Estimation of Stochastic Volatility Models with Implied Volatility Indices and Pricing of Straddle Option

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Outline

- Introduction : Stochastic Volatility Models, Volatility Index and Straddle Option
- Methodology
- Data and Results
- Conclusion

Stochastic Volatility Models

- Asset variance is assumed to follow a stochastic process

- Constant Elasticity of Variance (CEV)
- Heston
- GARCH(1,1)

Constant Elasticity of Variance (CEV) model

(Chan et al (1992), Chacko et al (2000) and Jones (2003))

- Under risk-aversion measure P, the logarithm of the asset price and its variance have the following process with CEV model

$$
\begin{aligned}
ds_t &= \left[r_t - d_t + \left[\lambda_1(1-\rho^2) - 1/2\right] v_t\right] dt + \sqrt{(1-\rho^2)v_t}\, dW_1^P(t) + \rho\sqrt{v_t}\, dW_2^P(t)) \\
dv_t &= \delta_v(\bar{v} - v_t)dt + k_v v_t^{\beta} dW_2^P(t).
\end{aligned}
$$

Heston and GARCH Model

- The model of Heston is obtained when $\beta = 0.5$

$$dv_t = \delta_v(\bar{v} - v_t)dt + k_v\sqrt{v_t}dW_2^P(t),$$

- The model of GARCH is obtained when $\beta = 1$

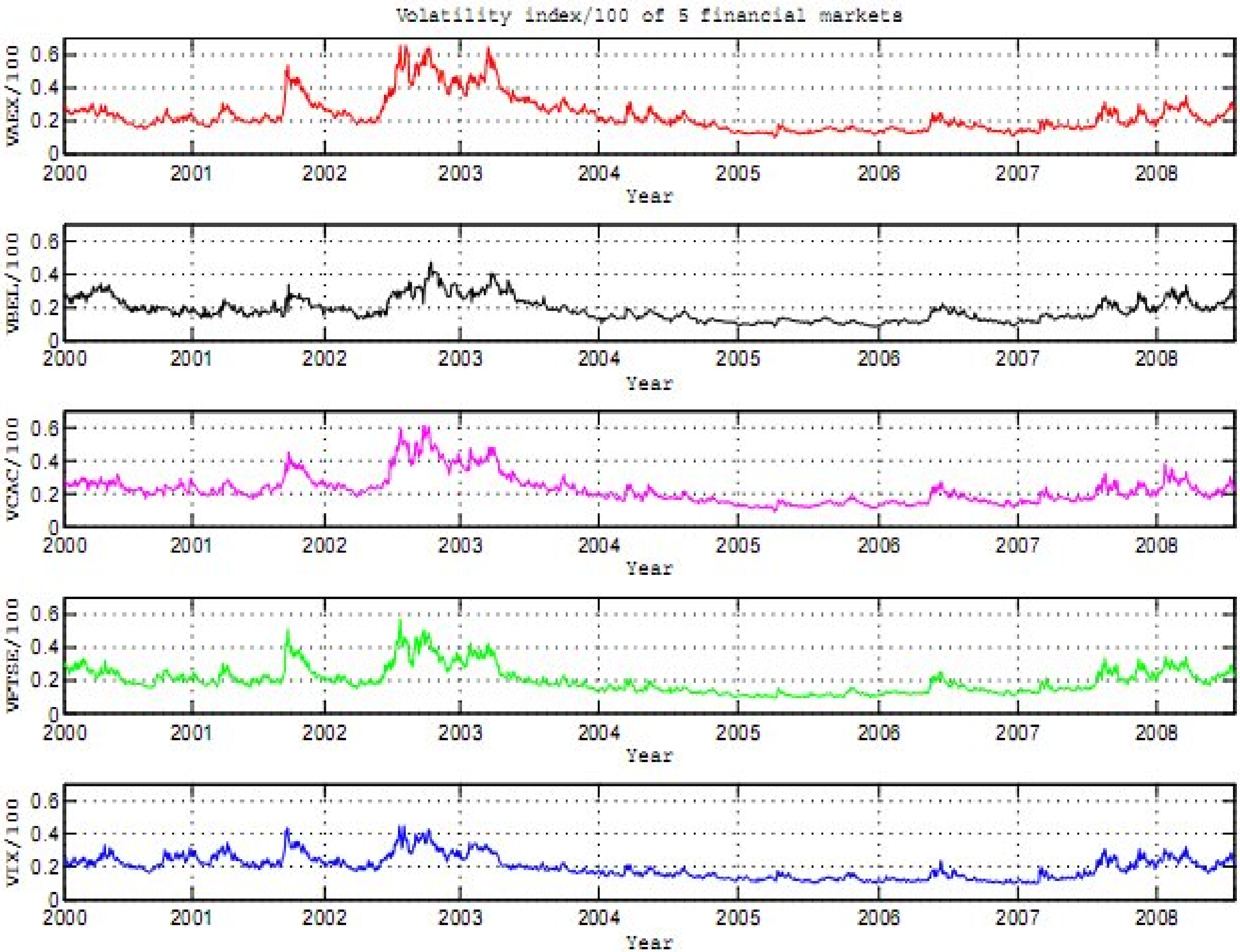
$$dv_t = \delta_v(\bar{v} - v_t)dt + k_v v_t dW_2^P(t),$$

Volatility Index

- It is a market expectation of short-term future volatility, based on prices of short-term options.

- The volatility index is used as a proxy for the true but unobserved instantaneous volatility.

Figure 1: Time series of 5 volatility indices (VAEX, VBEL, VCAC, VFTSE and VIX) from 04th January 2000 to 23rd July 2008.



ATM Forward Straddle

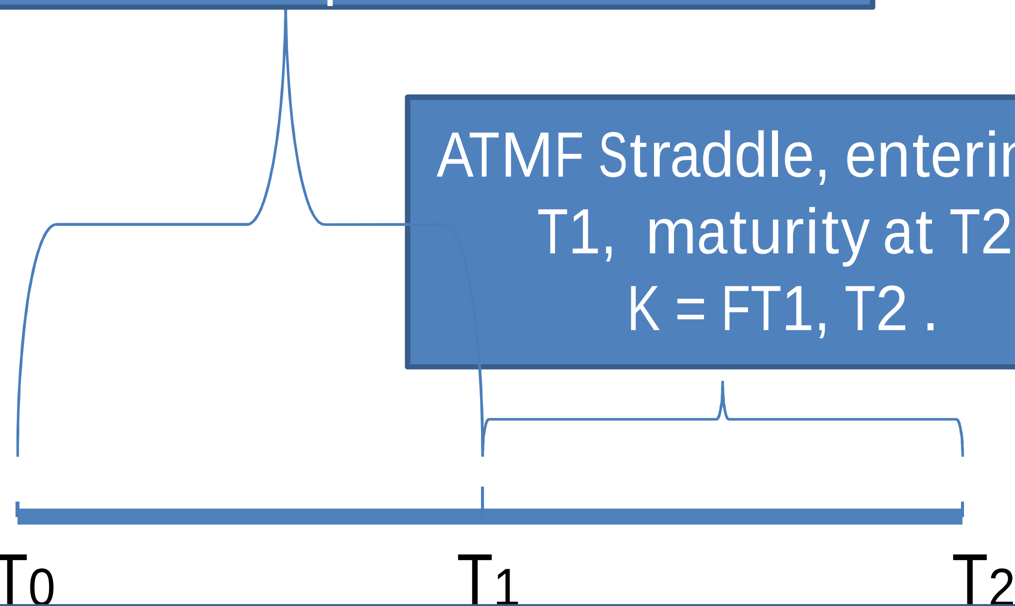
- A straddle -- a call option and a put option, with the same strike price and the same maturity time on the same underlying asset.

- An at-the-money forward straddle -- the strike price is equal to the corresponding forward price, $K = F_{t,T}$

ATM Forward Straddle Option (STO)

ATMF STO, entering at T0 , maturity at T1, KSTO is pre-specified.

ATMF Straddle, entering at T1, maturity at T2, K = FT1, T2 .



At T1, the buyer of ATMF STO has the option to buy a ATMF straddle with price equal to KST 0 . He receives a call and a put with strike price equal to the forward price FT1, T2.

ATM Forward Straddle Option (STO)

- Brenner et al (2006) give the closed form solution of ATMF STO with Stein and Stein(1991) model.

- We use Monte Carlo method to price ATMF STO with three stochastic volatility models in 5 stock markets.

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- Methodology: Maximum Likelihood Expansion
- Data and Results
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Maximum-Likelihood Estimation

- The state variables:

  s(t)--the logarithm of the level of stock index

  v(t)– instantaneous variance (the square of the volatility index)

The close-form expansion of stochastic volatility models

- The logarithm of the conditional density is

$$L\left(x_{t_n}|x_{t_{n-1}},\theta\right)=\ln\left[f\left(x_{t_n}|x_{t_{n-1}},\theta\right)\right].$$

- The true joint likelihood function

  x(t) = [s(t), v(t)]’ is unknown. We use closed form expansion instead of the true one

The close-form expansion of stochastic volatility models

Ait-Sahalia and Kimmel (2007) derive the following analytic expansion of order K of $L\left(x_{t_n}|x_{t_{n-1}},\theta\right)$:

$$L^{(k)}\left(x_{t_n}|x_{t_{n-1}},\theta\right) = -\frac{M}{2}ln(2\pi\Delta t_n)-\frac{1}{2}ln(det[v(x_{t_n};\theta)]) + \frac{C_x^{(-1)}(x_{t_n} \mid x_{t_{n-1}};\theta)}{\Delta t_n}+\sum_{i=0}^{K} C_x^{(i)}(x_{t_n} \mid x_{t_{n-1}};\theta)\frac{\Delta t_n^i}{i!},$$

where, $v(x_{t_n}) = \sigma(x_{t_n})\sigma(x_{t_n})$. The order k can be arbitrary. The coefficients $C_x^{(i)}$, $i = -1,0,1\ldots,k$ are unknown. However, each $C_x^{(i)}$, can be approximated with a Taylor series in $(x_{t_n}-x_{t_{n-1}})$ of order $l_i = 2(K-i)$. The approximation of the coefficient is denoted by $C_x^{(l,i)}$.

