Periodicities of FX Market Activities in Intrinsic Time

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Outline

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1. Introduction
2. Methodology
3. Data and Results
4. Concluding Remarks

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1. Introduction
   - Motivation
   - Goal

Empirical Observations

Physical Time

- Trading activity tends to vary depending on the time of day
- The presence of different patterns of trading activity makes the flow of physical time discontinuous

Intrinsic Time

- Time scale based on events
- Here this intrinsic time scale is defined by directional-change events, i.e. price movements exceeding a given threshold

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Empirical Observations

Scaling Laws

- A rich set of empirical scaling laws in intrinsic time has been discovered in UHF FX data [Glattfelder at al., 2008]
- This includes a scaling law for directional-change events considered here

Directional Changes

The time series of directional-change events

- includes fewer observations
- incorporates significant information via its associated scaling law
- is non-periodic

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Objective

- Detect potential periodic patterns of UHF FX market data

Method

The Lomb-Scargle Fourier transform (LSFT) is the natural tool to analyse UHF data in the frequency domain as:

- It is especially designed for non-periodic data
- It does not require any data transformation
- It reduces the computational effort required when analysing large data sets
- It avoids complex model specifications

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- Intrinsic Time and Physical Time
- Spectral Analysis of Tick-by-tick Order Book Data

Definitions

Here we analyse in the frequency domain an intrinsic-time process defined by a directional-change event.

Total Price Movement

The absolute price change Δx_{tm} between two local extremal values is decomposed into a directional change Δx_{dc} and an overshoot Δx_{os} (price movement beyond a fixed threshold)

Directional-change Events

Dissection (cut-off) points between Δx_{dc} and Δx_{os}

Intrinsic Time vs Physical Time

These events are independent of the notion of physical time!

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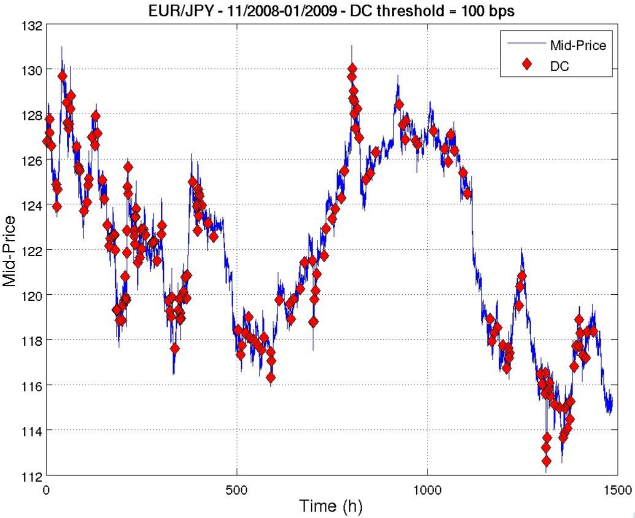

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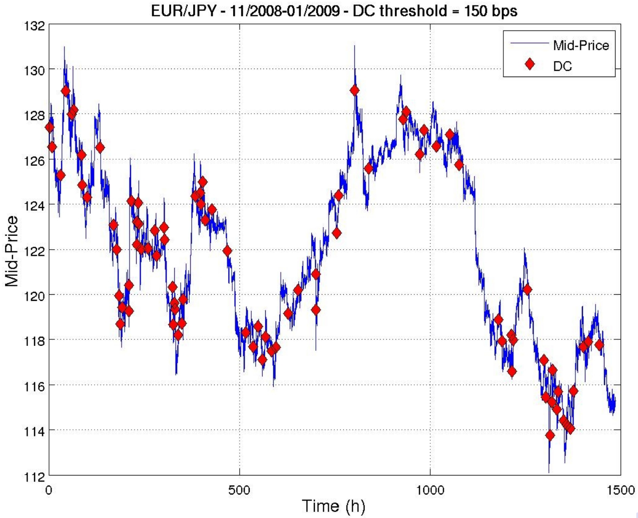

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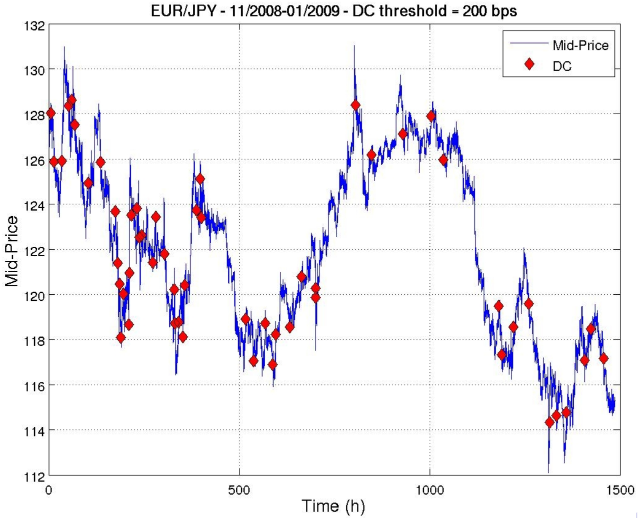

Empirical Scaling Law

Why Focus on DC events?

