impact of major events to the markets is introduced by Zumbach *et al.* and applied in foreign exchange markets Zumbach et al. (2000), it is so called the scale of market shocks. It quantifies market movements on a tick-by-

[7]When there is not an overshoot period between two Directional Change events.

tick basis. Later on, Maillet and Michel Maillet & Michel (2003) applied the scale of market shocks to the stock market, and it is designed to detect and to compare the severity of various crises. Inspirited by Zumbach et al. (2000), another unpublished work by Subbotin (2008) is also mentioned in Bisig et al. (2009). This later study proposed a probabilistic indicator for volatilities, of which seems usable for detecting crises and regime shifts rather than quantifying impact of individual events.

Although, there is no right or wrong when choosing metric of measuring market evolution (Bisig et al. 2009), and it shoulds like a natural choice to use volatility; Bisig et al. (2009) claimed that using volatility fails to maximise the criteria of simplicity and the ability of incorporating all details of the prise evolution, as aggregating activities into a volatility measurement mingles different price scales. For example, Bouchaud *et al.* Bouchaud et al. (2008) showed the dynamics of the market slowly 'digesting' the changes in supply and demand[8] involving market order book dynamic and market maker profits, of which is certainly interesting. However, because of using volatility as the measurement of market dynamics, it is still not clear that what impact a event brings to the market, as the volatility is a measure calculated from all past prices of which from various scales.

Therefore, to quantify the trajectory of market prise evolution, Bisig et al. (2009) proposed a frame work so called the scale of market quakes, in which the physical time no longer exists, instead, time ticks at every confirmation of price Directional Changes. By calculating the average Overshoot and comparing the overshoot-at-event, a quake at a certain magnitude/scale can be calculated. Testing at major news announcement and analysing the evolution of those scales, Bisig et al. (2009) claim that the SMQ response to news announcements or a mismatch of demand and supply. And the SMQ is believed to be the first step to build a global information system Bisig et al. (2009).

[8] Mainly, how transactions impact the market

2.7 Potential of Directional Changes

2.7.1 DC-based Summaries Reflect Properties of Original Raw Data

Although financial raw data are in high quality, it seems that there is not an efficient way to deal with the raw data but summarise them into either Interval-based or event-based summaries. When summarising data, it loses some information of the original data for sure. Therefore, it is important that the summary reflects the original data's properties/features.

When raw data are summarised as Interval-based summaries, without considering the properties/features, data are sampled at certain time points with fix-interval lengths. This mechanism makes the summary actually regardless to the properties/features of original data. Although, one may claim that with smaller intervals, it has bigger probability to capture market *significant* movements; still there is no guarantee (see figure 4). What is more, with smaller intervals it faces the problem of handling large size of data, which is one of the reasons that we summarise raw data. In other aspects, when dealing with smaller-interval data such as high frequency data, we may focus on more micro movements. But, the data are still time series with fixed interval lengths. This means that even with small intervals, Interval-based summaries still have the chance to miss significant movement at a much more micro scale (as in figure 4).

However, with DC-based summaries, once the threshold is decided, the summary captures these movements reaching or exceeding the threshold. Unlike Interval-based summaries, all DC based summaries' sampling are at extreme points, (as shown in figure 3 and figure 4). As a result, when markets have significant move, they are represented as a big Overshoots. And the size of the Overshoot actually reflects how significant a movement is. DC-based summaries work when facing high frequency data, and all need to be done is, depending on the frequency, to use a smaller threshold.

2.7.2 A Longer Coastline Potentially Offers More Profits

As mentioned in section 4.2, comparing to Interval-based summaries, DC-based summaries have longer coastlines. Because Directional Changes are always confirmed on extreme points (see figure 2). And Interval-based summaries can sample at potentially anywhere on the original price curve (see figure 1). Horizontally, both DC-based and Interval-based summaries pass the same path; vertically, because DC-based summaries are always at extreme which Interval-based summaries are not, DC-based summaries are considered more volatile than Interval-based summaries (see figure 4). i.e. DC-based summaries' curves are longer than Interval-based summaries'.

Considering the measurement of return $\frac{p_t - p_{t-1}}{p_{t-1}}$, in a market allowing short selling with proper trading strategies (as mentioned in section 4.2), higher volatility means potentially higher profitability. Compare with Interval-based summaries, DC-based summaries are the ones giving longer coastlines. Yet, the problem becomes how to find a proper trading rule.

2.7.3 DC-based Summaries Prevent Distorting Raw Data

Although there are methods to fill data holes, doing it makes the data distorted. Because, when a data hole is filed, artificial data has been added. One may claim that a big enough observation number could eliminate the artificial data's effects. However, there is no guarantee that the number of artificial data is not proportional to the observation number. While using DC-based summaries does not need to worry about this issue at all. By skipping data holes, DC-based summaries make sure that all data used to run statistical analyses exist.

2.7.4 A Potential New Risk Measure

Volatility gives investors an idea that the amplitude of change of price. According to a unrealistic assumption that the change or price is subject to normal distribution, predictions of future volatilities can be made. However, a high volatility does not only mean that a investor is like to have a higher chance to loose, it also indicates that there is a chance to gain more.

While interval-based summaries fix time intervals and change of price's amplitude is changing; DC-based summaries choose a constant threshold while time is varying. This means that the change of price is fixed and it gives the idea of how likely the price is to move a certain rate in a certain direction. This is helpful to traders to decide whether to open or close a position.

Although volatility tells us the general environment of the market, we are actually more interested in the timing of our trades. (Dacorogna et al. (1993))

2.7.5 DC-based Summaries Reveal Regularities

Actually, there are works on regularities in DC-based summaries. As mentioned in section 5.1, although trading strategies are not necessary to be built on regularities. But if there are regularities found, then trading strategies can be made upon.

According to Guillaume *et al.* Guillaume et al. (1997), scaling laws, as regularities in complex systems, are found in foreign exchange markets. And empirical works are done in foreign exchange markets showing positive results Guillaume et al. (1997), Glattfelder et al. (2010b,a). Based on the scaling laws found in DC-based summaries (see section 5.1), trading strategies are possible to be made.

2.7.6 DC-based Summaries as the Building Brick of Global Economical/Financial Information System

The Scale of Market Quakes (SMQ) is introduced by Bisig *et al.* Bisig et al. (2009). This system is built on DC-based summaries and it is for detecting the market dynamics. This system detects the quake scales of market by comparing the overshoot at event to the average overshoot to give a description of market status. Inspirited by the work, a further development can perhaps be using the overshoot distribution to make a value-at-risk-like risk measurement. Because this new measurement is based on DC-based summary, it may not have the drawback that volatility has (price activities at different scale are mingled). However, further works need to be done.

2.8 Trading

Technical analysis is one of the most important methods which traders use, aiming to predict the trend of the financial market. Technical analysis, which involves making investment decisions using past prices or other past statistics. Much of technical analysis involves pattern recognition using specific frequency (intra-day, daily, weekly) charts that display opening, high, low, and closing prices, as well as trading volume in some form. (Kavajecz, 2004)

Although, technical analysis has been doubted by the traders, because the technical analysis aim to grasp the trading opportunities when the price patterns appear again. However, it is too late to take an action when observing the similar price patterns. Besides, the basic elements of technical analysis widely used in everyday work do not behave the same way as they were described in textbooks and publications. Difficulties arise when technical analysis is used in daily short-term trading because of minor market fluctuations that, in essence, are just the market noise. This noise can be compared with radio interference hindering clear reception. Unfortunately, the amplitude of this interference is too high to be ignored in short-term trading, and it disturbs the market harmony. (Toshchakov, 2006)

Technical analysis is very popular with the investment and financial markets, all major brokerage firms publish technical commentary on the market and many of the advisory services are based on technical analysis. Nowadays, the many excellent traders and fund managers make profits according to technical analysis. In its simplest form, technical analysis uses information about historical price movements, summarized in the form of price charts, to forecast future price trends. This approach to forecasting originated with the work of Charles Dow in the late 1800s, and is now widely used by investment professionals as input for trading decisions. (Neely, Weller, & Dittmar, 1997). Technical analysis theory tends to become an industry in the financial market, covering the stocks, bonds, futures, and options.

2.9 Scaling Laws

Research on the origins of power-law relations, and efforts to observe and validate them in the real world, is an active topic of research in many fields of science, including physics, computer science, linguistics, geophysics, neuroscience, sociology, economics and more.

Scaling phenomena can be widely found in many systems from geophysical to biological, Mantegna & Stanley (1995) Some large-scale dynamical properties of these systems depend on the dynamical evolution of a large number of nonlinearly coupled subsystems.

West et al. (1997*b*) conducted a study of Allometric scaling relations. It provides a complete analysis of scaling relations for mammalian circulatory systems.

Piccinato et al. (1997) compared the behaviour of piratical trading prices and bid/ask quote prices both with intra-day and intra-week data.

In a 2005 study Matteo et al. (2005), they showed that the scaling properties are associated with characteristics of the markets. By examining 89 various markets and instruments, they found that the scaling behaviours are quite universal across financial markets. In addition, they found that emerging markets' scaling behaviours are more likely to be affected by the central bank decisions.

Stanley et al. (1996) argue that when a large number of microscopic elements interact without a characteristic scale, scaling laws may be found independent on the microscopic details.

Bouchaud et al. (2008) discussed different models in order to find the origin of scaling laws in financial time series. Complex, collective phenomenon often generates universal scaling laws. They are independent of the microscopic details. Scaling laws emerge from collective action which do not exhibit in individual behaviour. Examples are phase transitions and fluid turbulence. Although much less efforts have been devoted to understand the scaling laws on a microscopic level, the scaling laws are also found in financial data. For pedagogical interest they illustrate how and when scaling laws can arise.

Economy or financial systems can be seen as a many-body or a complex system. Such as exchange markets which display scaling properties. In 1997 Galluccio et al. studied the scaling behaviour in currency exchange rates with satisfying results. And found it qualitatively differs from a random walk. They also claim that the Foreign exchange markets are qualitatively different from stock exchange markets. A system with a large amount of interaction and interconnection could exhibit a high level of complexity due the high amount of correlations between individuals resulting a collective behaviour.

In the study of scaling behaviours, the early work done by Mandelbrot (1963). Later Mantegna & Stanley (1995) studied the scaling behaviours on a stock index.

Mandelbrot (1983) gave the fractal point of view, that is analysing objects on different scale levels.

Glattfelder et al. (2008) had discovered 17 new empirical scaling laws in FX data across 13 currency exchange rates, which give an accurate estimation of the length of the surprisingly long price-curve coastline. The new laws introduce more stylised facts. The scaling law provides the invariance of scale and insights of complex systems.

Glattfelder et al. (2010a) discovered 12 independent new empirical scaling laws in foreign exchange markets based on an event-based approach so called Directional Changes. The Scaling Laws estimates the length of the price curve accurately. The scaling-law relations could also identify key empirical patterns. They believe the universal laws could potentially enhance the understanding of markets.

Although the reason that scaling laws exit is not clear (Bouchaud 2001, Barndorff-Nielsen & Prause 2001, Farmer. & Lillo 2004, Lux 2006, Joulin et al. 2008)

The scaling invariance that proved by the scaling laws are essential in describing complex system. The scaling laws could apply to such as risk management and volatility modelling

The financial markets such as foreign exchange markets could be seen as complex networks made of interacting agents, for example corporations institutional, retail traders and brokers (Glattfelder et al. 2010a).

The Scaling Law based on Directional changes could be traced back to Müller et al.

(1990). It is mentioned that the tested four FX series follow a scaling law measured by the absolute mean price changes of logarithmic prices.