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  Summary Statistics

  Parameter Estimation

  Option Pricing
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Table 1: Summary statistics of 5 volatility indices (VAEX, VBEL, VCAC, VFTSE and VIX) from 04th January 2000 to 23rd July 2008.

	VAEX	VBEL	VCAC	VFTSE	VIX
Min	0.101	0.086	0.092	0.091	0.099
Max	0.657	0.474	0.615	0.571	0.451
Median	0.209	0.174	0.211	0.184	0.192
Mean	0.236	0.186	0.227	0.200	0.198
STD	0.104	0.069	0.089	0.080	0.068
Skewness	1.602	0.918	1.464	1.199	0.707
Kurtosis	5.456	3.450	5.415	4.370	3.063

Table 2: Summary statistics of realized volatility (RV) of 5 financial markets (AEX, BEL 20, CAC 40, FTSE 100 and S&P 500) from 04th January 2000 to 23rd July 2008.

	AEX	BEL 20	CAC 40	FTSE 100	S&P 500
Min	0.060	0.050	0.073	0.053	0.059
Max	0.680	0.571	0.594	0.527	0.462
Median	0.165	0.129	0.173	0.141	0.148
Mean	0.197	0.164	0.199	0.161	0.162
STD	0.119	0.098	0.101	0.083	0.074
Skewness	1.856	1.660	1.567	1.328	1.023
Kurtosis	6.489	5.672	5.475	4.881	3.911

Table 3: Correlations between returns on five stock market indices (AEX, BEL 20, CAC 40, FTSE 100 and S&P 500) from 04th January 2000 to 23rd July 2008.

	ΔAEX	ΔBEL 20	ΔCAC 40	ΔFTSE 100	ΔS&P 500
ΔAEX	1	0.8142	0.9148	0.8372	0.4896
ΔBEL 20	0.8142	1	0.7732	0.7255	0.4459
ΔCAC 40	0.9148	0.7732	1	0.8532	0.5130
ΔFTSE 100	0.8372	0.7255	0.8532	1	0.4680
ΔS&P	0.4896	0.4459	0.5130	0.4680	1

Table 4: Correlations between the five implied volatility indices (VAEX, VBEL, VCAC, VFTSE and VIX) from 04th January 2000 to 23rd July 2008.

	VAEX	VBEL	VCAC	VFTSE	VIX
VAEX	1	0.8576	0.9624	0.9074	0.8365
VBEL	0.8576	1	0.8883	0.8814	0.8267
VCAC	0.9624	0.8883	1	0.9468	0.8879
VFTSE	0.9074	0.8814	0.9468	1	0.9329
VIX	0.8365	0.8267	0.8879	0.9329	1

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Table 7: ML Estimation of CEV model

$$ds_t = \left[(r_t - d_t) + \left(\lambda\left(1-\rho^2\right) - \tfrac{1}{2}\right)v_t\right]dt + \sqrt{v_t}\,dW^s_{tP},\ dv_t = -\delta_v(v_t - \bar{v})dt + k_v v_t^{\beta} dW^v_{tP},\ E_t dW^s_{tP} dW^v_{tP} = \rho dt$$

	AEX		BEL 20		CAC 40		FTSE 100		S&P 500	
	Est.	s.e.	Est.	s.e.	Est.	s.e.	Est.	s.e.	Est.	s.e.
δ_v	2.126	1.176	6.704	1.862	2.179	1.450	1.721	1.335	2.774	1.217
$\overline{v}$	0.059	0.022	0.037	0.007	0.061	0.027	0.052	0.031	0.048	0.017
k_v	1.985	0.051	1.069	0.026	2.383	0.069	2.767	0.088	1.246	0.024
β	1.016	0.008	0.739	0.006	1.064	0.009	1.074	0.009	0.811	0.004
ρ	-0.763	0.007	-0.215	0.016	-0.664	0.007	-0.817	0.005	-0.821	0.005
λ	0.280	1.391	0.611	1.798	0.152	1.487	0.239	1.806	-0.621	1.987
$\mathcal{L}\left(\hat{\theta}\right)$	15084.4		14733.9		14737.9		16041.1		15833.6	

Table 5: ML Estimation of Heston model,

$$ds_t = \left[\left(r_t - d_t\right) + \left(\lambda\left(1-\rho^2\right) - \frac{1}{2}\right)v_t\right]dt + \sqrt{v_t}dW_{tP}^s,\ dv_t = -\delta_v(v_t - \bar{v})dt + k_v\sqrt{v_t}dW_{tP}^v,\ E_t dW_{tP}^s dW_{tP}^v = \rho dt$$

	AEX		BEL 20		CAC 40		FTSE 100		S&P 500	
	Est.	s.e.	Est.	s.e.	Est.	s.e.	Est.	s.e.	Est.	s.e.
δ_v	4.483	0.875	15.719	2.752	6.515	1.192	5.664	1.079	5.578	1.110
$\overline{v}$	0.065	0.016	0.039	0.006	0.058	0.012	0.045	0.011	0.043	0.010
k_v	0.642	0.007	0.702	0.008	0.642	0.008	0.634	0.006	0.575	0.005
ρ	-0.754	0.007	-0.282	0.017	-0.665	0.008	-0.804	0.005	-0.827	0.005
λ	-0.797	1.581	0.569	1.852	-0.126	1.616	-1.073	2.060	-0.482	2.071
$\mathcal{L}\left(\hat{\theta}\right)$	10737.4		10763.0		10478.1		11276.8		11431.8	

Table 6: ML Estimation of GARCH model

$$ds_t = \left[(r_t - d_t) + \left(\lambda\left(1-\rho^2\right) - \frac{1}{2}\right)v_t\right]dt + \sqrt{v_t}dW^s_{tP},\ dv_t = -\delta_v(v_t - \bar{v})dt + k_v v_t dW^v_{tP},\ E_t dW^s_{tP} dW^v_{tP} = \rho dt$$

	AEX		BEL 20		CAC 40		FTSE 100		S&P 500	
	Est.	s.e.	Est.	s.e.	Est.	s.e.	Est.	s.e.	Est.	s.e.
δ_v	2.256	1.112	7.603	7.068	2.619	1.298	2.196	1.243	0.089	1.901
$\bar{v}$	0.059	0.020	0.058	0.039	0.058	0.019	0.048	0.021	0.990	20.752
k_v	1.884	0.018	4.989	0.043	1.934	0.019	2.172	0.020	2.601	0.022
ρ	-0.765	0.007	0.035	0.032	-0.670	0.007	-0.820	0.005	-0.782	0.007
λ	0.340	1.380	0.559	1.919	0.223	1.481	0.030	1.844	-3.243	1.923
$\mathcal{L}\left(\hat{\theta}\right)$	15099.4		13788.0		14774.6		16047.8		15403.9	

- The values of log likelihood function for GARCH and the CEV are quite close. Heston gives much lower values.

- The estimated values of mean reversion speed have large standard errors. The estimated values for the diffusion terms of variance are always significant.

- The correlation between stock price and variance of BEL 20 market is much lower and less significant than the other markets. It seems there is not a clear leverage effect for the Belgian stock index.

Table 8: Heston Hypothesis

Statistic	AEX	BEL 20	CAC 40	FTSE 100	S&P
Likelihood Ratio	8694.0	7941.8	8519.6	9528.6	8803.6
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Wald	102.4	37.6	91.4	97.9	132.7
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)

Table 9: GARCH Hypothesis

Statistic	AEX	BEL 20	CAC 40	FTSE 100	S&P
Likelihood Ratio	-30.00	1891.80	-73.40	-13.40	859.40
	-	(0.0000)	-	-	(0.0000)
Wald	0.10	44.79	1.18	1.63	49.00
	(0.7537)	(0.0000)	(0.2780)	(0.2020)	(0.0000)

- Heston is rejected for all five markets.
- GARCH is rejected for the BEL 20 and S&P 500 indices.
- These two markets are the least correlated with other markets. Their volatility indices have a much lower kurtosis and skewness.

- So the reason for this is most likely a reflection of the difference across the volatility indices, but rather an artifact of the methodology.

- For the S&P 500 index the results are in line with Ait-Sahalia and Kimmel (2007), who find that both the Heston and the GARCH model are rejected.

