# A Formal Definition of Directional Change

An updated version of this document has been published as Appendix A, J. Chen & E.P.K.Tsang, *Detecting Regime Change in Computational Finance*, Data Science, Machine Learning and Algorithmic Trading, CRC Press, September 2020; This document replaces E.P.K.Tsang, *Directional Changes, Definitions*, CCFEA Working Paper WP050-10, University of Essex 2010

This appendix gives a formal definition of Directional Change. As the definitions are mutually recursive, some terms are used before they are well-defined. Directional Change takes an event-based view of the market. This Appendix also relates this event-based view to the point-based view, which may be more familiar to some readers.

## A.1. Definitions of Directional Change Events

At any time, the market is either in a **Downward Run** or an **Upward Run**. A Downward Run comprises a **Downturn Event** and a subsequent **Downward Overshoot Event**. An Upward Run comprises an **Upturn Event** followed by a subsequent **Upward Overshoot Event**.

In a Downward Run, a **Last Low** is constantly updated to the minimum of (a) the current price and (b) the Last Low. In an Upward Run, a **Last High** is constantly updated to the maximum of (a) the current price and (b) the Last High.

Given a **Threshold** $\theta$ (a percentage), a Downward Run is confirmed to have ended when a price P is found to be higher than the Last Low by $\theta$. The event of price change from the Last Low to P is called the **Upward Directional Change Event**. The Last Low (which is now confirmed to be a **Trough**) terminates the preceding Downward Run and starts the next Upward Run. P is called the **Upward DC Confirmation Point** for the Upward Run.

Given a Threshold $\theta$, an Upward Run is confirmed to have ended when a price P' is found to be lower than the Last High by $\theta$. The event of the price change from the Last High to P' is called the **Downward Directional Change Event**. The Last High (which is now confirmed to be a **Peak**) terminates the preceding Upward Run and starts the next Downward Run. P' is called the **Downward DC Confirmation Point** for the Down Run.

The event of price change from the Downward DC Confirmation point to the Trough in a Downward Run is called the **Downward Overshoot Event**. The Event of price change from the Upward DC Confirmation point to the Peak in an Upward Run is called the **Upward Overshoot Event**.

The above definitions are mutual recursive. Operationally, before one knows whether the market is in an Upward Run or Downward Run, one sets both the Last High and Last Low to the price at the beginning of the sequence.

Time is therefore defined by cycles of four events, as shown in Figure A1.

… → Downward Directional Change Event →
Downward Overshoot Event →
Upward Directional Change Event →
Upward Overshoot Event →
Downward Directional Change Event → …