At fixed time intervals, the mean absolute price change is a function of the fixed time interval selected. The line fitting is also mentioned in this chapter. It is appropriate to be employed to my study.

$$\overline{|\Delta x|} = c\Delta t^{1/E}$$

Later in the chapter conducted by Glattfelder et. al. (2010a), the new scaling law was discovered and clearly stated in Glattfelder et. al. (2010b):

$$\left\langle |\Delta x|^{OS} \right\rangle \approx \left(\frac{\Delta x_{dc}}{C} \right)^{E} \tag{2.1}$$

However, it is not clear in the chapter how the scaling was tested. Therefore, we are employing the methods used in Müller et. al. (1990), which is the line fitting approach introduced by Mosteller and Tukey (1977).

2.10 Summary

This chapter introduces the concept of Directional Changes, and also concepts that built on Directional Changes. First, two ways of summarising raw data were introduced. Interval-based summarising and DC-based summarising. As a well known method, interval-based summarising is not redundantly explained. Focusing on DC-based Directional Change events are firstly introduced, as the summarising based on Directional Change events, its uses are stated mainly included: DC-based summaries focus on periods matters more, they offer longer price coast lines, skip data holes and potentially can be a new risk measure telling investors the timing of closing or opening a position.

In addition, researches based on DC-based summaries are introduced – the discovery of scaling laws and the Scale of Market Quakes (SMQ). Scaling laws can be seen as regularities in the market, with which, trading strategies can be built. SMQ is a market indicator, it shows the affects that major events bring to the market.

If the Directional Changes would bring potential trading methods, then of course it would be necessary to test whether these methods or strategies could make profits. Therefore, the following section introduced few works in trading strategies.

Following this, the scaling laws are found in multiple disciplines. They are considered to provide insights of complex systems. Since financial systems are also seen as complex systems, we would be also interested to see whether certain scaling laws holds in certain markets. One of the pioneer work would be Müller's scaling law, based on which the Average Overshoot Length was also introduced.

Chapter 3

The Average Overshoot Length Scaling Law in the Stock Market

3.1 Introduction

Inspired by Müller's scaling law (Müller et al. 1990), along with other scaling laws, Glattfelder et al. (2010a) discovered one specific scaling law[1] in the foreign exchange market – the Average Overshoot Length Scaling Law (the AOL Scaling Law).

The AOL Scaling Law is built on the concept of Directional Change. A Directional Change is an event defined by a pre-determined price change – threshold (denoted as θ), when the certain amount of price change (θ) from a extremum is found, then there is a Directional Change. Following a Directional Change, there is an Overshoot,

[1] This is the so-called Average Overshoot Length Scaling Law, it will be referred as *the AOL scaling law* in the following context

which covers the period from the end of a Directional Change to the next start of a Directional Change. And together, a Directional Change and an Overshoot make a Total Movement (TM).

Glattfelder et al. (2010a) states that, with the concept of Directional Changes, in a Total Movement (TM), the mean absolute price change in Overshoots would approximately equal to the threshold used to define the Directional Changes.

However, the AOL Scaling Law is reported in the foreign exchange markets. And there is no existing literature reporting the same AOL Scaling Law in stock markets. Therefore, this is chapter is going to examine the AOL Scaling Law in the stock market. This chapter splits the examining of the AOL Scaling Law into 3 parts.

First of all, if the scaling law is to hold, there must be a scaling relationship between the average Overshoot length, or AOL (denoted $\langle|\Delta x^{OS}|\rangle$) and the Directional Change threshold (denoted θ). That is, $C_{x,OS}$ and $E_{x,OS}$ in the assumed scaling law relationship $\langle|\Delta x^{OS}|\rangle = \left(\frac{\theta}{C_{x,OS}}\right)^{E_{x,OS}}$ can not be 0. Therefore, this chapter is going to estimate the parameters $C_{x,OS}$ and $E_{x,OS}$ using linear regression applied by (Müller et al. 1990).

Second, as the AOL Scaling Law suggests, the average Overshoot length should be approximately equal to the threshold that defined the Directional Changes and the Overshoots. Yet this is reported in the foreign exchange markets. Therefore, this chapter is also going to look this property by comparing the estimated average Overshoot lengths and the threshold.

Third, as Müller et al. (1990) suggests, the exponent $E_{x,OS}$ could also be referred as the characteristic exponent, and potentially different markets may have significant different characteristic exponents. By comparing the exponents $E_{x,OS}$ from 5 different markets: FTSE 100, Hang Seng, Nasdaq 100, Nikkei 225, S & P, conclusions could be drawn on this matter.

In order to examine the above three questions, five sets of data representing 5 stock markets are tested with a linear regression between $ln\langle|\Delta x^{OS}|\rangle$ and $ln\theta$. As a result, $E_{x,OS}$ and $C_{x,OS}$ are obtained, and further comparison, observations could be conducted.

The remainder of this chapter is organised as following. The second section is one introducing the methodology about the Müller's scaling law, the AOL Scaling Law, and the way this chapter is going to set-up the experiments as well as the data that is used. The third section is going to present the results obtained from the experiments. And these results are interpreted in section four. Lastly is a conclusion section.

3.2 Methodology and Experiment set-up

3.2.1 Müller's Scaling Law

Scaling laws are widely reported in many disciplines, they are seen as important tools in studying complex systems. As financial systems are also considered as complex systems, the study of scaling laws in the financial systems are conducted by researchers as well. Among which, Müller et al's study (1990) was one of the pioneers.

A study of foreign exchange markets (Müller et al. 1990), based on 15 years' foreign exchange prices, shows that the mean absolute changes of logarithmic prices and the time interval in which the price changes are measured follow a particular scaling law. This scaling law suggests that the price changes (mean absolute changes) have a power-law relationship with the time interval (in which the price changes are measured). In other words, the relationship between the price change and the time interval is reported in their paper. Müller et al. (1990) further stated that this relationship is a scaling law relationship (or power law), and it was originally denoted in the paper (Müller et al. 1990) as:

$$\langle|\Delta x|\rangle = c\Delta t^{1/E}$$

where the average operator $\langle\rangle$ indicates the mean value over the entire sample period (in which the mean absolute changes of logarithmic prices are measured). And $|\Delta x|$ is the absolute changes of logarithmic prices. Correspondingly, Δt is the time interval in which the price changes are measured. In this case, Δt is a pre-determined parameter as series sampled in regular time periods have only one fixed time interval, and Δt is determined when the time interval is decided. c and E are to be determined by a regression, which are going to describe the scaling relations between $\langle|\Delta x|\rangle$ and Δt.

If we have a series of Δx_i generated by a random process with stable distributions, refer the series as raw data. To determine this relationship, one needs to first sample raw data with a time interval (Δt). With the time interval decided, $\langle|\Delta x|\rangle$ could be calculated. With a regression, c and E are also able to be calculated and are going to be constants. If the relationship does hold (Müller et al. 1990), E could be referred as the characteristic exponent.

3.2.2 The Average Overshoot Length Scaling Law

Inspired by Müller's Scaling Law, an extended study had been conducted by Glattfelder et al. (2010a), in which, there are 17 scaling laws discovered. Amidst these scaling laws, the Average Overshoot Length Scaling Law (the AOL Scaling Law) also made its début (Glattfelder et al. 2010b).

The AOL Scaling Law is built on the concept of Directional Change. In a time series, a time interval, like a day or an hour in a daily series or an hourly series respectively, is a pre-demerited number to define the whole series. Correspondingly, instead of a time interval, the Directional Change threshold[2] – θ (a percentage price change) is the pre-determined number to define the whole DC series. A DC starts at a extremum and ends at where the price is $\theta\%$ from the extremum. Therefore, with a θ, each DC is defined by and covers an Extreme point (EXT) and a Directional Change Confirmation point (DCC). The DCC is where the price is $\theta\%$ from the extremum, and a DCC is also the starting point of an Overshoot. An Overshoot is the price change from the DCC to the start of the next DC, which is going to be an extremum (but not necessarily the next extremum is the start of the next DC or next EXT). Hence, once DCs are found, OSs are found naturally as OSs cover the gaps between two DCs. A Total Movement (TM) is consisted of a Directional Change (DC) and its corresponding Overshoot (OS) that shares the same DCC. And the price change of a DC is going to equal to θ by definition.

Furthermore, the thresholds of the Directional Changes also define a new intrinsic time series. Unlike the fixed time interval series, the series of time are not evenly distributed in physical time. In other word, the lengths of time periods are different, the series of the intrinsic time is a series of different time lengths. However, this is not the focus of this thesis.

Recall that Müller's Scaling Law introduced a scaling law between the mean absolute price change and the time interval. Similar to the Müller's Scaling Law, the AOL Scaling Law also suggests a Scaling Relationship between two variables, which are the price change and the Directional Change threshold.

Therefore, the AOL Scaling Law uses the threshold – θ instead of a time interval to obtain the scaling relationship. Naturally, the AOL Scaling Law describes not the relationship between the price change and the time interval but the relationship

[2] could be often referred as 'the threshold' in this thesis

between the price change and the threshold – θ.

Scaling Laws found by Glattfelder et al. (2010a) are formally presented in their paper as:

$$\langle|\Delta x^*|\rangle = \left(\frac{\theta}{C_{x,*}}\right)^{E_{x,*}}$$

where $\langle\rangle$ is the average operator. The superscript and subscript * stands for {*TM*, *DC*, *OS*}. *TM, DC* and *OS* denote Total Movement, Directional Change and Overshoot respectively. $\Delta x = (x_i - x_{i-1})/x_{i-1}$ and $x_i = x(t_i)$ is the price at time t_i (Glattfelder et al. 2010a). Δx^* is the price change in a period *, $\langle|\Delta x^*|\rangle$ is the mean absolute price change covering all time periods *s. θ denotes the Directional Change threshold. And parameters $C_{x,*}$, $E_{x,*}$ are constants to be determined; the subscripts $(x, *)$ indicate that the parameters are related to the price x and period $*$.

* denotes the period the price change takes place. For instance Δx^{TM} is the price change in a TM (Total Movement); similarly Δx^{DC} and Δx^{OS} are the price changes in a DC and a OS respectively. When * in Δx^* is substituted by TM, DC, or OS, $C_{x,*}$, $E_{x,*}$ need to change accordingly to $C_{x,TM}$, $E_{x,TM}$, $C_{x,DC}$, $E_{x,DC}$ or $C_{x,OS}$, $E_{x,OS}$

Substituting * with DC, we can obtain one of the scaling laws – $\langle|\Delta x^{DC}|\rangle = \theta$, which holds by definition, as it suggests the price change in a Directional Change would equal to the threshold, since the threshold is the percentage change that determines the price changes in the DCs.

Since the main focus is going to be examining the AOL Scaling Law, the AOL Scaling Law is presented as:

$$\langle|\Delta x^{OS}|\rangle = \left(\frac{\theta}{C_{x,OS}}\right)^{E_{x,OS}}$$

With the tested foreign exchange data in the paper (Glattfelder et al. 2010a), the AOL scaling law suggests, on average, a Directional Change is followed by an Overshoot with the same magnitude. That is the average length of the Overshoots is about the same size as the threshold (θ). To be more specific, that is, $\langle|\Delta x^{OS}|\rangle \approx \theta$. And according to Glattfelder et al. (2010a), $C_{x,OS} \approx 1.06$ and $E_{x,OS} \approx 1.04$. It, on average, makes the total movement double the size of the directional change it is associated with. Therefore this could also be denoted as: $\langle|\Delta x^{TM}|\rangle \approx 2\theta$.