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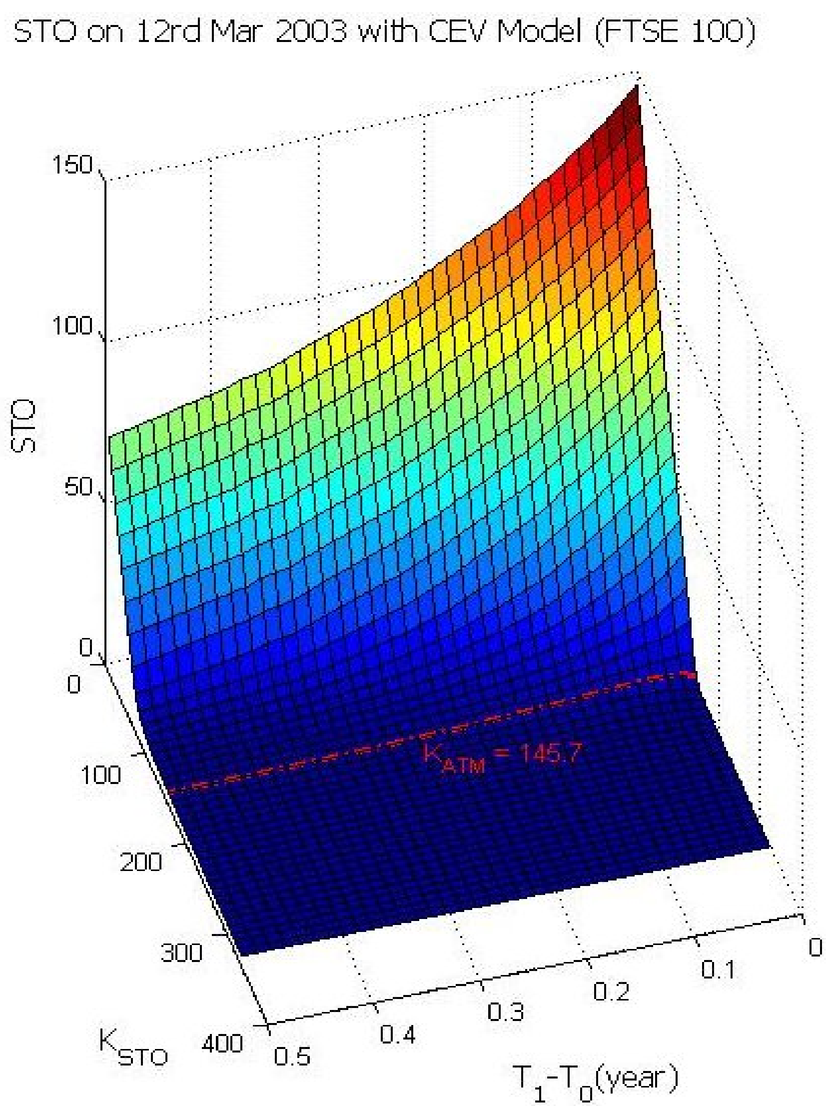
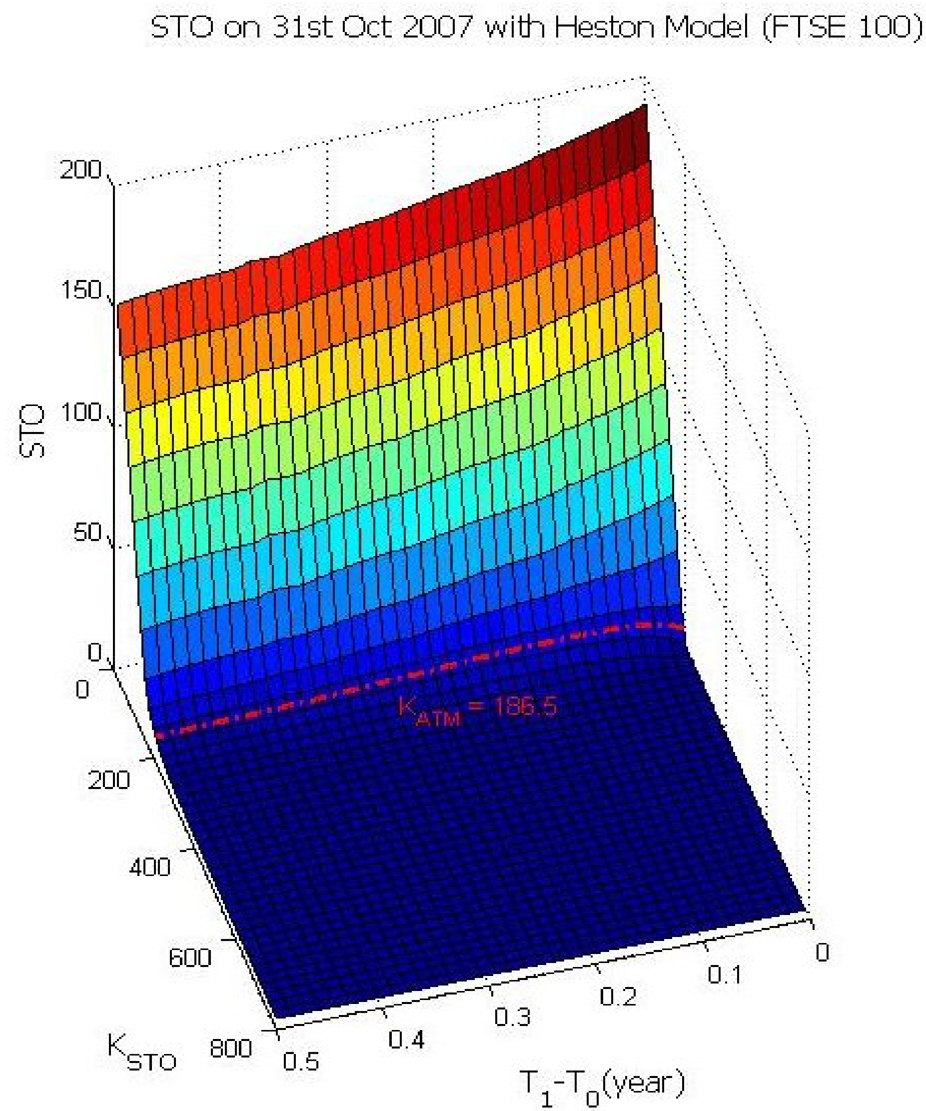
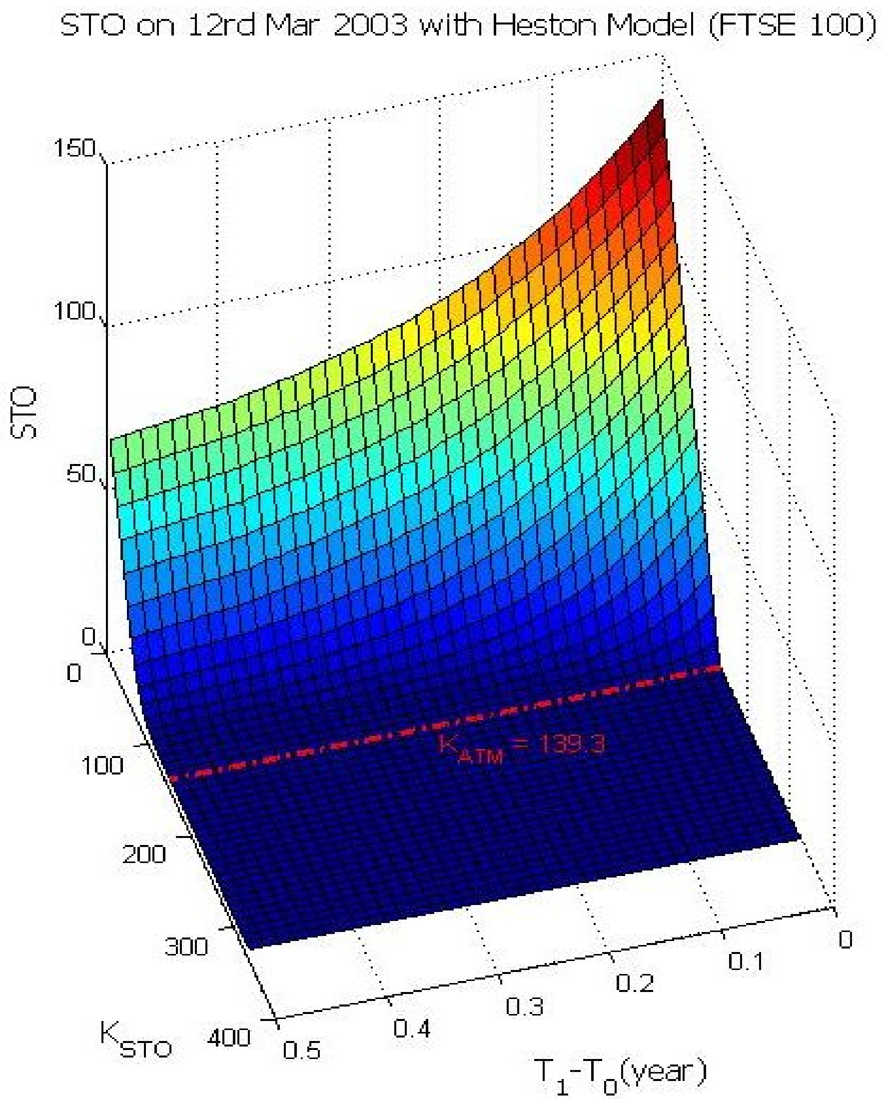
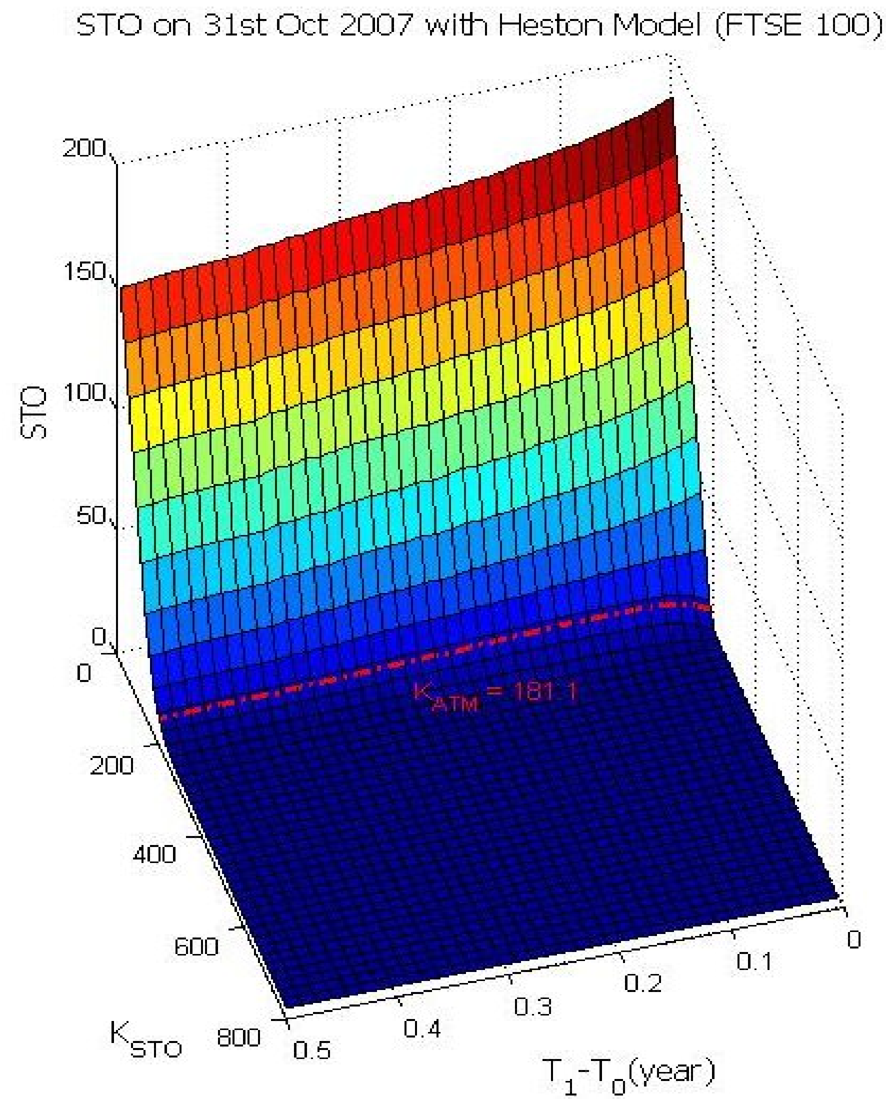
ATMF STO under CEV model (FTSE 100)