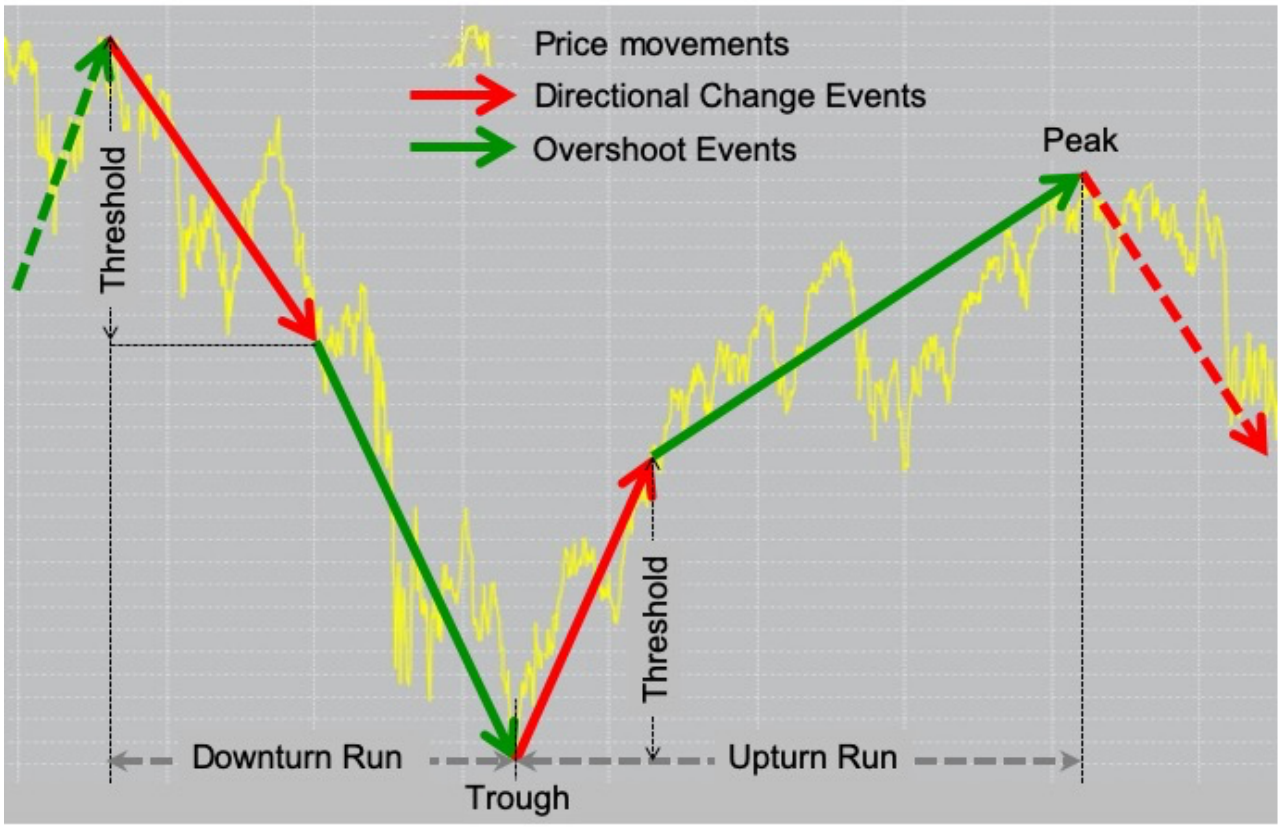


**Figure A1: Examples of Directional Change and Overshoot Events**

## A.2. Time Ontology

What is time exactly? The most studied ontologies were mainly based on **point**, **intervals** and **events** (Benthem 1983). Most people in finance are familiar with a point-based analysis. For example, people talk about the price or a commodity at a certain time (e.g. 1:23pm) on a certain day. Directional change research adopts the event ontology. Under this ontology, time is defined by events (one could say that when no events takes place, there is no notion of time).

A directional change event is a primary object in an event-based system. Intervals and points are secondary objects.

In physical time, which is popularly seen in a point-based system, an interval is a continuous set of points. In a point-based system, a Downward DC Event can be seen as a process that occupies an interval. One could define the starting point of a Downturn Event as a **Downturn Point** and the end of a Downward DC Event a **Downturn Confirmation Point**. The Downturn Point is the point at which the price last peaked. The Downturn Confirmation Point is the point at which the price has dropped by the threshold (percentage) from the Downturn Point. Similarly, one can define the start and end points of an Upward Directional Change Event as **Upturn Point** and **Upturn Confirmation Point**.

Under this stipulation, the **Downward DC Event Interval** (**DEI**) is the set of all points between the Downturn Point and the Downturn Confirmation Point. (Here one can debate whether the

interval should or should not include the Downturn Confirmation Point; we adopt the latter in our formal definition below.) The **Upward DC Event Interval** (**UEI**) is the set of all points between the Upturn Point and the Upturn Confirmation Point. Formally they are defined as follows:

$$DEI =_{def} \{t \mid \text{Downturn Point} \leq t < \text{Downturn Confirmation Point}\}$$
$$UEI =_{def} \{t \mid \text{Upturn Point} \leq t < \text{Upturn Confirmation Point}\}$$

The Downward Overshoot Event is a process that occupies an interval, which we refer to as the **Downward Overshoot Interval** (**DOI**), which is the set of all points between the previous Downturn Confirmation Point and the next Upturn Point. Similarly, an **Upward Overshoot Interval** (**UOI**) is the set of all points between the previous Upturn Confirmation Point and the next Downturn Point:

$$DOI =_{def} \{t \mid \text{Previous Downturn Confirmation Point} \leq t < \text{Next Upturn Point}\}$$
$$UOI =_{def} \{t \mid \text{Previous Upturn Confirmation Point} \leq t < \text{Next Downturn Point}\}$$

The relationship between our event-based system and the point-based system is shown in Figure A2. For a full account of time ontology, and relationship between different time systems, readers are referred to Van Benthem (1983).