As mentioned above, Scaling laws are considered to provide insights to the underlying complex system. The AOL scaling laws are considered to provide insights about the mechanism of the financial markets. As stated above the AOL Scaling law was discovered and tested within the foreign exchange market. And it is of interest to find out if the AOL Scaling Law also holds in the stock markets. and if the average length of the Overshoots are approximately equal to the threshold θ. Also, Müller et al. (1990) mentioned that potentially the characteristic exponent E could be used as a indicator to tell the difference between different markets.

3.2.3 Testing the AOL Scaling Law in the Stock Markets

To find out if the AOL Scaling Law holds in the stock market is to examine whether $\langle|\Delta x^{OS}|\rangle = \left(\frac{\theta}{C_{x,OS}}\right)^{E_{x,OS}}$ holds with stock markets' data. And to see if the average Overshoot length is also about the same size of θ is to look at the parameter $E_{x,OS}$ and $C_{x,OS}$. And by comparing $E_{x,OS}$, it would be known if it differs as markets varies.

First of all, for the scaling law to hold, variables $\langle|\Delta x^{OS}|\rangle$ (the average Overshoot length) and θ (the Directional Change threshold) need to have a scaling law relationship. That is, as the paper (Glattfelder et al. 2010a) suggests, $E_{x,OS}$ and $C_{x,OS}$ should not be 0 for the them to have the relationship. Therefore, to find out if the

AOL Scaling law holds in the stock markets is to find out whether the parameter $E_{x,OS}$ and $C_{x,OS}$ (the characteristic exponent according to Müller et al. (1990)) is significantly different from 0 with stock markets' data.

Secondly, to know whether the average Overshoot length is also about the same size of θ, it is necessary to know whether the characteristic exponent and parameter $C_{x,OS}$. That is, it is needed to know if $\langle|\Delta x^{OS}|\rangle \approx \theta$ with stock markets' data.

Thirdly, the comparison of characteristic exponents for different markets also requires obtaining $E_{x,OS}$ for each markets.

Therefore, to examine the AOL Scaling Law in the stock markets, it is essentially to examine the characteristic exponent $E_{x,OS}$ and $C_{x,OS}$ in the Law with the stock markets' data. As the methods used to justify the scaling laws are not obviously mentioned in Glattfelder et al. (2010a). Therefore, in order to examine the AOL Scaling Law, this chapter is going to employ the method used by Müller et al. (1990), which is the line fitting method introduced by Mosteller & Tukey (1977).

To test the AOL Scaling Law with the linear regression method employed by Müller et al. (1990), the Scaling Law needs to be re-arranged into a linear model $ln\left\langle|\Delta x^{OS}|\right\rangle = -E_{x,OS} \cdot lnC_{x,OS} + E_{x,OS} \cdot ln\theta$. In the linear model, $E_{x,OS}$ should not be zero. Otherwise there is no relationship between the threshold and the average overshoot length. Therefore, the null hypotheses for the test could be that $E_{x,OS} = 0$ and $lnC_{x,OS} = 0$. And alternative hypotheses are $E_{x,OS} \neq 0$ and $lnC_{x,OS} \neq 0$.

The relations between $\langle|\Delta x^{OS}|\rangle$ and θ could be double checked with a direct linear regression between these two variables. In this linear model, different parameters should be tested. For example, in linear model $\langle|\Delta x^{OS}|\rangle = a + e_{x,OS} \cdot \theta$, if for $\langle|\Delta x^{OS}|\rangle$ and θ to have a relationship, $e_{x,OS}$ should not be 0. Therefore, similar null hypotheses are $e_{x,OS} = 0$ and $a = 0$ and alternative hypotheses could be $e_{x,OS} \neq 0$ and $a \neq 0$.

If the AOL Scaling Law does hold in the stock markets, the it is possible to further look at the characteristic exponent $E_{x,OS}$, to see if it is the same in the stock market as in the foreign exchange market. That is if $\langle|\Delta x^{OS}|\rangle$ is approximately equal to θ. If this is the case, then the AOL Scaling Law would be the same as in the foreign exchange markets. If $\langle|\Delta x^{OS}|\rangle$ is not approximately equal to θ, then the AOL Scaling Law seems to not be the same as in the foreign exchange markets. That is, the average overshoot length is not going to be approximately equal to θ.

And if the characteristic exponent $E_{x,OS}$ is significantly different among the stock markets. Then this could potentially be used to distinguish the markets.

To sum up, by testing the regression model, we would know whether the characteristic exponent $E_{x,OS}$ and parameter $C_{x,OS}$ in the AOL Scaling Law would be significantly different from 0. And if the parameters are statistically significantly different from 0 tested by the stock markets' data, it could also be said the AOL Scaling Law holds is the tested stock markets. If the AOL Scaling Law tells us that $\langle|\Delta x^{OS}|\rangle \approx \theta$, then the AOL Scaling Law is the same as in the foreign exchange market. And if $E_{x,OS}$ is approximately the same in all the tested markets, then the characteristic exponent can not be used to tell different markets. As a result, we would be able to answer 1) whether the AOL Scaling Law holds in the stock markets; 2) what is the explaining exponent of the AOL Scaling Law in the stock markets; 3) if the characteristic exponent $E_{x,OS}$ is the same among the markets.

3.2.4 Experiment Set-up

In order to apply the line fitting method used by Müller et al. (1990) in testing the hypothesis listed in the previous section. The equation of the two variables is going to be re-arranged. The AOL Scaling Law is:

$$\langle|\Delta x^{OS}|\rangle = \left(\frac{\theta}{C_{x,OS}}\right)^{E_{x,OS}}$$

And according to Glattfelder et al. (2010a), to establish scaling law relations from two variables Y and X that have a linear relationship:

$$Y = A + BX$$

the scaling law relationship could be constructed:

$$y = \left(\frac{x}{C}\right)^{E}$$

where $y = e^{Y}$, $x = e^{X}$, $E = B$ and $C = e^{-A/B}$

Therefore, if we take the log for both sides for the AOL Scaling Law:

$$ln\,\langle|\Delta x^{OS}|\rangle = ln\left[\left(\frac{\theta}{C_{x,OS}}\right)^{E_{x,OS}}\right]$$

now the linear model could be obtained:

$$ln\,\langle|\Delta x^{OS}|\rangle = -E_{x,OS}\cdot lnC_{x,OS} + E_{x,OS}\cdot ln\theta$$

let $c = -E_{x,OS}\cdot lnC_{x,OS}$

it becomes the model going to be tested:

$$ln\,\langle|\Delta x^{OS}|\rangle = c + E_{x,OS}\cdot ln\theta \tag{3.1}$$

With the linear model, two sets of series of Δx^{OS} and θ are needed. In this chapter the tested series are going to be a series of 100 thresholds (from 0.005 to 0.1) and 100 corresponding average Overshoot lengths.

Hence, with stock markets' data, the experiments are going to be testing the linear model listed above as 3.1 with the:

- Null hypothesis: $E_{x,OS}$ is equal to 0;
- Alternative hypothesis: $E_{x,OS}$ is not equal to 0;
- Null hypothesis: c is equal to 0;
- Alternative hypothesis: c is not equal to 0.

That is, if $E_{x,OS} = 0$, there is no power-law relationship between $\left\langle|\Delta x^{OS}|\right\rangle$ and θ. However, $c = 0$ does not necessarily mean there is a power-law relationship between $\left\langle|\Delta x^{OS}|\right\rangle$ and θ.

When the regression is conducted, by observing the parameter $E_{x,OS}$, it would be answered that whether the AOL Scaling Law stands in the stock markets and whether it is the same as in the foreign exchange markets.

A linear model between $\langle|\Delta x^{OS}|\rangle$ and θ could also be tested to confirm the relations between the average overshoot lengths and the threshold:

$$\langle|\Delta x^{OS}|\rangle = a + e_{x,OS} \cdot \theta \tag{3.2}$$

where a and $e_{x,OS}$ are parameters to be determined. $\langle|\Delta x^{OS}|\rangle$ is the average price change in the Overshoots. θ is the Directional Change Threshold.

Similar hypotheses are:

- Null hypothesis: $e_{x,OS}$ is equal to 0;
- Alternative hypothesis: $e_{x,OS}$ is not equal to 0;

- Null hypothesis: a is equal to 0;
- Alternative hypothesis: a is not equal to 0.

3.2.5 Data

As the aim is to justify whether the scaling law stands in the stock markets. It is a necessity to use the stock markets' data. In this chapter, 5 sets of stock indices are going to be used testing the AOL Scaling Law. They are the FTSE 100, Hang Seng, Nasdaq 100, Nikkei 225 and S&P 500. These data are daily closing indices (treated as prices) starting from 02/01/09 to 01/11/13.

What is more, 100 thresholds, from 0.005 to 0.1 are selected to the law. 100 average Overshoot lengths corresponding to the thresholds are also calculated. The increment is calculated as : $(0.1 - 0.005)/99$ for there are 100 intervals.

To sum up, there are 5 indices with 100 thresholds from 0.005 to 0.1 are tested with their corresponding Average Overshoot Lengths.

3.3 Results

With the data and the models, experiments are conducted, the results are presented in this section. As the independent variable, with five markets' data are selected, 100 θs, and their corresponding average overshoot lengths for each market's data set are the dependant variable. $E_{x,OS}$ and c from testing model 3.1 for each market are obtained and listed in Table 3.1.

As can be seen in the Table 3.1, the first column lists the indices tested with the model (3.1). And from the second column, they are $E_{x,OS}$, $\Delta E_{x,OS}$, the P-value of $E_{x,OS}$, c (where $c = -E_{x,OS} \cdot lnC_{x,OS}$), Δc, the P-value of c, and the adjusted R^2 for

Table 3.1: Linear Regression between $ln \langle|\Delta x^{OS}|\rangle$ and $ln\theta$ from model 3.1 at 95 % Confidence Level

Index	$E_{x,OS}$	$\Delta E_{x,OS}$	P-value	c	Δc	P-value	Adj. R^2
FTSE 100	0.885	0.0379	1.92206E-68	-0.410	0.1226	1.80883E-09	0.9559
Hang Seng	0.889	0.0416	7.79264E-65	-0.284	0.1346	6.06335E-05	0.9477
Nasdaq 100	0.882	0.0408	2.5902E-65	-0.299	0.1320	1.86139E-05	0.9489
Nikkei 225	0.893	0.0252	1.05972E-91	-0.228	0.0816	2.42416E-07	0.9803
S & P 500	0.998	0.0661	4.19429E-51	0.050	0.2139	0.644568241	0.9004
Average	0.909			-0.234			

the line fittings. And the P-values of $E_{x,OS}$ is the probability that estimated $E_{x,OS}$ does not lie with in $E_{x,OS} \pm \Delta E_{x,OS}$, the same goes for c.

Therefore, by the coefficients obtained from the above table (Table 3.1), $C_{x,OS}$ is calculated from c and listed in Table 3.2.

