The impact of the UK government bank bail out on the CDS of major UK banks*

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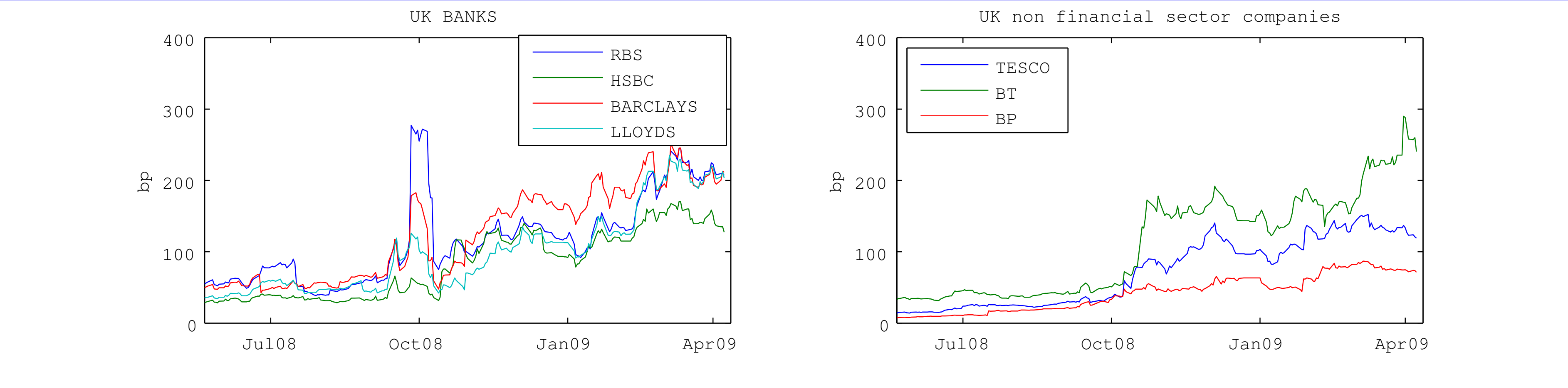
Motivation

Why the major UK banks Credit Default Swap (CDS) have been stable even since the Lehman brothers bankruptcy and the UK government's bail out?

It is almost impossible to specify the impact of probability of default (PD) and loss given default (LGD) on CDS itself. One of the most practical approach is the joint estimation with other securities such as CDS of different seniority, stock and stock options.



Outline of our Research(1)

Our research strategy is **using the common information among other financial assets to seperate PD and LGD in CDS** (joint estimation).

We assume the PD is common among all the financial assets by the same issuer. Then it is possible to interpret **stock as a defaultable security with zero recovery**.

Therefore, stock and stock options are useful products to estimate PD because we can observe **the "pure" PD (not the product of PD and LGD but PD itself)**.

Moreover, the listed options are counterparty risk free assets and provide multiple dimensional data to estimate the daily PD.

Outline of our Research(2)

Estimate the PD implied in equity options (implied PD).

1. To seperate credit risk and the other influence on option prices, we model the option prices of **defaultable stocks** under **stochastic volatility and interest rate**.

2. In the specification of volatility surface, we use a **singluar pertubation method**. It can provide more accurate approximation of volatility surface because it is essentially a model-free and non-parametric method.

Estimate the LGD implied in CDS (implied LGD)

1. We add the counterparty risk into the CDS valuation using **TED spread as "average" counterparty risk**.

Related Literatures

Jarrow (2001) and Jarrow et al. (2003) developed a methodology to estimate the PD implied in stock. Berndt et al. (2003) and Das and Hanouna (2009) estimate PD implied in stock prices and calculate the LGD implied in CDS.

Baba and Ueno (2006) and Schlafer and Uhrig-Homburg (2009) estimate PD and LGD from the senior and subordinate CDS.

Carr and Wu (2008&2009) estimate PD implied in stock options and calculate the LGD implied in CDS. Bayraktar (2008) and Bayraktar and Yang (2009) develop a non-parametric approach to estimate the PD implied in stock options using the sigular pertubation mthod.