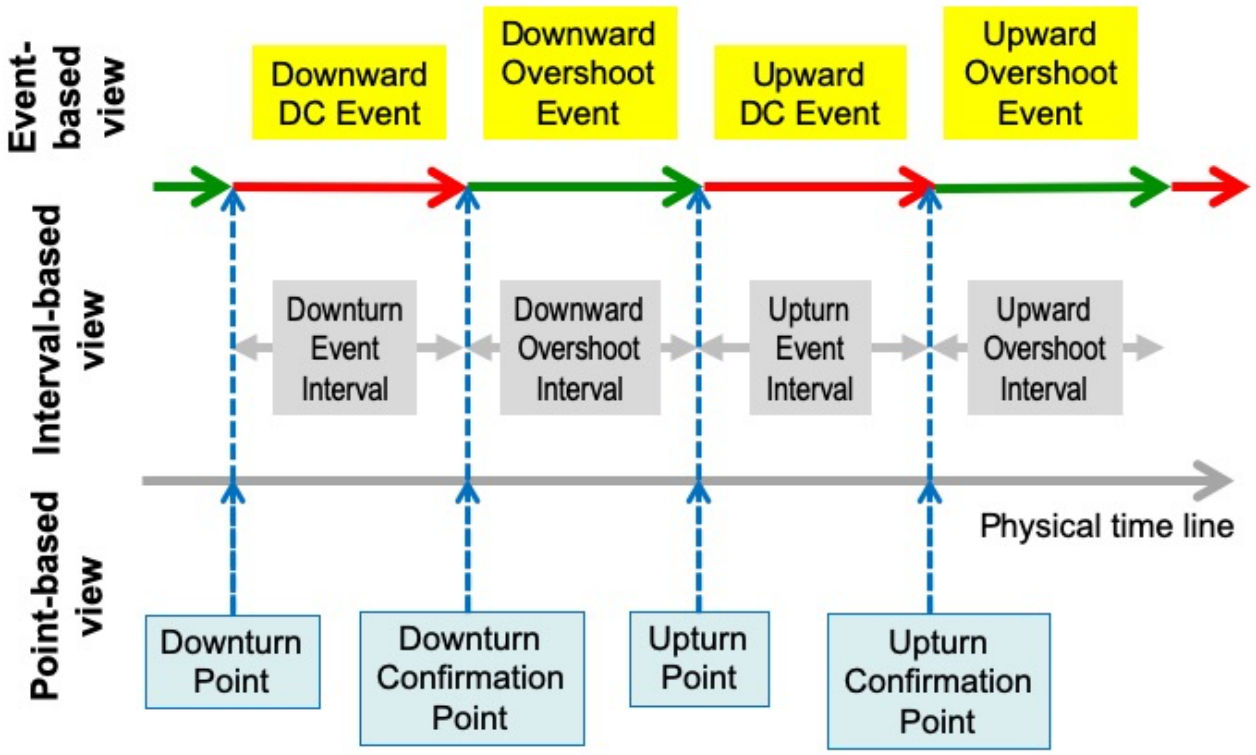


**Figure A2: Summary of terminology and relating events to points**

## A.3. Ex-Ante and Post-Ante views under different time systems

It is worth noting that in the point-based system, at the Downturn Point, we do not know that a Downturn Event has started. We would only know that a Downward DC Event has happened at the Downward DC Confirmation Point. Similarly, we only know that an Upturn Events has happened at the Upward DC Confirmation Point.

According to our definition, Overshooting Events are events in between Directional Change Events. For that reason, as soon as a Downturn Event is confirmed, we know that a Downward Overshoot Event has started. However, we will not know when it ends until the next Upward DC Event is confirmed. This means an Upturn Confirmation Point does not only confirm an Upward DC Event, it also confirms the end the last Upturn Point (trough), which is the end of the last Downturn Overshoot Event. The ex-ante and post-ante views are summarized in Table A1.

| **Events** | **Points** | **Post-ante view** | **Ex-ante confirmation views** |
|---|---|---|---|
| Downturn Event | Start | Downturn Point (peak) | Downward DC Confirmation Point |
| | End | Downward DC Confirmation Point | Downward DC Confirmation Point |
| Downturn Overshoot Event | Start | Downward DC Confirmation Point | Last Downward DC Confirmation Point |
| | End | Next Upturn Point (trough) | Next Upward DC Confirmation Point |

Table A1 – Summary of different views for different events (views on Upturn and Upturn Overshoot Events are similar and omitted here)

References:


[1] Glattfelder, J.B., Dupuis, A. & Olsen, R. Patterns in high-frequency FX data: discovery of 12 empirical scaling laws, Quantitative Finance, Volume 11 (4), 2011, 599-614

[2] Van Benthem, J., The logic of time, D Reidel Publishing Co, 1983

[3] J. Chen & E.P.K.Tsang, Detecting Regime Change in Computational Finance, Data Science, Machine Learning and Algorithmic Trading, CRC Press, September 2020