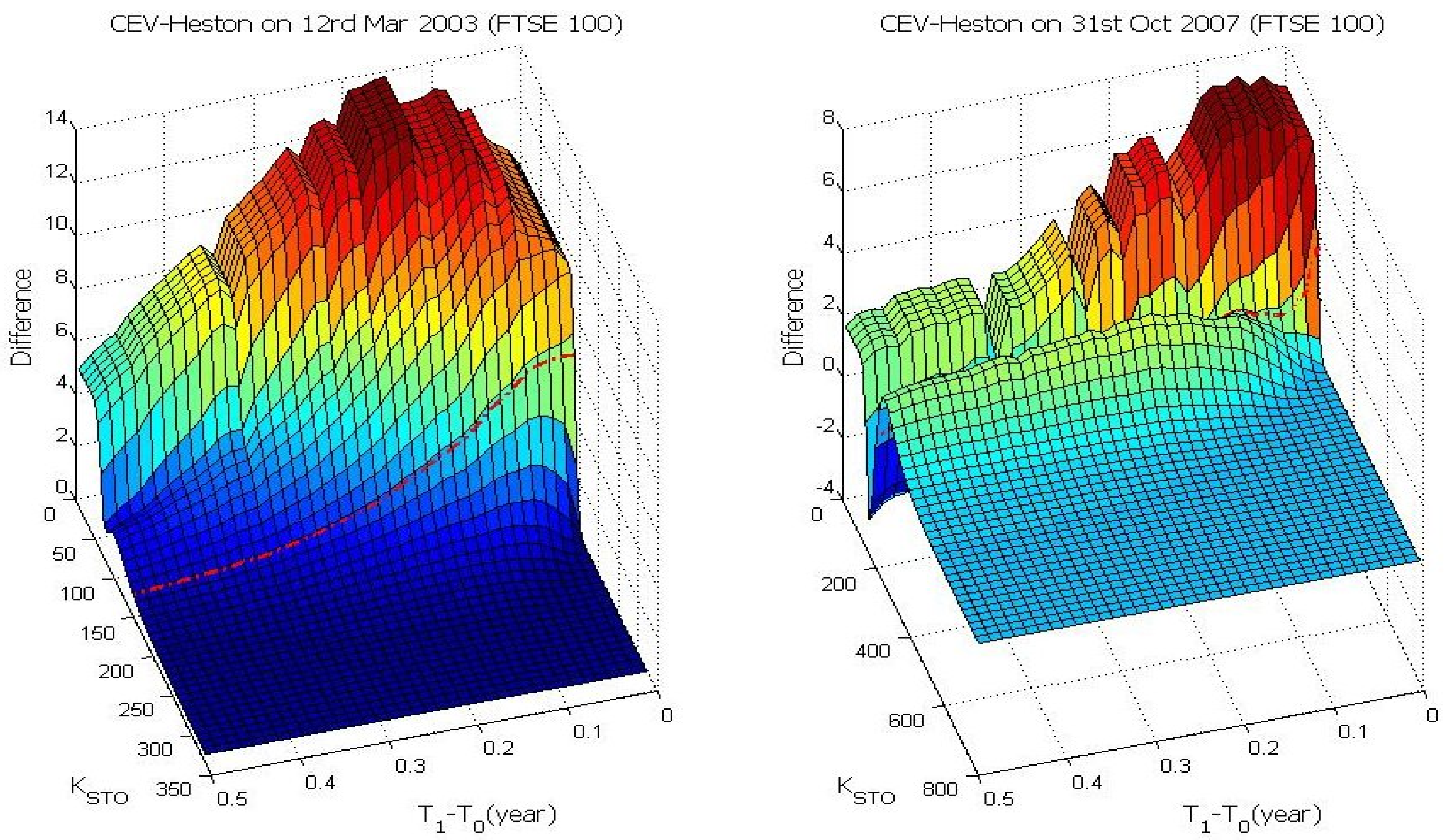


ATMF STO under Heston model (FTSE 100)