This event is chosen because of

- its scaling properties
- periodic behaviour of FX markets due to their market microstructure effects

Average Duration of a Directional Change

$$\langle \Delta t_{dc} \rangle = c(\Delta x_{dc})^k \tag{1}$$

$$\log(\langle \Delta t_{dc} \rangle) = \log(c) + k \cdot \log(\Delta x_{dc}) \tag{2}$$

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UHF Data and Alternative Models

Time Domain

UHF data

- Observed in real-time
- Characterised by irregularity of time intervals between two consecutive events (duration)

UHF Data as Point Processes

- ACD model
- Count models
- Intensity Models

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UHF Data and Alternative Models (Cont'd)

Time Domain

Limits

- Extension to a multivariate setting
- Loss of information
- Complexity
- Computational load

UHF Data and Alternative Models

Frequency Domain

FFT

- Requires evenly-spaced data, i.e.
  - Regular resampling or
  - Interpolation to a grid of evenly-spaced times

Limits

- Loss of information
- Generation of spurious data See below

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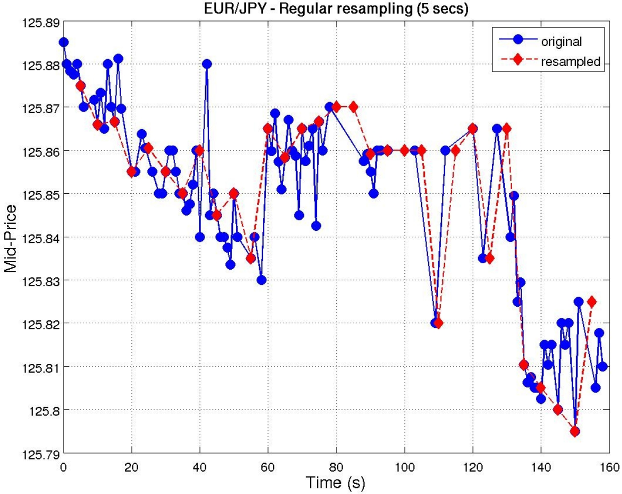
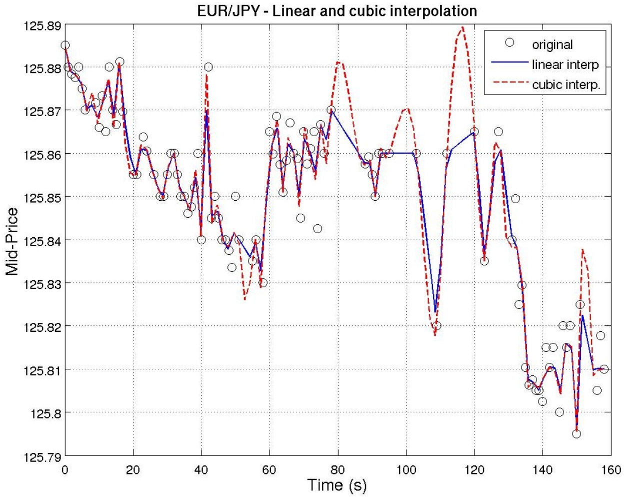
Regular Resampling



Linear and Cubic Interpolation



Lomb-Scargle Fourier Trasform (LSFT)

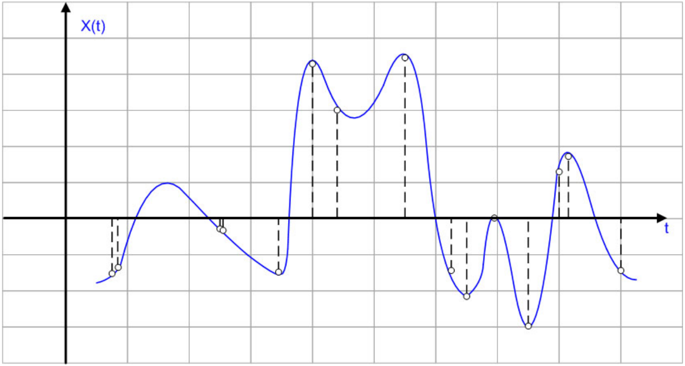


Figure: The application of the LSFT is equivalent to linear least-squares fitting sine waves to the data