Contribution of our Research

- Complete the methodology to estimate PD implied in stock options using the singular pertubation method.

  The previous works estimate the PD with parametric assumptions or fit the volatility surface using the PD implied in the corporate bonds.

- Develop a measure (the LGD implied in CDS) to analyze the liquidity of credit products and the risk preference in credit market.

- Specify the impact of PD and LGD on the CDS spread of major UK banks during the recent financial crisis.

Estimation the PD implied in stock options

For accurate and unbiased estimation, it is necessary to modify not only credit risk but also other assumptions.

- Option pricing under credit risk

  Add the intensity of hazard rate into the underlying stock price process.

- Modeling term structure of interest rate

  Previous works assume the flat or time homogeneous term structure like Vasicek (1977).

- Modeling volatility surface

  Nonparametric estimation using the singular pertubation method.

Model (Option Pricing Under Credit Risk)

The Cox process $\tilde{N}_t$ defines the time of default

$$\tilde{N}_t = \begin{cases} 0 & \tau > t \\ 1 & \tau \leq t \end{cases}$$

where

$$\tilde{N}_t = N_t \left(\int_0^t \lambda_s ds \right) \tag{1}$$

Stock price will become zero on default

$$d\bar{S}_t = \bar{S}_t \left(r_t dt + \sigma(\tilde{Y}_t) dW_t^{(0)} - d \left(N_t - \int_0^{t \vee \tau} \lambda_u du \right) \right) \tag{2}$$

Before default, stock price follows

$$dS_t = S_t \left((r_t + \lambda_t)\, dt + \sigma(\tilde{Y}_t) dW_t^{(0)} \right) \tag{3}$$

Model (Option Pricing Under Credit Risk (2))

Under the above setting European option prices are

$$
\begin{aligned}
Call(t,T) &= xN(d_1) - K\,E\left[\exp\left(-\int_t^T (r_s + \lambda_s)\,ds\right)\Big|\mathcal{F}_t\right] N(d_2)\,, \\
Put(t,T) &= -x\,N(-d_1) + K\,E\left[N(-d_2)\exp\left(-\int_t^T (r_s + \lambda_s)\,ds\right)\Big|\mathcal{F}_t\right] \\
&\quad + K\,E\left[\exp\left(-\int_t^T r_s\;ds\right) - \exp\left(-\int_t^T (r_s + \lambda_s) ds\right)\Big|\mathcal{F}_t\right]
\end{aligned}
$$

where $N()$ is the standard normal distribution function and

$$
\begin{aligned}
d_1 &= \frac{\log\left(\frac{x}{kB^c(t,T)}\right) + \frac{1}{2}\sigma(t,T)}{\sqrt{\sigma(t,T)}}, d_2 = d_1 - \sqrt{\sigma(t,T)}\,. \\
B^c(t,T) &= E\left[\exp\left(-\int_t^T (r_s + \lambda_s(l_T))\,ds\right)\Big|\mathcal{F}_t\right]\,.
\end{aligned}
$$

Model (Term Structure Modeling)

The exsiting researches assume term structure of interest rate is flat or time homogeneous. These simple models cannot capture the curvature of term structure especially in 2008.

Hull and White (1993) type term structure model can fit the observed term structure of spot rates perfectly and be solved analytically (the model is the special case of Hull and White (1990) model that $\beta_t = \beta$).

$$dr_t = (\alpha_t - \beta\, r_t)dt + \eta\, dW_t^{(1)} \tag{4}$$

Model (Volatility Surface Modeling)

We model the volatility surface using singular pertubation method. It is possible to rescale the stock price volatility and intensity of hazard process with the asymptotic expansions on ϵ and δ.