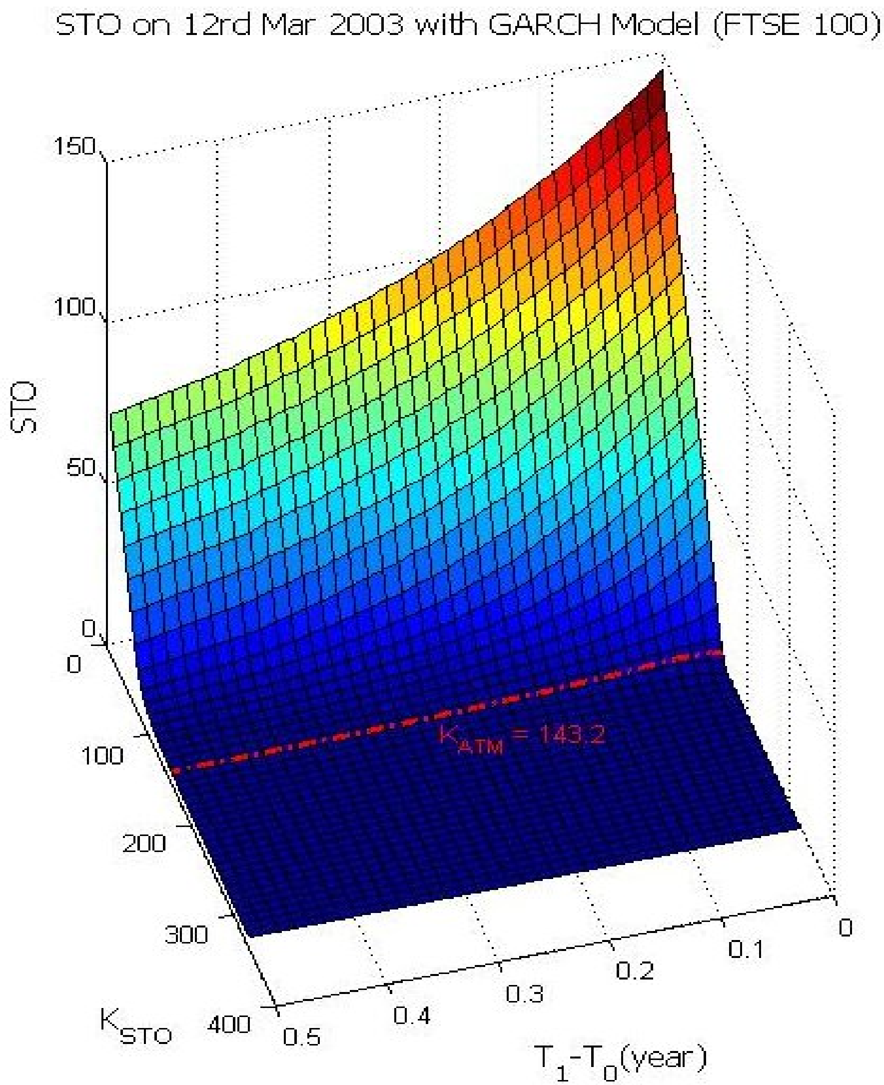


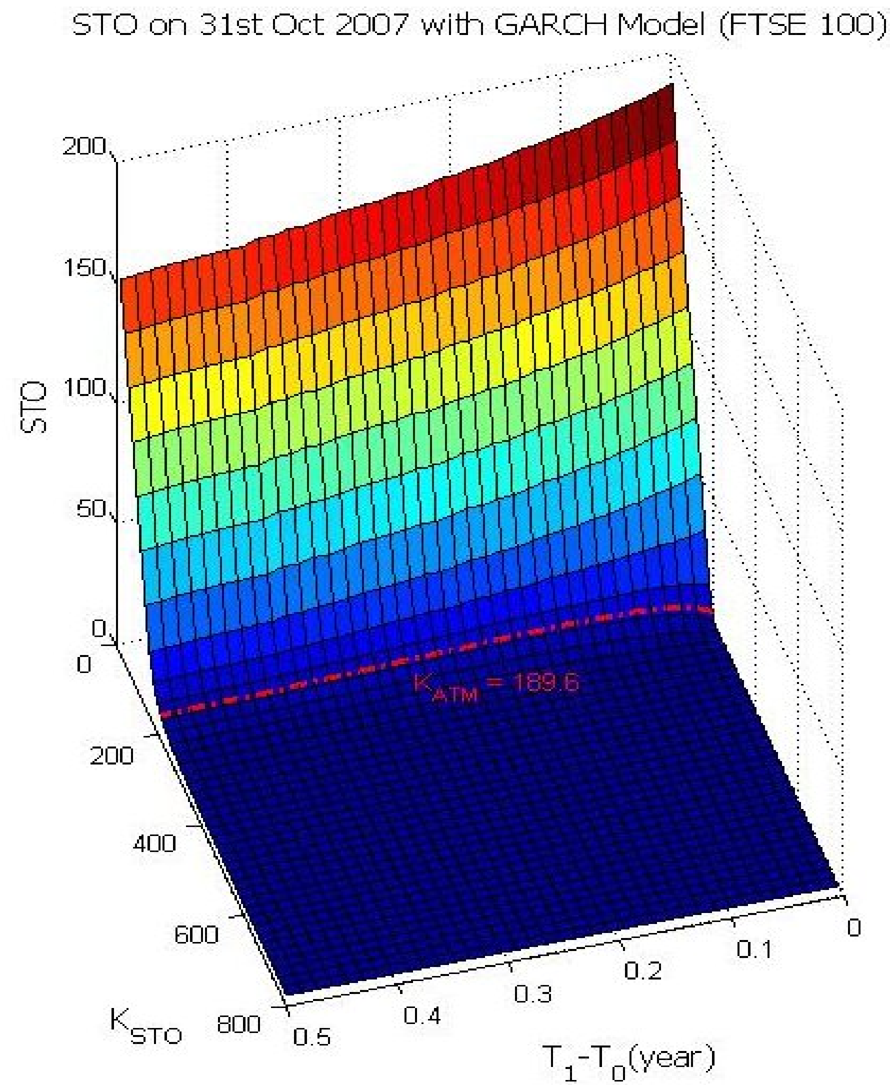


Difference between Heston and CEV (FTSE 100)

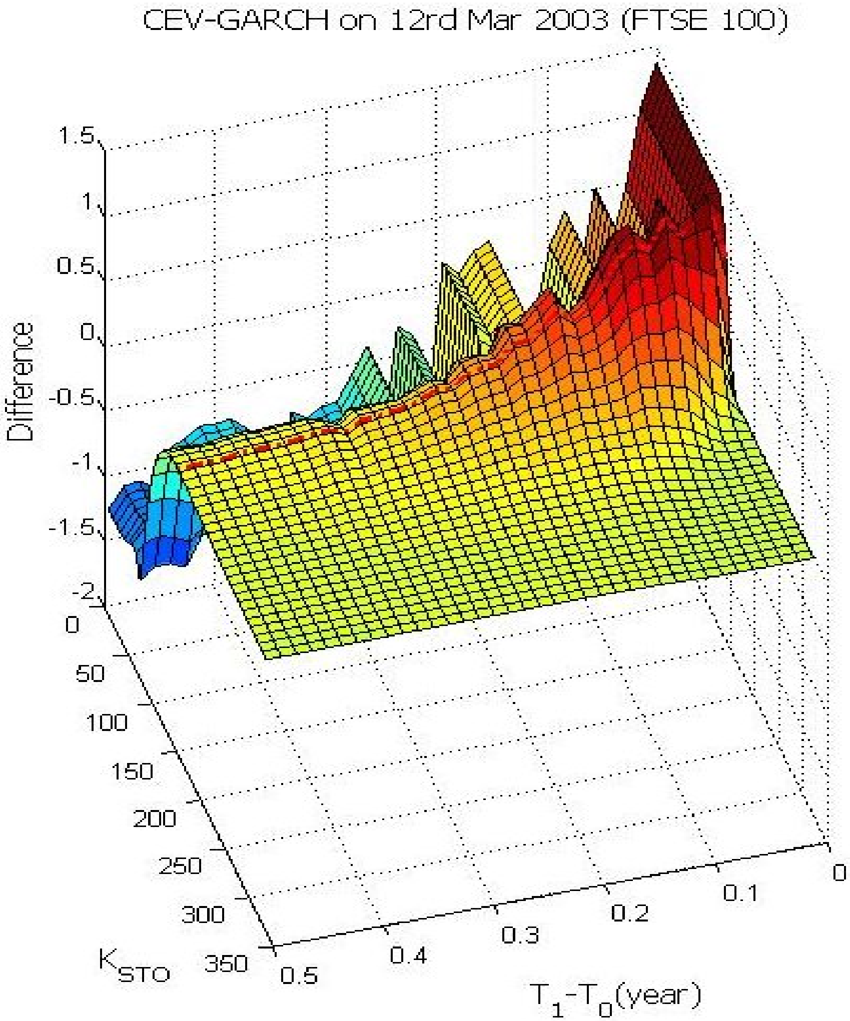


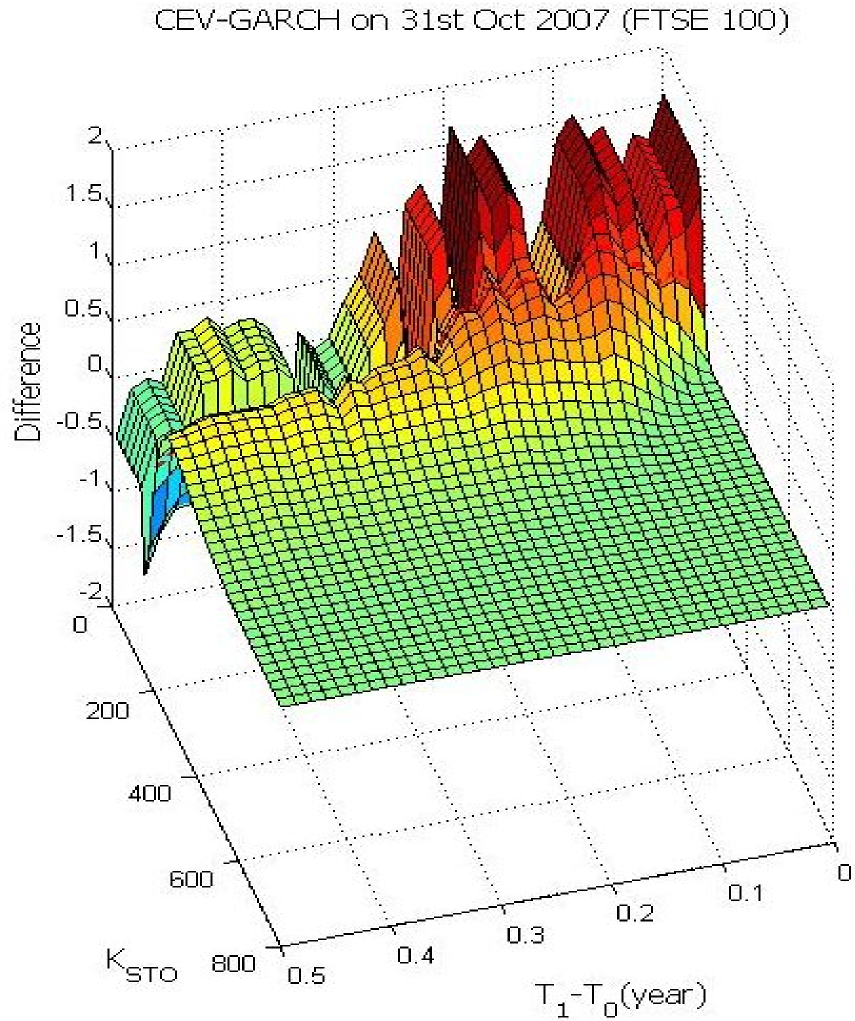
ATMF STO under GARCH model (FTSE 100)