Table 3.2: $C_{x,OS}$ calculated from c

Index	$C_{x,OS}$
FTSE 100	1.589
Hang Seng	1.377
Nasdaq 100	1.404
Nikkei 225	1.291
S & P 500	1.000
Average	1.332

With the AOL Scaling Law:

$$\langle|\Delta x^{OS}|\rangle = \left(\frac{\theta}{C_{x,OS}}\right)^{E_{x,OS}}$$

scaling relations between $\langle|\Delta x^{OS}|\rangle$ and θ for the each market are as following:

FTSE 100:

From the linear regression using the tested FTSE 100 data, the probability that $E_{x,OS}$ does not lie within 0.885 ± 0.0379 is 1.92206E-68. Similarly, the probability that c lies beyond -0.410 ± 0.1226 is 1.80883E-09. Therefore, the hypotheses that $E_{x,OS} = 0$ and $c = 0$ are going to be rejected in this case. And $C_{x,OS}$ could be obtained: $C_{x,OS} = 1.589$. And the adjusted R^2 for the line fitting is 0.9559. So the AOL Scaling Law in the FTSE 100 is:

$$\langle|\Delta x^{OS}|\rangle = \left(\frac{\theta}{1.589}\right)^{0.885}$$

Hang Seng:

From the linear regression using the tested Hang Seng data, the probability that $E_{x,OS}$ does not lie within 0.889 ± 0.0416 is 7.79264E-65. Similarly, the probability that c lies beyond -0.284 ± 0.1346 is 6.06335E-05. Therefore, the hypotheses that $E_{x,OS} = 0$ and $c = 0$ are going to be rejected in this case. And $C_{x,OS}$ could be obtained: $C_{x,OS} = 1.377$. And the adjusted R^2 for the line fitting is 0.9477. So the AOL Scaling Law in the Hang Seng is:

$$\langle|\Delta x^{OS}|\rangle = \left(\frac{\theta}{1.377}\right)^{0.889}$$

Nasdaq 100:

From the linear regression using the tested Nasdaq 100 data, the probability that $E_{x,OS}$ does not lie within 0.882 ± 0.0408 is 2.5902E-65. Similarly, the probability that c lies beyond -0.299 ± 0.1320 is 1.86139E-05. Therefore, the hypotheses that $E_{x,OS} = 0$ and $c = 0$ are going to be rejected in this case. And $C_{x,OS}$ could be obtained: $C_{x,OS} = 1.404$. And the adjusted R^2 for the line fitting is 0.9489. So the AOL Scaling Law in the Nasdaq 100 is:

$$\langle|\Delta x^{OS}|\rangle = \left(\frac{\theta}{1.404}\right)^{0.882}$$

Nikkei 225:

From the linear regression using the tested Nikkei 225 data, the probability that

$E_{x,OS}$ does not lie within 0.893 ± 0.0252 is 1.05972E-91. Similarly, the probability that c lies beyond -0.228 ± 0.0816 is 1.80883E-09. Therefore, the hypotheses that $E_{x,OS} = 0$ and $c = 0$ are going to be rejected in this case. And $C_{x,OS}$ could be obtained: $C_{x,OS} = 1.291$. And the adjusted R^2 for the line fitting is 0.9559. So the AOL Scaling Law in the Nikkei 225 is:

$$\left\langle |\Delta x^{OS}| \right\rangle = \left(\frac{\theta}{1.291}\right)^{0.893}$$

S & P:

From the linear regression using the tested S & P data, the probability that $E_{x,OS}$ does not lie within 0.998 ± 0.0661 is 4.19429E-51. Similarly, the probability that c lies beyond 0.050 ± 0.2139 is 0.644568241. Therefore, the hypothesis that $E_{x,OS} = 0$ is going to be rejected in this case. However, the hypothesis $c = 0$ is not going to be rejected. Therefore $C_{x,OS}$ could be obtained: $C_{x,OS} = 1$. And the adjusted R^2 for the line fitting is 0.9004. So the AOL Scaling Law in the S & P is:

$$\left\langle |\Delta x^{OS}| \right\rangle = \left(\frac{\theta}{1}\right)^{0.998}$$

On average, there is:

$$\left\langle |\Delta x^{OS}| \right\rangle = \left(\frac{\theta}{1.332}\right)^{0.909}$$

The relations between $\left\langle |\Delta x^{OS}| \right\rangle$ and θ could also be tested in a linear model between $\left\langle |\Delta x^{OS}| \right\rangle$ and θ (model 3.2), notice this is not the linear relations between $ln \left\langle |\Delta x^{OS}| \right\rangle$ and $ln\theta$ (model 3.1).

As can be seen in the Table 3.3, the first column lists the indices tested with the model (3.2). And from the second column, they are $e_{x,OS}$, the P-value of $e_{x,OS}$, the

Table 3.3: Linear Regression between $\langle |\Delta x^{OS}| \rangle$ and θ from model 3.2 at 95 % Confidence Level

Index	$e_{x,OS}$	P-value	a	P-value	Adj. R^2
FTSE 100	1.000	6.17149E-58	-0.003	0.074515105	0.9277
Hang Seng	1.152	6.40451E-64	-0.005	-0.008117526	0.9454
Nasdaq 100	1.116	3.5377E-56	0.002	0.140211143	0.9215
Nikkei 225	1.096	6.95119E-80	0.000	0.785616773	0.9742
S & P 500	1.474	2.9809E-40	-0.018	1.7604E-05	0.8343
Average	1.168		-0.006		

a, the P-value of a, and the adjusted R^2 for the line fittings.

According to the P-values shown in Table 3.3, $e_{x,OS} = 0$ is rejected for all data sets. However, it does not seem the case for a. $a = 0$ could be rejected for Nasdaq 100 and Nikkei 225. And for FTSE 100, Hang Seng and S & P, $a = 0$ is not rejected.

Therefore, there relations between $\langle |\Delta x^{OS}| \rangle$ and θ could also be presented as:

FTSE 100:

$$\langle |\Delta x^{OS}| \rangle = \theta$$

Hang Seng:

$$\langle |\Delta x^{OS}| \rangle = -0.005 + 1.152\theta$$

Nasdaq 100:

$$\langle |\Delta x^{OS}| \rangle = 1.116\theta$$

Nikkei 225:

$$\left\langle |\Delta x^{OS}| \right\rangle = 1.096\theta$$

S & P:

$$\left\langle |\Delta x^{OS}| \right\rangle = -0.018 + 1.474\theta$$

3.4 Interpretation

This section is going to interpret the results presented in the previous section. And this section is mainly consisted with three sub-sections. The first one is going to be an interpretation of the whether the AOL Scaling Law holds in the stock markets. The second sub-section is going to discuss the relations between the average Overshoot length and the threshold. And the third sub-section is going to look at the difference of $E_{x,OS}$ among the markets tested.

3.4.1 The AOL Scaling Law

First of all, from the AOL Scaling, we know that for the AOL Scaling Law to hold in the stock market, $E_{x,OS}$ and $C_{x,OS}$ can not be 0. That is, in model (3.1), $E_{x,OS}$ should not equal to 0. c is ok to be 0, as $c = -E_{x,OS} \cdot lnC_{x,OS}$, and $c = 0$ means that in the AOL Scaling Law $C_{x,OS} = 1$, and that means the AOL Scaling Law holds.

From Table 3.1, we know that the all P-values for the $E_{x,OS}$ indicate that $E_{x,OS}$ is not equal to 0 at 95% confidence level, as the P-values are far lower from 5%, which means that $E_{x,OS}$ lies outside $E_{x,OS} \pm \Delta E_{x,OS}$ is lower than 5%.

P-values for c are mostly far lower than 5% which means that the probability to reject that $c = 0$ are high enough, except the P-value for S & P is roughly 0.645. This means that c for S & P is not significantly different from 0.

With c values, we can directly calculate $C_{x,OS}$ from c in Table 3.1 for each data set except $C_{x,OS}$ for S & P which is calculated as 1 as c for S & P is count as 0. They are all listed in Table 3.2. As can be seen in the table, it is clear that all data sets have ranged mostly from 1.291 to 1.589, except S & P has a 1.

The above relations are concluded from the linear regressions of $ln \langle|\Delta x^{OS}|\rangle$ and $ln\theta$. And it seems like that the AOL Scaling Law does hold in these five tested stock markets, as all $E_{x,OS}$ shown in Table 3.1 are significantly different from 0. This means that the exponent $E_{x,OS}$ does have explaining power over $\langle|\Delta x|^{OS}\rangle$ and there is a scaling relationship between $\langle|\Delta x|^{OS}\rangle$ and θ.

3.4.2 The Relations Between the Average Overshoot Length and the Threshold

As we know that in the foreign exchange markets, the average Overshoot length is about the same size of its Directional Change threshold. A number of thresholds and their corresponding estimated AOL are listed in the Table 3.4.

Table 3.4: Estimated AOLs obtained from model 3.1

	θ	0.005	...	0.024	...	0.043	...	0.063	...	0.082	...	0.1
FTSE 100	AOL	0.006	...	0.025	...	0.041	...	0.057	...	0.072	...	0.087
Hang Seng		0.008	...	0.028	...	0.044	...	0.059	...	0.073	...	0.085
Nasdaq 100		0.011	...	0.032	...	0.047	...	0.061	...	0.74	...	0.085
Nikkei 225		0.014	...	0.036	...	0.052	...	0.066	...	0.77	...	0.088
S & P 500		0.014	...	0.037	...	0.053	...	0.066	...	0.78	...	0.088
Average		0.010	...	0.031	...	0.048	...	0.062	...	0.075	...	0.087

From this table (3.4), at a glance, we cant see that the estimated AOLs are not very different from their thresholds. To examine this further, we can measure the AOL over θ ratio. That is $\frac{\langle|\Delta x^{OS}|\rangle}{\theta}$.

Table 3.5: Average Estimated AOL over θ (derived from model 3.1)

FTSE 100	0.958
Hang Seng	1.076
Nasdaq 100	1.074
Nikkei 225	1.106
S & P 500	1.007

Table 3.5 shows the average estimated AOL over θ ratios for each data set. This is the mean value of $\frac{\langle|\Delta x^{OS}|\rangle}{\theta}$, where $\theta = 0.005, ...0.1$ for each market. That is, for each data set, every threshold has a corresponding estimated AOL[3] from the model 3.1. And their average is what listed in the table.

And from the table, we can see that the average estimated AOL over θ is ranged from 0.958 to 1.106, which means that the estimated AOL is roughly 0.958 to 1.106 times bigger than θ. And this could also be double checked by a linear regression using model 3.2.

Table 3.6: Estimated AOLs obtained from model 3.2

	θ	0.005	...	0.024	...	0.043	...	0.063	...	0.082	...	0.1
FTSE 100	AOL	0.005	...	0.024	...	0.043	...	0.063	...	0.082	...	0.100
Hang Seng		0.001	...	0.023	...	0.045	...	0.067	...	0.089	...	0.110
Nasdaq 100		0.006	...	0.027	...	0.048	...	0.070	...	0.091	...	0.112
Nikkei 225		0.005	...	0.027	...	0.048	...	0.069	...	0.090	...	0.110
S & P 500		-0.010	...	0.018	...	0.046	...	0.075	...	0.103	...	0.130
Average		0.001	...	0.024	...	0.046	...	0.067	...	0.091	...	0.112

In Table 3.6, the estimated AOLs are derived from testing model 3.2. Results are similar to what in Table 3.4. And Table 3.7 shows the average estimated AOL to θ ratios for each data set derived from model 3.2.

Table 3.7: Average Estimated AOL over θ (derived from model 3.2)

FTSE 100	0.100
Hang Seng	0.996
Nasdaq 100	1.112
Nikkei 225	1.096
S & P 500	0.903

[3]Notice that AOL is $\langle|\Delta x^{OS}|\rangle$

And similar results are obtained as the average estimated AOL over θ is between 0.903 and 1.112.

As mentioned above, the characteristic exponent alone cannot really tell the size of AOL. Therefore, combining the characteristic exponent $E_{x,OS}$ and the other parameter $C_{x,OS}$, we know that within the tested thresholds, the AOL is about the same size as its corresponding threshold by testing model 3.1. And this 1:1 relationship between $\langle|\Delta x^{OS}|\rangle$ and θ is also confirmed by the test results of 3.2. This means, on average, the Overshoot goes about the same size of the threshold, this makes a TM (Total Movement) twice the size of the threshold θ.