This is a model free method. It just assumes that the volatility and intensity follow mean reverting processes. Fouque et al. (2000&2003) proved that the asymptotics are independent of the level of Y_t and $\tilde{Y}_t$.

$$\begin{aligned}
\lambda_t &= f(Y_t, Z_t), \\
dY_t &= \frac{1}{\epsilon}(m - Y_t)dt + \frac{\upsilon\sqrt{2}}{\sqrt{\epsilon}} dW_t^{(2)} \\
dZ_t &= \delta c(Z_t)dt + g(Z_t)dW_t^{(3)} \\
d\tilde{Y}_t &= \left(\frac{1}{\epsilon}\left(\tilde{m} - \tilde{Y}_t\right) + \frac{\tilde{\upsilon}\sqrt{2}}{\sqrt{\epsilon}}\Gamma(\tilde{Y}_t)\right) dt + \frac{\tilde{\upsilon}\sqrt{2}}{\sqrt{\epsilon}} dW_t^{(4)}
\end{aligned}$$

Model (Volatility Surface Modeling (2))

The approximation of (put and call) option prices P_t is

$$\tilde{P}_{\epsilon,\delta} = P_0 + \sqrt{\epsilon} P_{1,0} + \sqrt{\delta} P_{0,1}\,, \tag{5}$$

where

P_0 is the analytical solution of option price.

$$\begin{aligned}
\sqrt{\epsilon} P_{1,0} &= -(T-t)\left(V_1^{\epsilon}(z) x^2 \frac{\partial^2 P_0}{\partial x^2} + V_2^{\epsilon} x \frac{\partial}{\partial x}\left(\frac{\partial^2 P_0}{\partial x^2}\right)\right) \\
&\quad + V_3^{\epsilon}(z)\left(-x \frac{\partial^2 P_0}{\partial x \partial \alpha} - \frac{P_0}{\partial \alpha}\right) + V_4^{\epsilon} x^2 \frac{\partial^3 P_0}{\partial x^2 \partial \alpha} + V_5^{\epsilon} x \frac{\partial^2 P_0}{\partial \eta \partial x} + V_6^{\epsilon} x \frac{\partial^2 P_0}{\partial x \partial \alpha} \\
\sqrt{\delta} P_{0,1} &= V_1^{\delta}(z) \frac{(T-t)^2}{2}\left(x^2 \frac{\partial^2 P_0}{\partial x^2}\right) + V_2^{\delta}\left[\left(x \frac{\partial^2 P_0}{\partial \alpha \partial x} - \frac{\partial P_0}{\partial \alpha}\right)\right. \\
&\quad \left. -(T-t)\left(x \frac{\partial^2 P_0}{\partial r \partial x} - \frac{\partial P_0}{\partial r}\right) + \frac{(T-t)^2}{2}\left(x^2 \frac{\partial^2 P_0}{\partial x^2} - x \frac{\partial P_0}{\partial x} + P_0\right)\right]
\end{aligned}$$

Model (CDS Spread under Couterparty Risk)

Counterparty risk free CDS spread is

$$CDS_F(t,T) = \frac{E\left[\exp\left(-\int_t^T r_s\ ds\right)|\mathcal{F}_t\right] - E\left[\exp\left(-\int_t^T (r_s + l_s\lambda_s)ds\right)|\mathcal{F}_t\right]}{\sum_{m=1}^{M} E\left[\exp\left(-\int_t^{T_m}(r_s + \lambda_s)ds\right)|\mathcal{F}_t\right]}$$

Crepey et al.(2009) derived CDS spread under counterparty risk

$$CDS_C(t,T) = \frac{E\left[\exp\left(-\int_t^T r_s\, ds\right) - \exp\left(-\int_t^T \left(r_s + (\lambda_s^1 + \lambda_s^2)l^*\right) ds\right)\Big| \mathcal{F}_t\right]}{\sum_{m=1}^{M} E\left[\exp\left(-\int_t^{T_m}(r_s + \lambda_s^1 + \lambda_s^2)\ ds\right)|\mathcal{F}_t\right]}$$