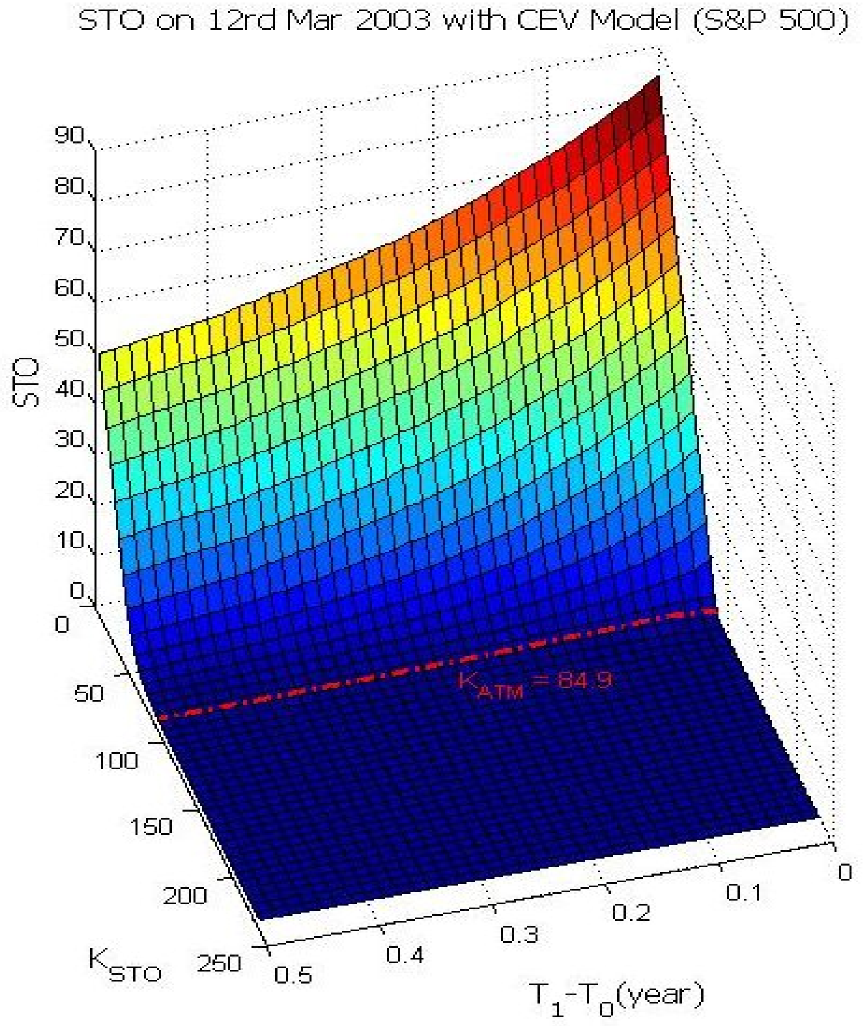


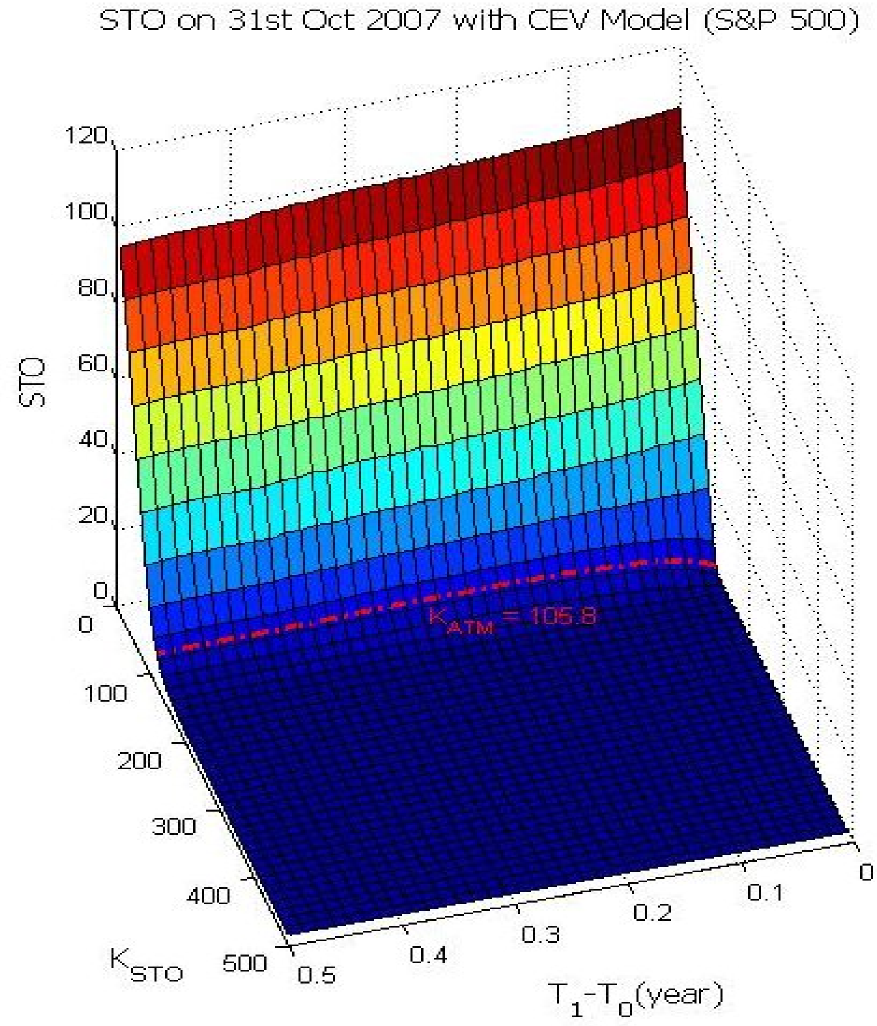
Difference between GARCH and CEV (FTSE 100)



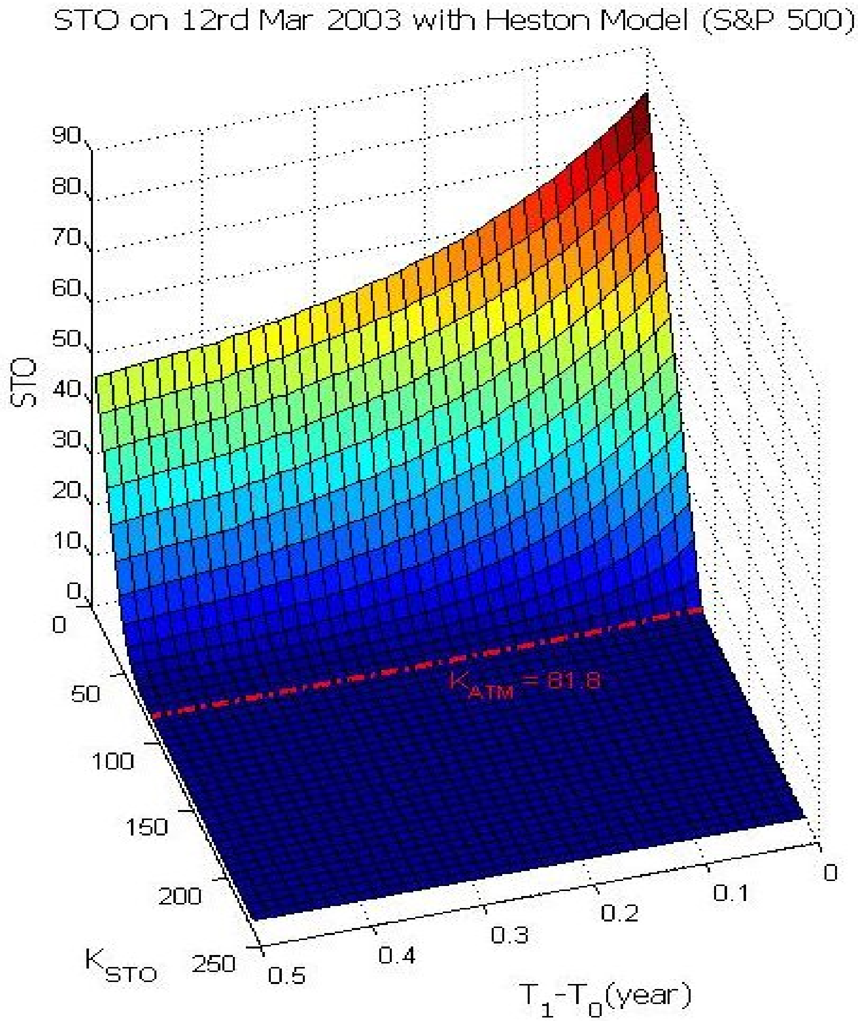


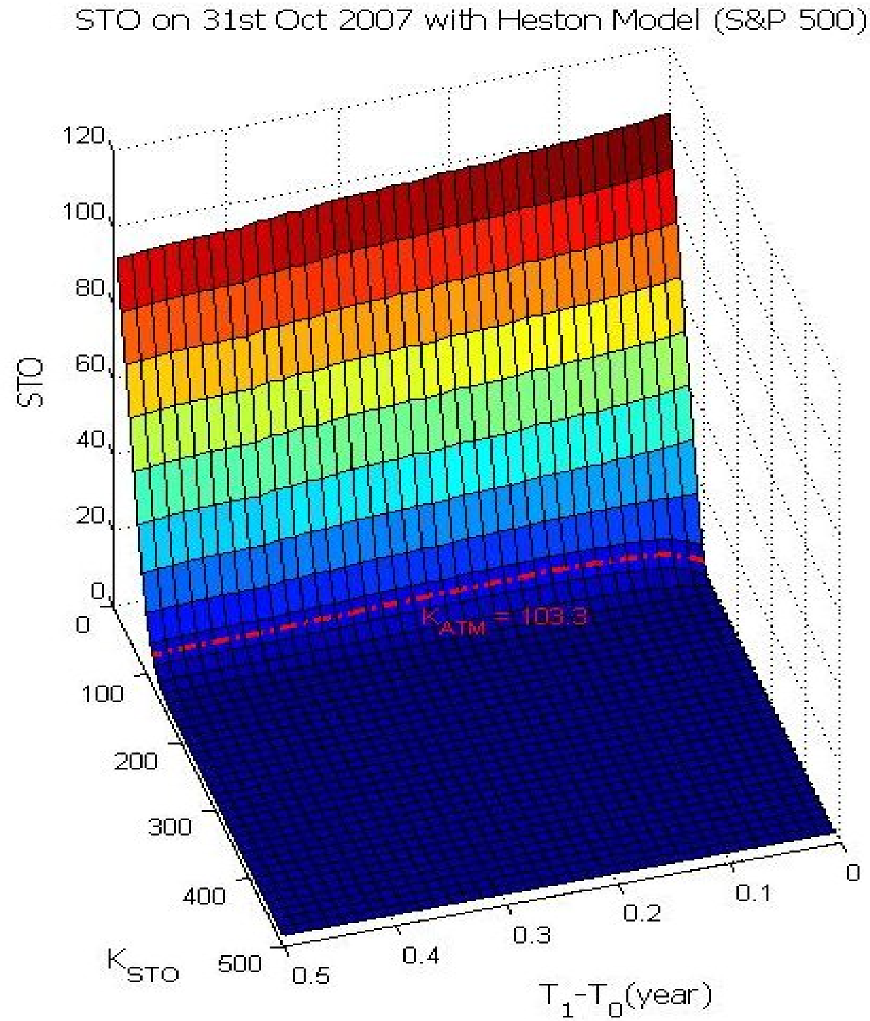
ATMF STO under CEV model (S&P 500)