Lomb-Scargle Fourier Trasform (cont'd)

$$SDF_{LS}(\omega_k) = \frac{1}{2\sigma_x^2}\left\{ \frac{\left[\sum_{j=1}^{N}(x_j-\bar{x})\cos\omega_k(t_j-\tau)\right]^2}{\sum_{j=1}^{N}\cos^2\omega_k(t_j-\tau)} + \frac{\left[\sum_{j=1}^{N}(x_j-\bar{x})\sin\omega_k(t_j-\tau)\right]^2}{\sum_{j=1}^{N}\sin^2\omega_k(t_j-\tau)} \right\} \quad (3)$$

where $\bar{x} = N^{-1}\sum_{j=1}^{N} x_j$ and with

$$\tau(\omega_k) = \frac{1}{2\omega_k}\arctan\left(\frac{\sum_{j=1}^{N}\sin(2\omega_k t_j)}{\sum_{j=1}^{N}\cos(2\omega_k t_j)}\right) \quad (4)$$

Lomb-Scargle Fourier Trasform (cont'd)

Advantages

- Can be easily applied to multivariate data
- Reduces computational load
- No data manipulation
- Well defined statistical properties:
  - SDF_{LS} has an exponential probability distribution with unit mean [Scargle, 1982]
  - The false-alarm probability of the null hypothesis (the probability that a given peak in the periodogram is not significant) is $P(Z > z) \equiv 1 - (1 - e^{-z})^M$

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UHF FX data

Data

- 6 currency pairs:
  - AUD-HKD (4’472’222)
  - AUD-JPY (18’821’980)
  - EUR-JPY (32’250’932)
  - EUR-USD (23’057’152)
  - HKD-JPY (6’052’923)
  - USD-JPY (19’010’622)
- From November 1, 2008 to January 31, 2009

Definitions

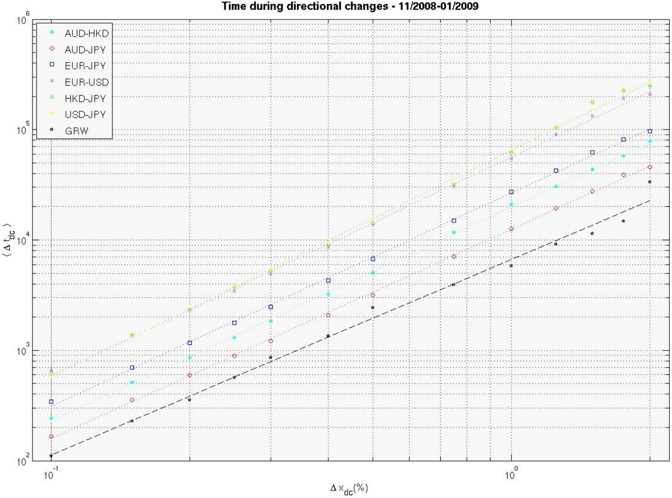
Mid-price

$$x_t = (bid_t + ask_t)/2 \tag{5}$$

Thresholds

$$\Delta x_{dc} = \{0.1\%, 0.15\%, 0.2\%, 0.25\%, 0.3\%, 0.4\%, 0.5\%, 0.75\%, 1\%, 1.25\%, 1.5\%, 1.75\%, 2\%\} \tag{6}$$

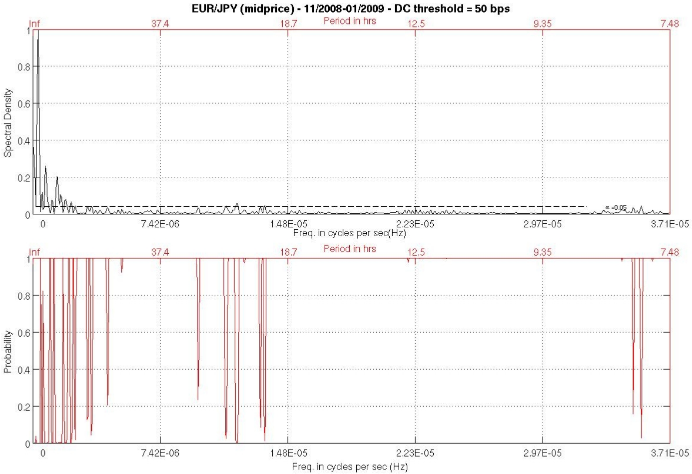
Scaling Law Regression Lines



Estimated Regression Parameters

Currency	Intercept (s.e.)	Slope (s.e.)	R^2	MSE
AUD-HKD	8.1806 (0.0258)	1.9407 (0.0111)	0.9996	2.7E-4
AUD-JPY	7.8952 (0.0284)	1.8985 (0.0123)	0.9995	3.3E-4
EUR-JPY	8.2895 (0.0342)	1.9316 (0.0148)	0.9994	4.8E-4
EUR-USD	8.7042 (0.0445)	1.9768 (0.0192)	0.9990	8.2E-4
HKD-JPY	8.9131 (0602)	2.0520 (0.0260)	0.9982	0.0015
USD-JPY	8.9539 (0.0604)	2.0656 (0.0260)	0.9983	0.0015
GRW	7.3671 (0.1121)	1.7717 (0.0484)	0.9919	0.0052