3.4.3 Characteristic Exponents among Different Stock Markets

From Table 3.1, we know that the estimated $E_{x,OS}$ lie within 0.885 and 0.998. However, if we exclude $E_{x,OS}$ for S & P, which is 0.998. We have $E_{x,OS}$ lie within 0.885 and 0.893. Therefore, not a big difference could be observed. The reason that $E_{x,OS}$ for S & P is excluded is because that c for S & P is not significantly different from 0. Therefore, it does not seem like the characteristic exponents could be used to distinguish different markets.

3.5 Conclusion

This chapter has introduced the Scaling law discovered by Müller et al. (1990), which states that the absolute price change of logarithmic prices follow a scaling law to the time on which they are measured. Inspired by Müller, Glattfelder et al. (2010a) introduced the Average Overshoot Length (AOL) Scaling Law, which is similar idea based on the concept of Directional Changes. Instead of the time interval, this AOL scaling law are relations between a two price changes. One is the threshold, the other one is the average price changes during the Overshoots.

Like Müller's scaling law, the AOL Scaling Law is also reported in the foreign exchange markets, and there is no existing literature testing the AOL Scaling Law in the stock markets. Therefore, this chapter has examined the AOL Scaling Law in the stock markets with 5 indices.

This chapter explicitly presented the way to test the AOL Scaling Law in the stock markets using the methods employed by Müller et al. (1990). We used 5 data sets representing five stock markets to test two hypotheses. They are FTSE 100, Hang Seng, Nasdaq 100, Nikkei 225 and S&P daily closing prices from 02/01/09 to 01/11/13. As a result, the following can be concluded.

First of all, from the test results of model 3.1 shown in Table 3.1 and calculated $C_{x,OS}$ shown in 3.2, the AOL Scaling Law does hold in the stock markets with the tested threshold. As $E_{x,OS}$ for all 5 indices are significantly different from 0 at 95% confidence level. Although c for S & P is not significantly from 0, $c = 0$ translates into $C_{x,OS} = 1$ for S & P.

Second, now that we know the characteristic exponent alone can not really tell the size of the AOL. Substituting parameters $E_{x,OS}$ and $C_{x,OS}$ with coefficients listed in Table 3.1 and 3.2, we can obtain the estimated AOLs. Subsequently, estimated AOL to θ ratio could be calculated, and they are listed in Table 3.5, which indicates the average Overshoot length is about the same size of its corresponding threshold θ, that is $\langle|\Delta x^{OS}|\rangle = 1.04 \cdot \theta$ on average. And this is also confirmed by a linear regression between $\langle|\Delta x^{OS}|\rangle$ and θ (model 3.2), which gives us $\langle|\Delta x^{OS}|\rangle = 1.02 \cdot \theta$.

Third, the characteristic exponents $E_{x,OS}$ is lying between 0.885 and 0.893, except that $E_{x,OS}$ for S & P is 0.998. And c for S & P is 0, which leads to $C_{x,OS} = 1$ for S & P. Therefore, no significant difference can be observed among them, as a result, it does not seem like the characteristic exponents could be used to distinguish different markets.

Chapter 4

Trading Strategy Built on Directional Changes

4.1 Introduction

This chapter is going to introduce two trading strategies – Trading Strategy 1 (TS1) & Trading Strategy 2 (TS2), which are built based on Directional Changes. TS1 is consisted with three rules. It opens a long position at a Directional Change confirmation point (DCC), and hold the position till the price either goes up by another $\theta\%$ or goes down by $\alpha\%$. In the former scenario, the strategy makes money. And in the later scenario the strategy loses money. Similar to TS1, TS2 also opens a position at an upward DCC, and hold the position till either the price goes up by $\beta\%$ or goes down by $\alpha\%$, where β is the median of Overshoot lengths. And among the tested data sets, medians are smaller than θ.

And the results using different defining arguments are show of TS1 and TS2 are going to be shown to see if they generate positive outcome (making money). The reason of using different arguments is to see if their performance could be changed by adjusting the arguments.

By looking into the distribution of Overshoot values. We also try to use medians instead of AOL. And medians, in the tested data, are less than thresholds.

As Rule 3 is the one makes money, and Rule 2 is the one controls losses, this chapter also introduces the Rule 3/Rule 2 ratio. And it is clear that the higher Rule 3/Rule 2 ratios could lead to higher returns of the trading strategies as the returns of TS1 or TS2 could be summation of all the money made by Rule 3 minus all the money lost by Rule 2 if the price is continuous. But when it is not continuous the price where the positions close are likely to be bigger (Rule 3) or less (Rule 2) than what we expect. Therefore, this could be a good measure of performance of the trading strategies, and by improve the ratio, the performance is also improved.

The correlations of the returns of the trading strategies and the Rule 3/Rule 2 are calculated. And correlations between the returns of the trading strategies and the overall price changes of each index is also obtained.

The remainder of the chapter is as follows: the second section introduces the concept of Overshoot value, the Trading Strategy 1 & 2. The third section explains the thoughts behind the experiments and the how the experiments is going to be set-up. The fourth section lists the results obtained from the experiments. While the fifth section interprets the results. Lastly, there is a conclusion section.

4.2 Methodology

4.2.1 OSV & OSV^{EXT}

This chapter is going to use the concept of Overshoot Value (OSV). An Overshoot value is a price change within the Overshoot divided by the threshold θ. It could be used to measure Overshoot lengths without considering the effect of θ, so that Overshoot lengths can be compared across different thresholds. So, OSV is defined as:

$$OSV_i^c = \frac{P_i^c - P_i^{DCC}}{P_i^{DCC} \cdot \theta}$$

where P_i^{DCC} is the price where the ith Directional Change is confirmed (ith DCC). P_i^c is the current price in ith Overshoot at time c, c could be any physical time between $t(P_i^{DCC})$ and t(P_i^{EXT+1}). And θ is the threshold used to looking for Directional Changes. OSV_i^c is the Overshoot value at physical time c within ith Overshoot. The OSV varies dynamically as the price changes within an Overshoot.

OSV reaches its maximum or minimum when the price reaches the next extremum point – EXT, where OSV^{EXT} is calculated. Therefore, ith OSV_i^{EXT} is shown as:

$$OSV_i^{EXT} = \frac{P_i^{EXT+1} - P_i^{DCC}}{P_i^{DCC} \cdot \theta}$$

where P_i^{DCC} is the price where the ith Directional Change is confirmed (ith DCC). P_i^{EXT+1} is the price at $i+1$ EXT. And θ is the threshold used to looking for Directional Changes. OSV_i^{EXT} is the Overshoot value at physical time ith EXT.

Notice that there could be multiple OSV in an Overshoot, and if the price series is continuous, there would be infinite OSVs in an Overshoot. However, there is always only one OSV^{EXT} in an Overshoot.

4.2.2 Trading Strategy 1

As mentioned in the previous chapter, Scaling Laws may provide insights of the underlying markets. In particular the AOL Scaling Law may provide information for Directional Changes based trading algorithms. And Trading Strategy 1 (TS1) would be one of the attempts.

Trading Strategy 1 (TS1) is built on DC, backed by the idea that on average the Overshoots are approximately equal to the threshold θ that defines that Directional Changes and Overshoots. TS1 is considered as a proof of concept (that DC based trading algorithms could generate positive profits), for the sake of simplicity, we make the strategy take only long positions in this chapter.

The strategy TS1 is consisted of three trading rules, one opening rule and two closing rules. The opening rule would be: opening a long position at an upward Directional Change Confirmation point (a DCC). When there is a position, no longer open another one. Then hold the position until one of the following (two closing rules) happens. First closing rule is: the price goes down by α; second closing rule is: the price goes up by another θ.

Trading Strategy 1 could be presented as:

$$TS1 \equiv (\theta, \alpha)$$

where θ is the threshold used to find the Directional Changes and Overshoots, α is a pre-set number to control the loss. In TS1, we make $\alpha < \theta$. Both α and θ are bigger than 0.

As shown above TS1 is defined by two arguments, θ and α. To better illustrate the rules, denote the current price as P^t, the ith extreme price as P_i^{EXT}, and the price at the ith DCC as P_i^{DCC}.

Therefore, the above three rules could be listed below:

Rule 1. When $\frac{P^t - P_i^{EXT}}{P_i^{EXT}} \geq \theta$, open a long position;

Rule 2. When $\frac{P^t - P_i^{DCC}}{P_i^{DCC}} \leq -\alpha$, close the position;

Rule 3. When $\frac{P^t - P_i^{DCC}}{P_i^{DCC}} \geq \theta$, close the position.

The first rule is the entry rule for TS1, that is opening a long position when there is an upward Directional Change confirmation. In other words, when the current price P^t is θ% higher than the price at an EXT (P_i^{EXT}), take a long position.

The second rule is a closing rule. If the situation expected in rule 3 does not happen before the next Directional Change, then it's the time to stop losing. As the price never go up by another θ, the strategy would hold the position till the price to go down by α. This is when the price P^t is α% lower than P_i^{DCC}

According to the AOL scaling law, on average the price is expected to increase after a DCC by at least another θ. The third rule assumes the exact situation that the price would go up by another θ. That is when P^t is θ% higher than P_i^{DCC}. Therefore, this rule would take the advantage and close the position if the price goes up another θ or more, as the price may not be continuous. This is a rule trying to make profits.

These three rules make sure that a position would be opened when there is a upward Directional Change confirmed. And this position would be closed either when the price goes up by another θ or more, or decrease by $-\alpha$. For example, if θ is set to 0.05 and α is set to 0.025. The strategy would open a long position when there is a upward 5% Directional Change confirmed. This position would be held until one of the follows happen: 1) the price goes down by 2.5% or more; 2) the price goes up by another 5% or more[1]. In this case the price either goes up by another 5% or it hits the 2.5% downward marker, as a result, either Rule 2 of 3 is going to be triggered

[1] Prices may not be continuous, the position would be closed at the closest price that makes the price change greater or equal to 5%

before the next upward Directional Change takes place.

4.2.3 Trading Strategy 2

The major difference between Trading Strategy 1 (TS1) and Trading Strategy 2 (TS2) is that TS1 always expect the mean value of the Overshoots – AOL. TS2 is going to use the median of Overshoot lengths.

The rules could be shown as:

$$TS_2 \equiv (\theta, \alpha, \beta)$$

where θ is the threshold used to find the Directional Changes and Overshoots, α and β are pre-set numbers to cut losses and make profits respectively. In TS2, we make $\alpha < \theta$, β is the median of the OSV^{EXT} with the chosen threshold – θ. α, β, θ are bigger than 0.

As shown above, TS2 is defined by three arguments: θ, α and β. θ is the threshold used to find Directional Changes. α is a parameter used to stop loss. θ and α are just like their counterparts in TS1. β, however is the argument to take profit.

TS2 is built similar to TS1, it is also consisted with 3 trading rules. They are 1) the opening rule: opening a long position at an upward Directional Change Confirmation point (a DCC); 2) first closing rule is: the price goes down by α; 3) second closing rule is: the price goes up by another β. When there is a long position, TS1, TS2 no longer take another long position. Then hold the position until one of the closing rules triggered.

Rule 1. When $\frac{P^t - P_i^{EXT}}{P_i^{EXT}} \geq \theta$, open a long position;

Rule 2. When $\frac{P^t - P_i^{DCC}}{P_i^{DCC}} \leq -\alpha$, close the position;

Rule 3. When $\frac{P^t - P_i^{DCC}}{P_i^{DCC}} \geq \beta$, close the position.

These rules allow the strategy to open a long position when a downward Directional Change event is confirmed, it expects the median of Overshoots length – β, with a certain tolerance (α) of down-going of the price.