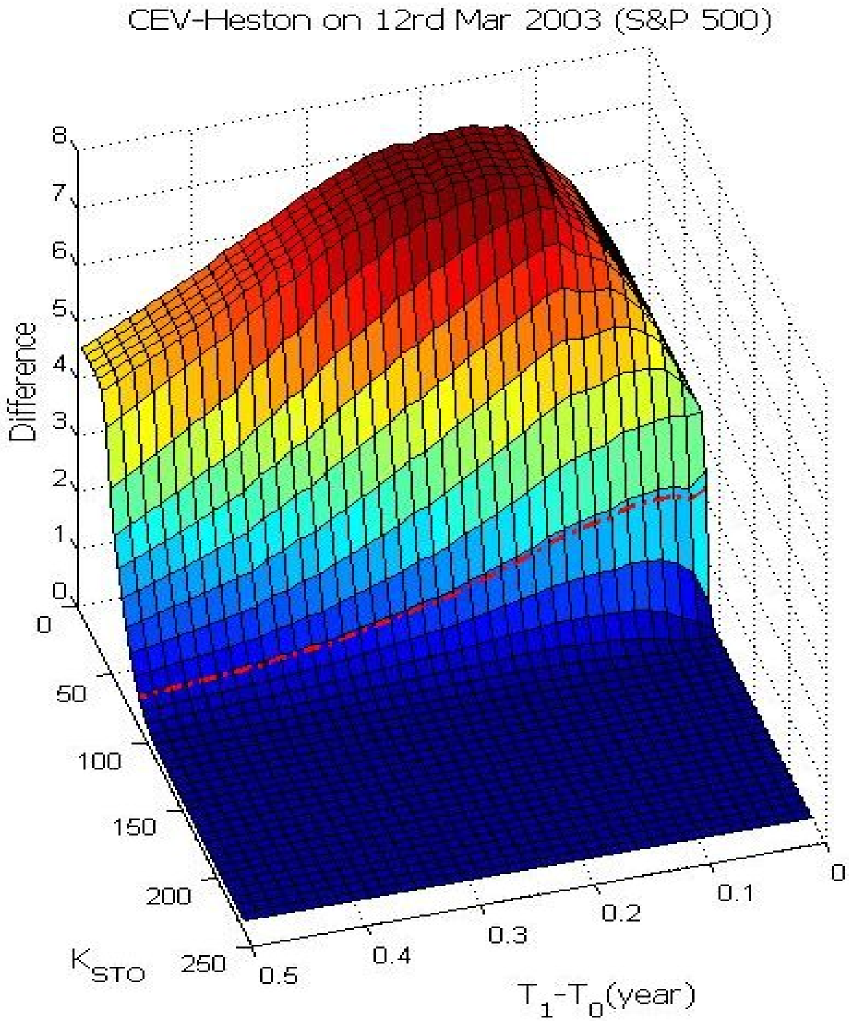


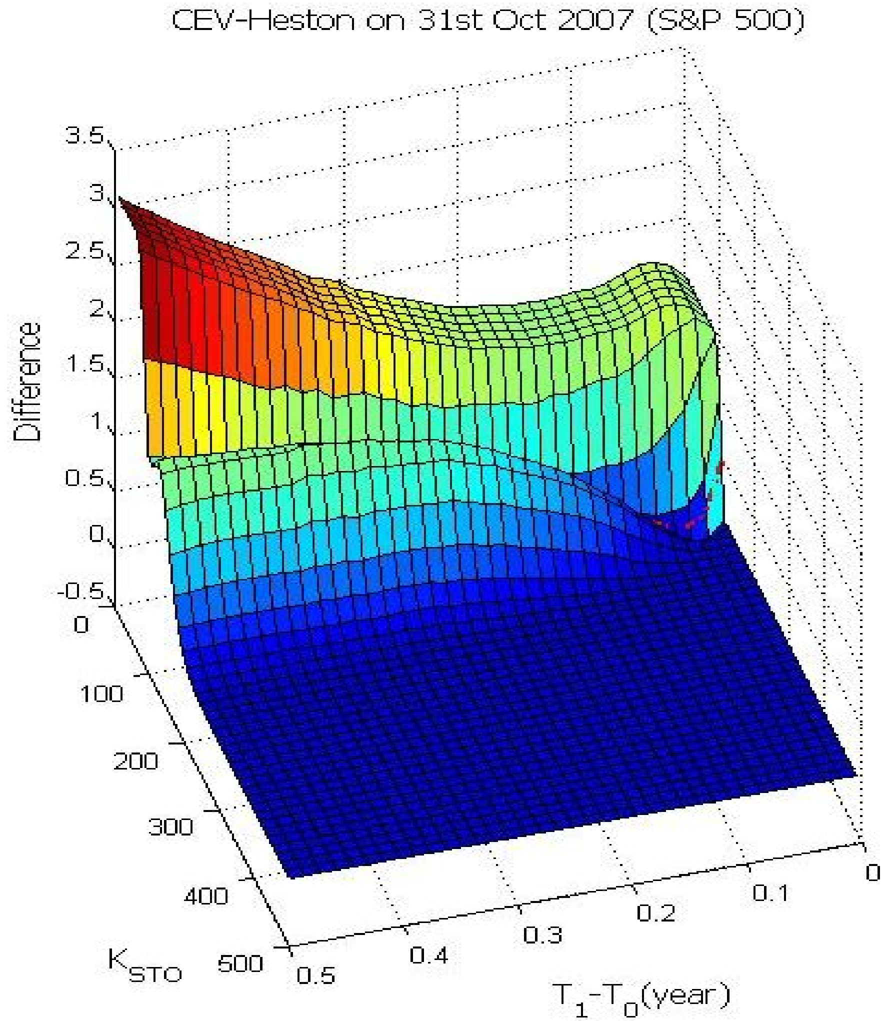
ATMF STO under Heston model (S&P 500)