Empirical Spectral Densities



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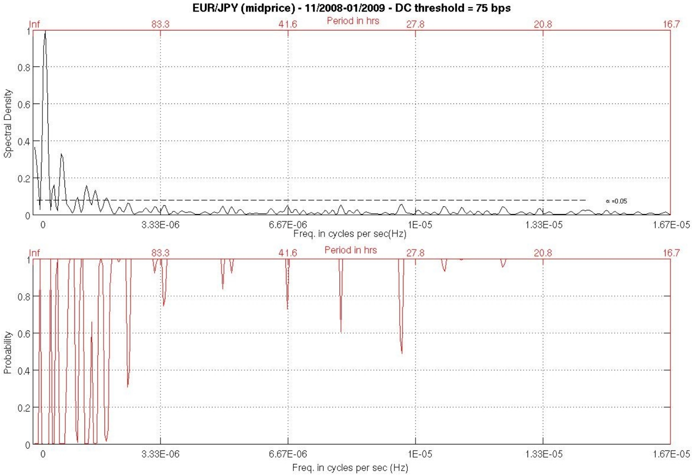

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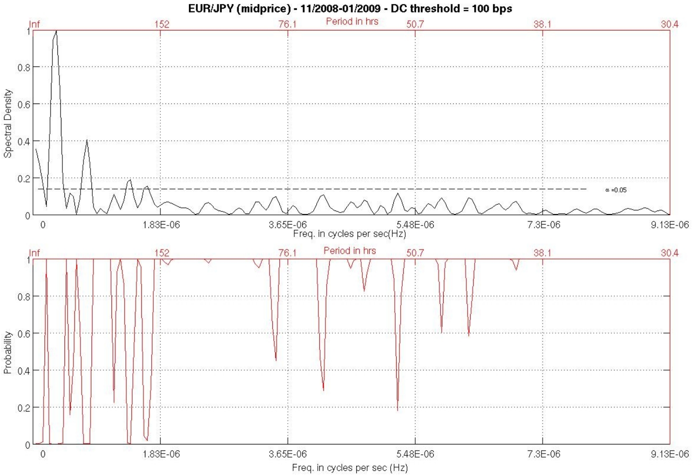

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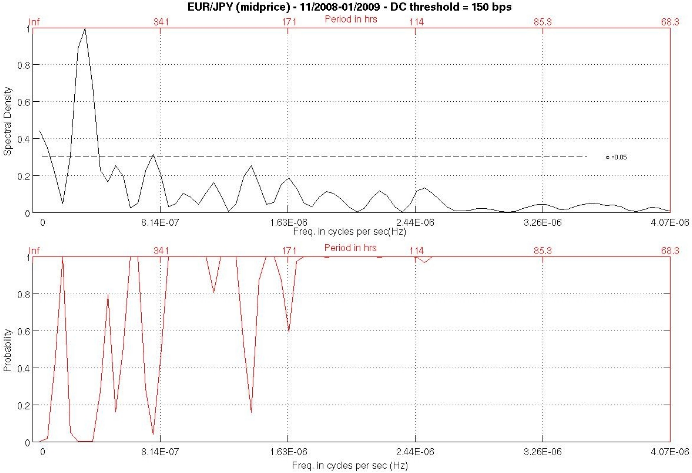

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Results

We combined the LSFT and an event-based approach, to analyse foreign exchange tick-by-tick data

Empirical Findings

- The price process displays different periodic patterns, revealed by the energy of the process in the frequency domain.
- The period associated with these patterns tends to increase as the directional-change threshold increases, confirming similar results in other studies [Glattfelder at al., 2008].

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Applications and Future Research

Related Application

- Trading strategies and decision support for traders

Further Research

- FFT in intrinsic trading time:
  - Periodicity as recurrence after a certain number of events
- Analyse dependencies between different variables (e.g. price, volume, etc.) in the frequency domain
  - Generalisation of LSFT framework to a multivariate scheme

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References

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Thank you for your attention !

Questions ?

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