By design, Trading Strategy 2 opens a long position at a upward Directional Change confirmation point. And if the price goes down and reaches the cut-losing point where the price P^t is $\beta\%$ lower than P_i^{DCC}, Rule 2 will be triggered, the long position is closed. Or it does not close the position till the price P^t is $\beta\%$ higher than the price at the DCC (P_i^{DCC}), similarly.

4.3 Experiment Setup

4.3.1 Data

In this chapter, 5 sets of stock indices used in the previous chapter to test the AOL scaling law are going to be employed to test the Trading Strategies. They are the FTSE 100, Hang Seng, Nasdaq 100, Nikkei 225 and S&P 500. These data are daily closing indices (treated as prices) starting from 02/01/09 to 01/11/13. Thresholds used to calculate medians of OSV^{EXT} are 0.05 and 0.1.

To test the Trading Strategies, there are 4 sets and 5 sets of arguments are used for TS1 and TS2 separately:

Table 4.1: Tested Arguments of TS1

θ	0.05		0.1	
α	0.02	0.025	0.02	0.05

Table 4.2: Tested Arguments of TS2

θ	0.05		0.1		
α	0.02	0.025	0.02	0.05	0.05
β	0.0361	0.0361	0.0903	0.06	0.0903

4.3.2 Evaluating the Trading Strategies

First of all, there medians of OSV_{EXT} with threshold 0.05 and 0.1 are calculated before testing TS2, so that β could be decided accordingly. Average daily return of each of the data sets tested are all roughly around 1% (0.01). Therefore, we choose the arbitrary threshold 0.05 and 0.1 so that there are a reasonable number of transactions take place.

The evaluation on the trading strategy would essentially be calculating the rate of returns, as the main goal of this chapter is to judge if the trading strategies are able to make positive profits. To evaluate the trading rules, the most important measure is the rate of return. The rate of return is the ratio of profits or losses on an investment relative to the amount of money invested. It is all known that the rate of return is widely used in the financial analysis. It is one of the simple but most direct ways to measure the profits.

To test the effectiveness of the trading strategy, testing a trading strategy is to see whether it works, that is, produces a profit (Pardo 2008). The most and foremost goal of testing a trading strategy is to make sure it has a profit potential. Therefore, the trading strategy TS1 and TS2 is tested using the arguments listed in Table 4.1 and 4.2 respectively.

In addition, we also calculate the ratio of Rule 3 to Rule 2 ratio. This ratios divide the times that Rule 3 is triggered by the times that Rule 2 is triggered. As the Rule 3 in both TS1 and TS2 is the rule makes profits and the rule 2 is the one to cut losses. According to the definition of both trading strategies there is always either a Rule 2 or a Rule 3 following Rule 1. This means that a long position is always closed by a money-making Rule 3 or a stop-losing Rule 2. By calculating the Rule 3 to Rule 3 ratio, we hope that there are insights obtained from it to evaluate the trading strategies.

Lastly, in order to find out if the performance of the trading strategies is affected by the overall price change for the indices, the correlations between the returns obtained from the trading strategies and the overall price changes of the markets and the the correlations between the returns obtained from the trading strategies and the Rule 3 to Rule 2 ratios are calculated respectively. And a comparison between these two correlations is going to be made.

4.4 Experiment Results

4.4.1 Medians of OSV at EXTs

Table 4.3 lists the medians of OSV^{EXT}, and their equivalent medians of Overshoot lengths in a percentage form.

Table 4.3: Medians of OSV^{EXT} and Medians of Overshoot Lengths

	Median of OSV^{EXT}		Median of OS^{EXT}	
	$\theta = 0.05$	$\theta = 0.1$	$\theta = 0.05$	$\theta = 0.1$
FTSE 100	0.687	0.6193	3.44%	6.19%
Hang Seng	0.8055	0.7899	4.03%	7.90%
Nasdaq 100	0.9048	0.5977	4.52%	5.98%
Nikkei 225	0.8017	1.0305	4.01%	10.31%
S & P 500	0.4149	1.476	2.07%	14.76%
Average	0.72278	0.90268	3.61%	9.03%

The first row divides the table into two. The left half are the medians of OSV^{EXT} and the right half are the medians of Overshoot Lengths shown as OS^{EXT} in the table. The first column are the data sets going to be tested. The second the column are the medians of the OSV^{EXT} with threshold 0.05 for each data set, and the average across all data sets in listed at bottom. The third column is similar to the second except the threshold is 0.1. The fourth column are the medians of Overshoot lengths with threshold 0.05 shown in percentage. And the bottom is the average. Fifth column is similar to the fourth except the threshold is 0.1.

4.4.2 Trading Strategy 1

Table 4.4 shows the rate of returns of Trading Strategy 1 with the arguments listed in Table 4.1.

Table 4.4: Rate of Returns of TS1

	Rate of Return			
	$\theta = 0.05$		$\theta = 0.1$	
	$\alpha = 0.02$	$\alpha = 0.025$	$\alpha = 0.02$	$\alpha = 0.05$
FTSE 100	6.91%	2.49%	-14.93%	22.26%
Hang Seng	25.60%	15.11%	21.14%	24.54%
Nasdaq 100	52.36%	62.60%	3.94%	5.95%
Nikkei 225	-2.27%	-9.39%	14.80%	6.94%
S & P 500	3.62%	16.42%	4.58%	-0.22%
Average	17.24%	17.45%	5.91%	11.89%

The first column are the data sets tested with TS1. Apart from the first column, the left half is tested with threshold $\theta = 0.05$, the right half is tested with $\theta = 0.1$. The number 6.91% in second column is rate of return of TS1 using threshold $\theta = 0.05$ and $\alpha = 0.02$.

4.4.3 Trading Strategy 2

Similar to the previous TS1, Table 4.5 shows the rate of returns of Trading Strategy 2 with the arguments listed in Table 4.2.

Also similar to the previous Trading Strategy, Table 4.5 shows the test results using TS2 with threshold 0.05 and 0.1, and $\alpha = 0.02$, $\alpha = 0.025$. However, the difference

Table 4.5: Rate of Returns of TS2

	Rate of Return				
	$\theta = 0.05$		$\theta = 0.1$		
	$\alpha = 0.02$ $\beta = 0.0361$	$\alpha = 0.025$ $\beta = 0.0361$	$\alpha = 0.02$ $\beta = 0.0903$	$\alpha = 0.05$ $\beta = 0.06$	$\alpha = 0.05$ $\beta = 0.0903$
FTSE 100	1.66%	-4.30%	-14.93%	7.84%	18.88%
Hang Seng	23.58%	14.51%	20.08%	17.97%	23.45%
Nasdaq 100	44.94%	54.13%	3.47%	-2.90%	4.79%
Nikkei 225	0.90%	-5.68%	13.45%	17.52%	22.47%
S & P 500	1.33%	11.07%	3.33%	-1.41%	-1.41%
Average	14.48%	13.95%	5.08%	7.81%	13.64%

is that there is one more argument β, they are listed in row 4, right below values of α. For example, the number 1.66% in the 5th row second column is the result of TS2 using threshold 0.05, $\alpha = 0.02$ and $\beta = 0.0361$ with data FTSE 100.

4.4.4 Rule 3 to Rule 2 Ratio & Overall Return of the Data Sets

Table 3.7 shows the overall returns for each data set tested.

Table 4.6: Overall Returns of Each Tested Data Set

FTSE 100	47.63%
Hang Seng	54.56%
Nasdaq 100	167.45%
Nikkei 225	57.04%
S & P 500	89.06%
Average	83.15%

Table 4.7 and 4.8 are tables showing how many times Rule 2 and Rule 3 are triggered in TS1 with threshold 0.05 and threshold 0.1 respectively. In each table α is set to either 0.02 or half of the threshold.

Similar to the previous two tables, Table 4.9 and 4.10 are the TS2 equivalents. And the tested threshold are 0.05 and 0.1 as well. However, what different is that the TS expects the median instead of the mean of Overshoot lengths. And in Table 4.10,

Table 4.7: Rules Triggered and Rule 3 to Rule 2 Ratios for TS1 with Threshold = 0.05

	$\theta = 0.05$					
	$\alpha = 0.02$			$\alpha = 0.025$		
	Rule2	Rule3	Rule3/2	Rule2	Rule3	Rule3/2
FTSE 100	13	8	0.615385	12	9	0.75
Hang Seng	14	12	0.857143	14	12	0.857143
Nasdaq 100	8	12	1.5	7	13	1.857143
Nikkei 225	17	9	0.529412	17	9	0.529412
S & P 500	13	8	0.615385	10	10	1
Average	13	9.8	0.823465	12	10.6	0.998739

Table 4.8: Rules Triggered and Rule 3 to Rule 2 Ratios for TS1 with Threshold = 0.1

	$\theta = 0.1$					
	$\alpha = 0.02$			$\alpha = 0.05$		
	Rule2	Rule3	Rule3/2	Rule2	Rule3	Rule3/2
FTSE 100	6	0	0	2	3	1.5
Hang Seng	4	3	0.75	3	4	1.333333
Nasdaq 100	5	2	0.4	4	3	0.75
Nikkei 225	5	3	0.6	4	3	0.75
S & P 500	2	1	0.5	2	1	0.5
Average	4.4	1.8	0.45	3	2.8	0.966667

there is an additional β tested which is the a number approximately equal to the median of Overshoot lengths for FTSE 100.

Table 4.9: Rules Triggered and Rule 3 to Rule 2 Ratios for TS2 with Threshold = 0.05

	$\theta = 0.05$					
	$\alpha = 0.02$			$\alpha = 0.025$		
	$\beta = 0.0361$			$\beta = 0.0361$		
	Rule2	Rule3	Rule3/2	Rule2	Rule3	Rule3/2
FTSE 100	12	9	0.75	11	10	0.90909091
Hang Seng	13	14	1.076923077	13	14	1.07692308
Nasdaq 100	6	14	2.333333333	5	15	3
Nikkei 225	16	11	0.6875	16	11	0.6875
S & P 500	12	9	0.75	9	11	1.22222222
Average	11.8	11.4	1.119551282	10.8	12.2	1.37914724

Table 4.10: Rules Triggered and Rule 3 to Rule 2 Ratios for TS2 with Threshold = 0.1

	$\theta = 0.1$								
	$\alpha = 0.02$			$\alpha = 0.05$			$\alpha = 0.05$		
	$\beta = 0.0903$			$\beta = 0.06$			$\beta = 0.0903$		
	Rule2	Rule3	Rule3/2	Rule2	Rule3	Rule3/2	Rule2	Rule3	Rule3/2
FTSE 100	6	0	0	2	3	1.5	2	3	1.5
Hang Seng	4	3	0.75	3	5	1.666667	3	4	1.333333
Nasdaq 100	5	2	0.4	4	3	0.75	4	3	0.75
Nikkei 225	5	3	0.6	4	3	0.75	3	4	1.333333
S & P 500	2	1	0.5	2	1	0.5	2	1	0.5
Average	4.4	1.8	0.45	3	3	1.033333	2.8	3	1.083333

Lastly, there are the tables of correlations between the returns obtained from the trading strategies (TS1 and TS2) and the overall returns listed in Table 4.6, and the correlations between the returns of the strategies and the Rule 3 to Rule 2 ratio. They are Table 4.11 and 4.12. And in the tables, Corr. R stands for the correlations between the overall price change of the markets and the returns generated from the trading strategies. Similarly, Corr. 3/2 means the correlations between the Rule 3 to Rule 2 ratios and the returns generated from the trading strategies.