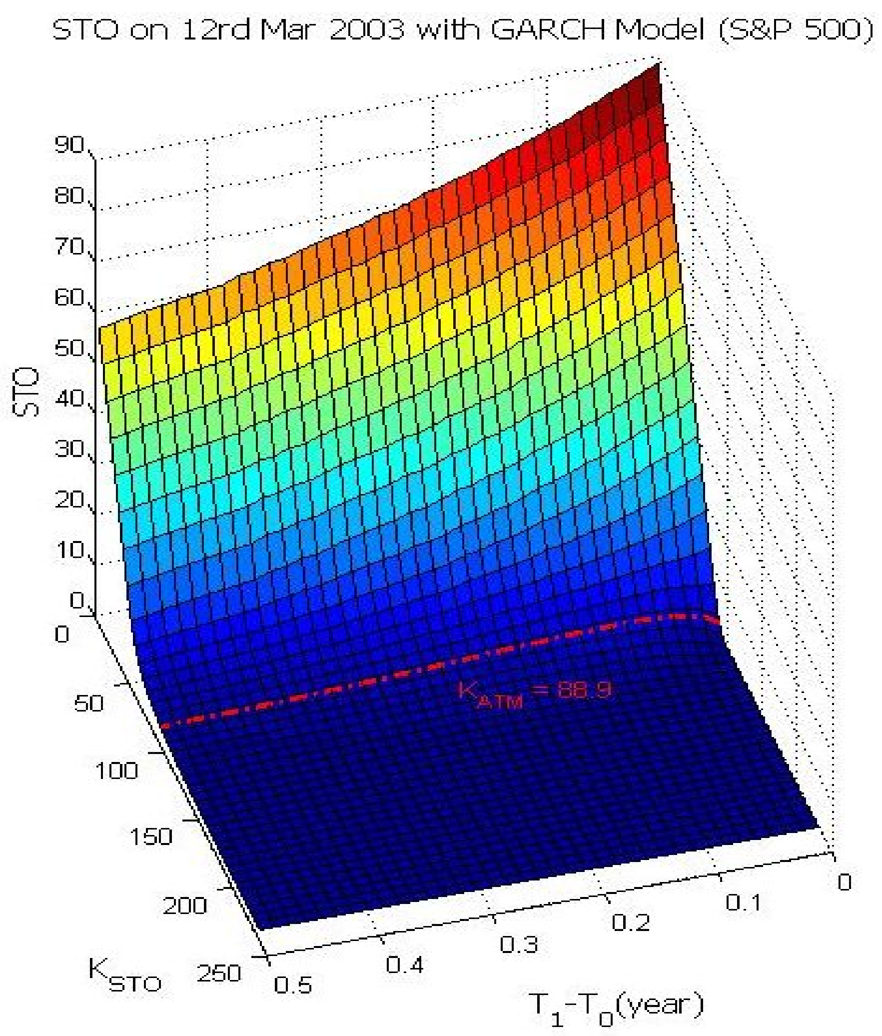


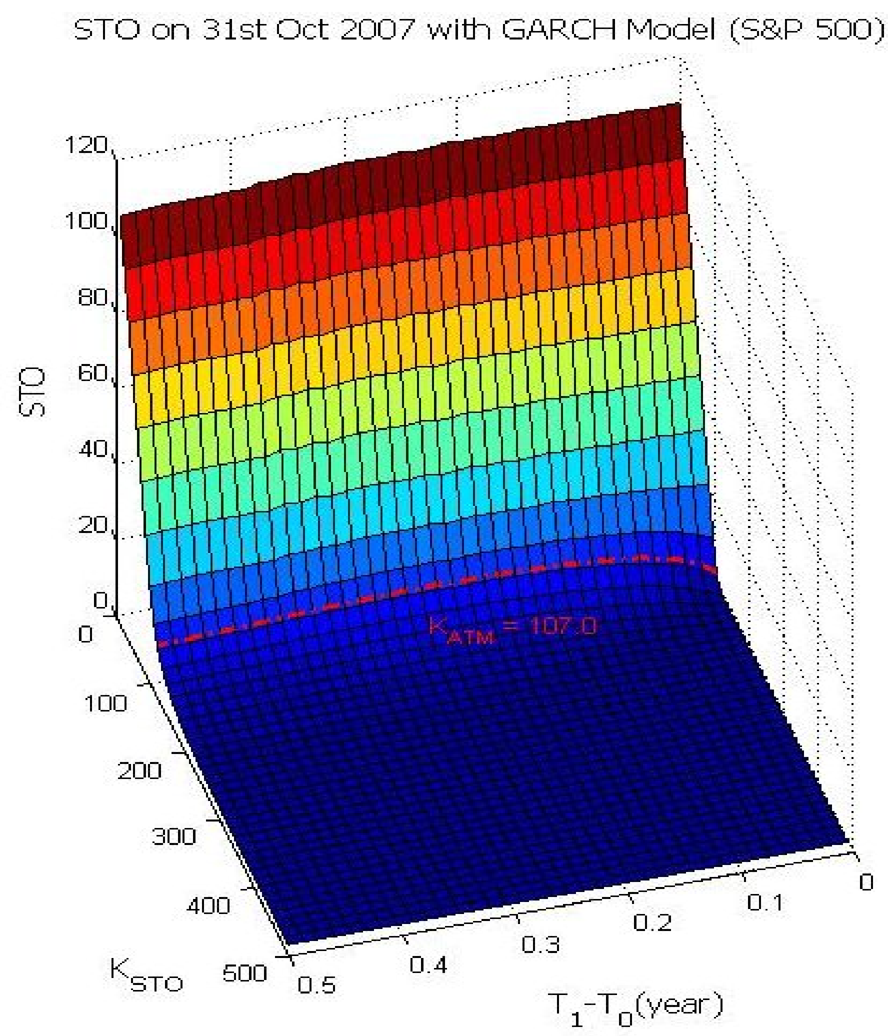
Difference between Heston and CEV (S&P 500)



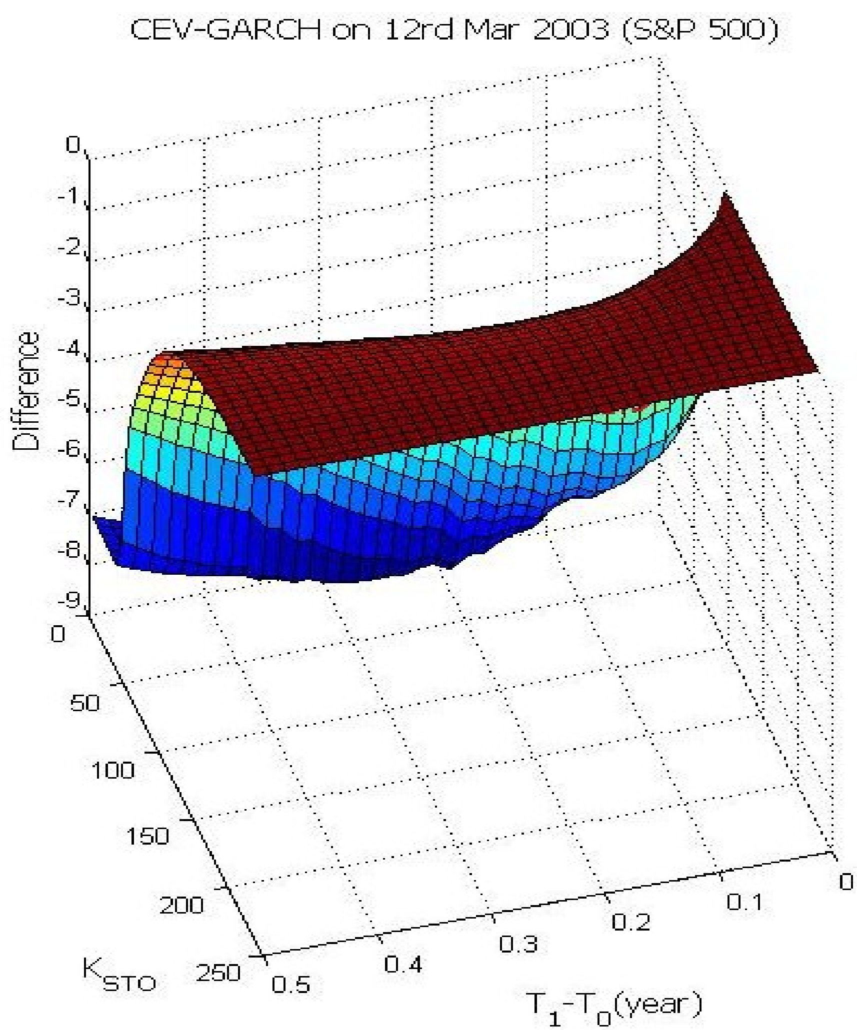


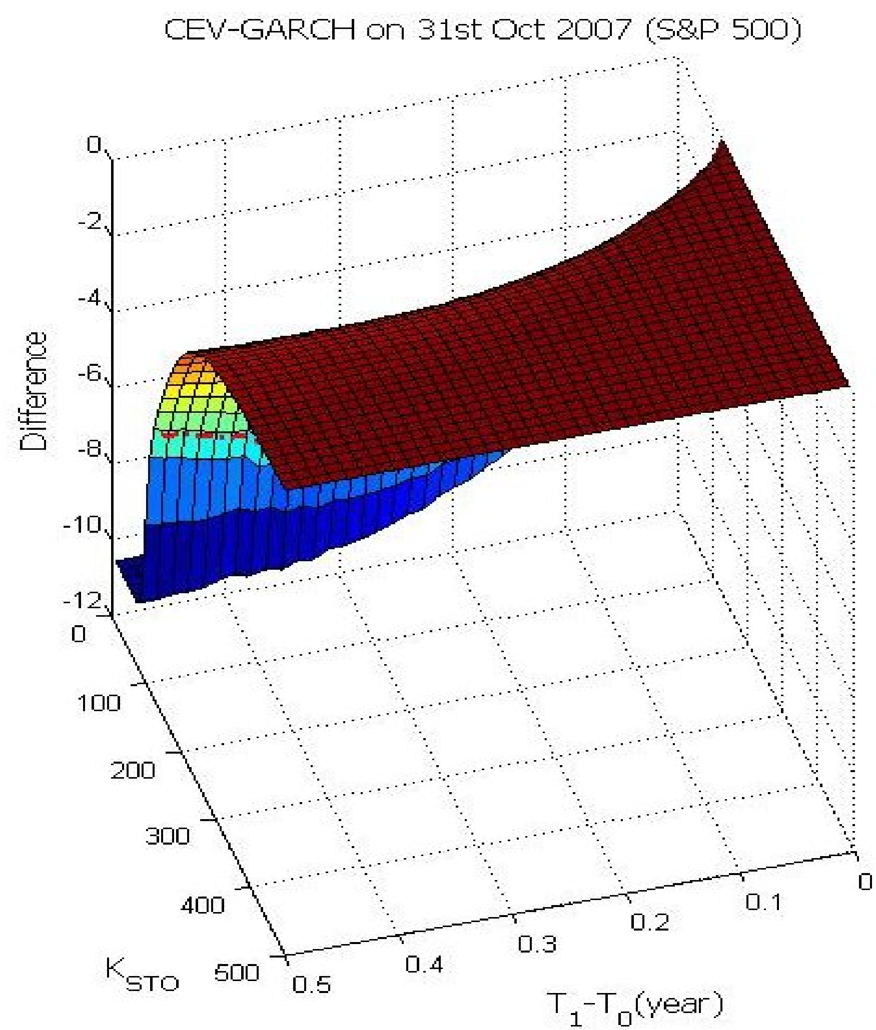
ATMF STO under GARCH model (S&P 500)





Difference between GARCH and CEV (S&P 500)





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Conclusion

- Heston is rejected for all markets. But GARCH model is not rejected for the FTSE 100, AEX and CAC 40.

- For the BEL 20 index we only find very little evidence of the leverage effect.

Conclusion

- The results of ATMF STO confirm the outcomes of hypothesis.
- The difference in parameter estimation across the different stock index markets translates in non-trivial difference in ATMF STO prices.

Thank you
for your attention
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