Table 4.11: Correlation of TS1

	$\theta = 0.05$		$\theta = 0.1$	
	$\alpha = 0.02$	$\alpha = 0.025$	$\alpha = 0.02$	$\alpha = 0.05$
Corr. R	0.795141758	0.937225276	-0.098249747	-0.57295
Corr. 3/2	0.983718169	0.994143918	0.970436128	0.93395

Table 4.12: Correlation of TS2

	$\theta = 0.05$		$\theta = 0.1$		
	$\alpha = 0.02$	$\alpha = 0.025$	$\alpha = 0.02$	$\alpha = 0.05$	$\alpha = 0.05$
	$\beta = 0.0361$	$\beta = 0.0361$	$\beta = 0.0903$	$\beta = 0.06$	$\beta = 0.0903$
Corr. R	0.778781	0.937533	-0.09468	-0.78433	-0.68685
Corr. 3/2	0.956535	0.976599	0.96525	0.81838	0.916764

4.5 Interpretation

The previous sub-section has listed two trading strategies based on the AOL Scaling Law introduced in the previous Chapter – TS1 and TS2. As proof of concept, TS1 and TS2 take only long position. This way, the comparison of the returns made by the trading strategies and the overall price change of the markets could be conducted.

For example, if the long-position-only strategies' return are highly correlated to the overall price change, then maybe the strategies are not really working but simply takes profit as the markets' prices go up. Otherwise, we may think the strategies are working. And when it took short positions, even the returns and the overall price changes are not highly correlated, it is harder to tell whether it is the rising market contributing to the trading strategies profit or they truly works, as potentially a working trading strategy makes profits either way.

There is a difference between the trading strategies. TS1 expects the Overshoot to approximately equal to the threshold – θ. Instead of expecting the mean of Overshoot lengths, TS2 takes medians of OSV^{EXT} as its profit making point. In other words, TS2 closes a position when the price rises to a point where the Overshoot reach the median while TS1 would take the same action at the average Overshoot length.

The main goal of this chapter is to prove that the trading strategies based on the Scaling Law discovered by Glattfelder et al. (2010a) are able to generate positive profits. Therefore the rate of return was used to evaluate the success of the trading strategies.

As could be seen in Table 4.4, in most scenarios, TS1 could make a profit except using $\theta = 0.05$ and $\alpha = 0.02$ with Nikkei 225, $\theta = 0.05$ and $\alpha = 0.025$ with Nikkei 225, $\theta = 0.1$ and $\alpha = 0.02$ with FTSE 100, $\theta = 0.1$ and $\alpha = 0.05$ with S & P. In these cases, TS1 loses 2.27%, 9.39%, 14.93% and 0.22%. While Nasdaq 100 makes biggest profits as 52.36% and 62.60%, with $\alpha = 0.02$ and $\alpha = 0.025$. And there are 16 out 20 (80%) θ and α combinations make profits.

As for TS2, as shown in Table 4.5, there are six cases that it loses money, hence the other 19 out 25 (76%) make profits. Among them, FTSE 100 and Nikkei 225 with $\theta = 0.05$, $\alpha = 0.025$, $\beta = 0.0361$; with $\theta = 0.05$, $\alpha = 0.025$, $\beta = 0.0361$; FTSE 100 with $\theta = 0.1$, $\alpha = 0.02$, $\beta = 0.0903$; Nasdaq 100 and S & P with $\theta = 0.1$, $\alpha = 0.05$, $\beta = 0.06$; S & P with $\theta = 0.1$, $\alpha = 0.05$, $\beta = 0.0903$ loses money. The rest all make profits. And similar to TS1, Nasdaq 100 at $\theta = 5\%$ makes most profits.

If we take the Rule 3 to Rule 2 ratios into consideration (as shown in Table 4.7 and 4.8), it is not hard to find that, in general, the higher the ratio, the higher the returns, and vise versa. For example, Nikkei at $\theta = 0.05$ with $\alpha = 0.02$ has 13 Rule 2 triggered while there are 9 Rule 3 triggered. As the Rule 3 is the profit-making rule, and Rule 2 is the stop-losing rule.

For both trading strategies, α uses two not very different numbers at $\theta = 0.05$. They are 0.02 and 0.025, but with this smell change, there are different results in Rule 3/Rule 2 ratio. And consequently there are different results in the returns. In fact, only S & P tested using TS1 give the same Rule 3/Rule 2 ratio regardless to the small amount change α. The same applies to Nikkei 225 using TS2. This potentially means that by adjusting α, the return of the trading strategies could be improved.

Another noticeable point is that using 0.1 θ give far fewer transactions than using a 0.05 θ. This indicates that if the strategies were to be used practically, θ needs to be chosen within a certain range with the overall trading time considered, so that there are enough transactions to take place. Otherwise, a trading strategy might be profitable over a long term, but may lose money simply due to small number of trades it could make in a short time period.

Using the median $-\ \beta$ is going to make less profits than using the mean value of

Overshoot θ only with two exceptions in the listed results (Table 4.4 and 4.5). And as shown in Table 4.2, the medians are less than θ.In the results above (Table 4.7 to 4.10), the number of trades (Rule 2 + Rule 3) are actually very similar. However, the Rule 3 in TS2 closes the position earlier than the Rule 3 in TS1. And this might be the reason that the TS2 is making less money than TS1 when have other arguments the same. Therefore, if we want to improve the trading strategies performance by adjusting the Rule 3's closing point, maybe using medians instead of mean of Overshoot lengths is not the best solution.

On one hand, as can be seen in Table 4.11 and 4.12, the correlations of the returns of the trading strategies to the overall price change are not uniformly positive. This could mean that the trading strategies are not able to catch the rising price of the underlying asset and make a profit with certain argument sets. However if we look closer, the correlations are between 0.78 and 0.94 with $\theta = 0.05$. And with $\theta = 0.1$, the results are either close to 0 or close to negative 0.5. However, this might due to the fact that with $\theta = 0.1$ there are only few transactions (maximum is 8) take place.

On the other hand, the returns of TS1 and TS2 are hight correlated with the Rule3/Rule2 ratio. If the number of triggered Rule 3 is smaller than the number of triggered Rule 2, it is expected to have a smaller return than those opposite. And if the price is continues the returns would simply be: $a^{R3} \cdot \theta - b^{R2} \cdot \alpha$ for TS1 and $a^{R3} \cdot \beta - b^{R2} \cdot \alpha$ for TS2, where a^{R3} is the count of how many times Rule 3 is triggered, and similarly b^{R2} is the count of how many times Rule 2 is triggered. $R3$ and $R2$ stands for Rule 3 and Rule 2 respectively. α, β and θ are the arguments defines the trading strategies.

As a result, it is clear that the trading strategies are able to make profits with certain conditions. And by adjusting the input of the arguments, the performance of the trading strategies could be modified. And if we want to achieve higher performance, we should always improve the Rule 3/Rule 2 ratio by adjusting the arguments, choosing fitter markets or by same other yet-to-be-found ways.

4.6 Conclusion

This chapter has introduced the two trading strategies – TS1 & TS2, which are built based on Directional Changes. TS1 is consisted with three rules. It opens a long position at a DCC, and hold the position till the price either goes up by another $\theta\%$ or goes down by $\alpha\%$. In the former scenario, the strategy makes money. And in the later scenario the strategy loses money. Similar to TS1, TS2 also opens a position at an upward DCC, and hold the position till either the price goes up by $\beta\%$ or goes down by $\alpha\%$, where β is the median of Overshoot lengths. And among the tested data sets, medians are smaller than θ.

And the results show that in most cases TS1 and TS2 are able to generate positive outcome (making money). And their performance could be changed by adjusting the arguments we used to define both strategies.

The thoughts behind using medians instead of AOL is expecting more positive closing Rules to be triggered. However using the median, in general, does not seems to provide not only money making closes of the positions but also stop-losing ones. That is, although using the median does not give more transactions. Therefore, with a smaller money-making closing point, the trading strategies tend to make less money.

As Rule 3 is the one makes money, and Rule 2 is the one controls losses, this chapter also introduces the Rule 3/Rule 2 ratio. And it is clear that the higher Rule 3/Rule 2 ratios could lead to higher returns of the trading strategies. Therefore, this could be a good measure of performance of the trading strategies, and by improve the ratio, the performance is also improved.

The correlations of the returns of the trading strategies and the Rule 3/Rule 2 are calculated. And it does seem like they are highly correlated as the returns could simply be a summation of all the money made by Rule 3 minus all the money lost by Rule 2. Correlations between the returns of the trading strategies and the overall price changes of each index is also obtained. And it seem like the with 0.05 threshold, the trading strategies are correlated with the overall change of the price, while they are not correlated when the threshold is 0.1.

Chapter 5

Directional Changes Indicators

5.1 Introduction

This chapter is going to sub Directional Changes to examine how the numbers of sub DCs change as the price get closer to EXTs. As EXTs are the turning points of a trend. If the numbers of sub DCs follow a certain pattern, then we can find a way to know if the price is getting to an EXT. And potentially this could contribute to the trading strategies built on Directional Changes.

The remainder of this chapter is: second, the methodology section which introduces the methods and terms used to obtain and test the sub DCs. Third section is where the results are presented. Fourth section is the interpretation of the results. And finally is the conclusion.

5.2 Methodology

5.2.1 Sub Directional Changes

In this chapter we would like to examine the properties of Directional Changes, particularly when the price moves towards an EXT. Like DCCs are the ends of Directional Changes, EXTs are the ends of Overshoots and Total Movements. When the price reaches a DCC, the price is expect to go further along with its trend. Unlike the DCCs, trends after EXTs are expected to change by definition. If somehow the price would exhibit some property before it reaches an EXT, it would potentially contribute to any trading strategies that built on Direction Changes.

The way we approach this is to observe the behaviour of DCCs on a smaller scale.

In order to make description in a clearer manner, with a time series at a certain Directional Change threshold θ, we define:

- t_i^{EXT} is the time at which there is the ith extremum (EXT), where i=(1,2,...,n)[1];
- t_i^{DCC} is the time at which ith Directional Change is confirmed (DCC), where i=(1,2,...,n);
- $\Delta t_i^{DC} = t_i^{EXT} - t_i^{DCC}$, is the time difference between the ith DCC and the EXT;
- $\tau_i^1 = t_{i+1}^{EXT} - \Delta t_i^{DC}$, is the time point which is Δt_i^{DC} before t_{i+1}^{EXT};
- $\tau_i^2 = t_{i+1}^{EXT} - \frac{\Delta t_i^{DC}}{2}$, is the time point which is half of Δt_i^{DC} before t_{i+1}^{EXT};
- With $\frac{\theta}{d}$ ($d \in \mathbb{N}$ and $d > 1$), we can have Directional Changes on a smaller scale, or we call them sub Directional Changes;
- define η_i^1 the number of sub Directional Changes in $(\tau_i^1, \tau_i^2]$;

[1] Assume there are n Directional Changes, and EXT_{n+1} is not known

- similarly η_i^2 denotes the number of sub Directional Changes in $(\tau_i^2, t_{i+1}^{EXT}]$;

- for the sake of consistency, also denote t_{i+1}^{EXT} as τ_i^3, therefore, η_i^2 is the number of small directional changes in $(\tau_i^2, \tau_i^3]$.

First of all, with any time series the time, we find the Directional Changes with threshold θ. Time intervals between ith EXT (t_i^{EXT}) and ith DCC (t_i^{DCC}) are calculated. These time intervals are not equal to each other, for the time it takes to confirm a Directional Change varies. As a result, the a series of time intervals could be obtained. As price change between ith EXT (EXT_i) and ith DCC (DCC_i) is the ith Directional Change event, therefore we use DC as the superscript to denote that this is the time interval of ith DC. So the time intervals could be denoted as: Δt_i^{DC}.

Consequently, we can count from t_{i+1}^{EXT} backwards by ith time interval (Δt_i^{DC}) we can get time point – τ_i^1. Similarly count backwards by half of the ith interval ($\frac{\Delta t_i^{DC}}{2}$)we can get two time point – τ_i^2. And if we denote t_{i+1}^{EXT} as τ_i^3 we can obtain two time periods – $(\tau_i^1, \tau_i^2]$ and $(\tau_i^2, \tau_i^3]$. These two time periods are two subsequent periods leads to the EXT_{t+1}, and the two time periods equal to each other.

Lastly, we would get sub Directional Changes, and count the numbers of them in both $(\tau_i^1, \tau_i^2]$ and $(\tau_i^2, \tau_i^3]$. Once the two numbers of sub Directional Changes in each of them are obtained we could run a linear regression to see whether the number of sub Directional Changes (η_i^2) in $(\tau_i^2, \tau_i^3]$ is depended on the number of sub Directional Changes (η_i^1) in $(\tau_i^1, \tau_i^2]$. And the empirical model could be:

$$\eta^2 = \beta_0 + \beta_1 \eta^1$$

where parameters β_0 and β_1 are to be determined by the linear regression.

5.2.2 Data

Although this thesis was aiming to examine the potential of Directional Changes in the stock markets. The stock markets data at hand was only daily prices. The nature of experiments in this chapter need a large number of sub Directional Changes in each Directional Change event. Therefore, it is better to have high frequency data over a long period of time.

As a result, the data sets tested are four high frequency minute-by-minute foreign exchange data, dated from 1st January 2009 to 9th October 2014. The foreign exchange pair are AUD/USD, GBP/USD, USD/CHF and USD/JPY.

The observations are made under threshold (θ): 0.02. And the sub Directional Changes are looked with $\frac{\theta}{20}$. These numbers are chosen so that there are both statistically enough Directional Changes and sub Directional Changes in each Directional Change.

Table 5.1 lists the basic information about the tested data sets, such as the number of prices, the number of DCs and the number of sub DCs.

Table 5.1: Data Size and Number of DCs and Sub DCs

	θ=0.02, d=20		
	Price	DC	subDC
AUD/USD	1048575	136	27598
GBP/USD	1048575	80	20572
USD/CHF	1048575	65	18631
USD/JPY	1048575	69	17683

5.3 Results

5.3.1 Linear Relations between η_i^1 and η_i^2

As it is not very practical to list such large numbers of sub Directional Changes (η_i^1 and η_i^2) in time periods $(\tau_i^1, \tau_i^2]$ and $(\tau_i^2, \tau_i^3]$ for each data set. This section is going to list the estimated linear relations between η_i^1 and η_i^2 for each data set.

AUD/USD:

$$\eta^2 = \underset{(2.3E-08)}{26.70} + \underset{(0.0002)}{0.42}\,\eta^1$$

where the P-values are listed in the brackets. The adjusted $R^2 = 0.095$.

GBP/USD:

$$\eta^2 = \underset{(2.6E-07)}{27.39} + \underset{(8.05E-08)}{0.60}\,\eta^1$$

where the P-values are listed in the brackets. The adjusted $R^2 = 0.304$.

USD/CHF:

$$\eta^2 = \underset{(7.6E-08)}{34.63} + \underset{(0.0003)}{0.40}\,\eta^1$$

where the P-values are listed in the brackets. The adjusted $R^2 = 0.181$.

USD/JPY:

$$\eta^2 = \underset{(9.5E-05)}{20.09} + \underset{(1.89E-07)}{0.53}\,\eta^1$$

where the P-values are listed in the brackets. The adjusted $R^2 = 0.329$.

5.3.2 Means and Medians of η_i^1 and η_i^2

Beyond the linear regression, the means and medians of η_i^1 and η_i^2 for each data set are listed below:

Table 5.2: Mean and Median of η_i^1 and η_i^2 at $\theta = 0.02$

		η_i^1	η_i^2
AUD/USD	Mean	29.2963	38.91852
	Median	21	27
GBP/USD	Mean	35.39241	48.46835
	Median	30	36
USD/CHF	Mean	38.625	50.23438
	Median	30.5	43
USD/JPY	Mean	37.51471	39.79412
	Median	29	29

5.4 Interpretation

From the linear relations of η_i^1 and η_i^2 listed in 5.3.1, We know that although the P-values suggest β_0 and β_1 are not likely to be 0, the adjusted R^2 tells us the estimated line is a very poor estimation to the underlying relations between η_i^1 and η_i^2. Therefore, it is very unlikely that there is a linear relationship between η_i^1 and η_i^2.

This means that as the price getting closer to a EXT, the numbers of sub Directional Changes measured in periods $(\tau_i^1, \tau_i^2]$ and $(\tau_i^2, \tau_i^3]$ are not seem to follow a noticeable pattern.

However, if we look at Table 5.2, it is noticeable that the means of η_i^2 are greater than η_i^1, and they are all roughly 30% to 37% greater than the means of η_i^1 except for USD/JPY which mean of η_i^2 is 6% greater than the mean of η_i^1. Similar properties also apply to medians. But it is not clear if this is a general phenomena across different thresholds, as this is only obtained with $\theta = 0.02$.

The reason that Δt_i^{DC} is chosen to obtain $(\tau_i^1, \tau_i^2]$ and $(\tau_i^2, \tau_i^3]$ is even if there are 0 Overshoots (OSV=0), the calculations of η_i^1 and η_i^2 are still able to process. As the calculation count backwards from t_{i+1}^{EXT}, if there is a 0 Overshoot, τ_i^1 would be t_i^{EXT} and τ_i^3 would be t_i^{DCC}, τ_i^2 is always the middle point between them.

This chapter examined how the sub Directional Changes behave as the price get closer to the EXTs. Although we find that η_i^1 and η_i^2 do not follow a noticeable pattern, this provides a new insight on examining the potential of Directional Change – the use of sub Directional Changes. And if we define $(\tau_i^1, \tau_i^2]$ and $(\tau_i^2, \tau_i^3]$ differently, there might be patterns to be found. for example excluding the 0 Overshoots and measure the sub Directional Changes in the earlier half and later half in Overshoots.

5.5 Conclusion

This chapter has conducted an experiments on testing the relations between two sub Directional Changes sets η_i^1 and η_i^2 in periods $(\tau_i^1, \tau_i^2]$ and $(\tau_i^2, \tau_i^3]$ respectively. The results of the experiments tell us that η_i^1 and η_i^2 do not seemingly follow a pattern, and there is no linear relations among them.

However, the means and medians of η_i^2 are greater than those of η_i^1 with tested data sets at $\theta = 0.02$, $d = 20$. A more universal experiment might be necessary to tell is this is a general property of η_i^1 and η_i^2.

Chapter 6

Conclusion

6.1 Summary

Directional Changes as a way of summarising information from complex systems. For financial markets are also seen as complex systems, this thesis aims to explore the potential of Directional Changes mainly in three ways. They are the AOL scaling laws the Trading strategies, and the sub Directional Changes.

First of all, the Average Overshoot Scaling law has been tested in the stock markets, which was not exist in current literature. With the results obtained in Chapter 3, we now understand that AOL Scaling Law does hold in the stock markets as well, which means that the average Overshoot length has a scaling law relationship with the threshold θ. And we also find out that the average Overshoot length is approximately the same as θ, which the same property was reported in the foreign exchange markets. And this approximation is confirmed by a linear regression as well. Thirdly, we also found out that the characteristic exponent $E_{x,OS}$ could not be used to distinguish different markets, as the relations of AOL and θ are defined by both $E_{x,OS}$ and $C_{x,OS}$ in the AOL Scaling Law.

Second, in Chapter 4, there were 2 trading strategies based on the AOL Scaling Law introduced. They are TS1 and TS2, both contain 3 trading rules. TS1 is defined by argument θ and α, where θ is the Directional Change threshold and α is an argument used to control losses. TS2 is built similarly and defined by θ, α and β, where β is the median of Overshoot lengths rather the mean.

In this chapter both TS1 and TS2 are able to generate positive outcomes (are able to make money) in most cases, and by changing the inputs of the arguments, the performance is able to be adjusted. Later in the chapter, the correlations of the Reuters generated by the strategies and the overall price change are calculated. And it suggest that only with certain combinations of arguments the returns are somehow correlated to the overall price change. Similarly, the correlations of the Reuters generated by the strategies and the Rule 3/Rule 2 ratios are also calculated. And they are highly correlated. As a results if these strategies are to perform well, the right combination of argument inputs are necessary.

Thirdly, we conducted a set of experiments on sub Directional Changes with high frequency foreign exchange markets' data. Although this was meant to be done with stock markets' data, there was no data suffice the task. For we statistically need enough both Directional Changes and sub Directional Changes. As a result, we find out that when the price get closer to an EXT, the number of sub Directional Changes measured as η_i^1 and η_i^2 do not seem to follow a noticeable pattern.

6.2 Contributions

First of all, this thesis has explained how an Average Overshoot Scaling Law is going to be tested explicitly. And this method could expanded to test other scaling laws.

Second, we have tested the AOL Scaling Law with stock markets, so that now we know the AOL Scaling Law stands not only in foreign exchange markets but also could be found in the stock markets.

Third, we have also found out the the average Overshoot lengths are approximately equal to the Directional Change threshold θ across five indices with multiple thresholds. And this was double checked by a linear regression between θ and AOL.

Fourth, the lengths of Overshoots are defined with both $E_{x,OS}$ and $C_{x,OS}$. $E_{x,OS}$ does not exhibit big difference among different markets.

Fifth, this thesis proposes two trading strategies built on Directional Changes as proof of concept – TS1 and TS2. Both of them are able to make profits in most tested scenarios. And we now understand that with different inputs to the arguments that define the trading strategies, the performance of them is able to be adjusted. And this could be a optimisation problem.

Sixth, we found out that when the price is getting to an EXT, the sub Directional Changes measured as η_i^1 and η_i^2 do not seem to present a certain pattern.

Seventh, although it is not tested comprehensively, with the tested $\theta = 0.02$ and $d = 2$, we know that the medians and means of η_i^2 are greater than η_i^1's.

6.3 Limitations

Although The Average Overshoot Length Scaling Law is tested with multiple thresholds across five different markets on the global. The tests have not been tested with individual stocks' data.

The trading strategies TS1 and TS2 are proof of concept. They do make money in most tested scenarios, but obviously, they could be made more sophisticated. For starter, taking short positions.

Limited by the data at hand, the tests of sub Directional Changes were not able to be tested in the stock markets. Even with the foreign exchange markets' data. Only limited thresholds and d values are able to generate sufficient number of Directional Changes as well as sub Directional Changes.

6.4 Future Work

First of all, the AOL Scaling Laws could be tested with individual stock's data to see whether this AOL Scaling Law stands for each individual stock. And if it stands, we can further examine if the AOL is approximately equal to the threshold for each individual stock. What is more, although the characteristic exponent Ex, OS does not seem to be able to tell the difference between different markets. This might not be the case for individual stocks.

There is a lot to explore for trading strategies built on the AOL Scaling Law, using median instead of mean was only one of the attempt. And both TS1 and TS2 are trend following trading strategies. It is still not clear what would it be if we take long positions at an estimated downward EXT, and close the position at an upward DCC.

Last but no least, this thesis only tried one way on exploring the potential of sub Directional Changes, and only one way of calculating the numbers of sub Directional Changes. There are far more ways to define the time periods right before EXTs left to be explored.

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