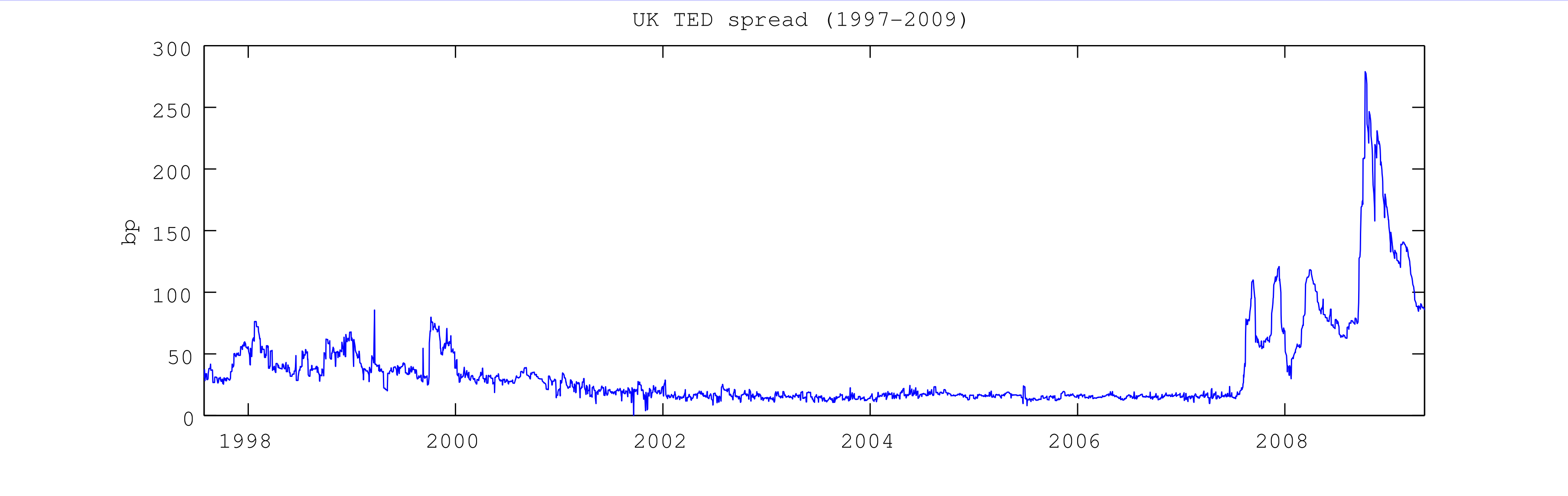
where

$$l^* = \begin{cases} l_1 & \tau_1 < \tau_2 \\ l_1 * l_2 & \tau_1 = \tau_2 \end{cases} \tag{6}$$

Model (CDS Spread under Couterparty Risk (2))

Counterparty risk is generally difficult to analyze quantitatively because the individual transaction data is not available.

As a simple approximation, we assume that TED spread implies the credit risk of financial institutions (the counterparty in CDS).



The LGD implied in CDS

The LGD implied in CDS is the potential indicator of risk appetite or market liquidity in credit market.

The LGD of CDS is, in some cases, determined in the auction of dealers after credit events while examiners investigate the present value of the firm's asset value and decide the LGD of the loan and bond.

The LGD of CDS does not necessarily match that of reference assets (bonds and loans). Helewege et al. (2009) showed the results of the auction did not match even the price of bonds particularly in 2008 while they were consistent with the market price of reference assets by 2007.

Data and Estimation

The sample period is June 2008- April 2009. We estimate UK major banks (HSBC, BARCLAYS, LLOYDS and RBS) and non-finacial sector companies (BP, BT and TESCO).

It includes the Lehman Brothers bankruptcy, the following turmoil in financial system and the bail out by governments.

The stock otions are listed in Euroclear. They are suitable to estimate the implied PD because they are free from counterparty risk.

The calibration is divided into two steps. (i)The first one is the estimation of the implied PD with minimization of the square error between (5) and actual option prices. (ii)The second one is the calculation from (6) and the implied PD.

Summary of Results

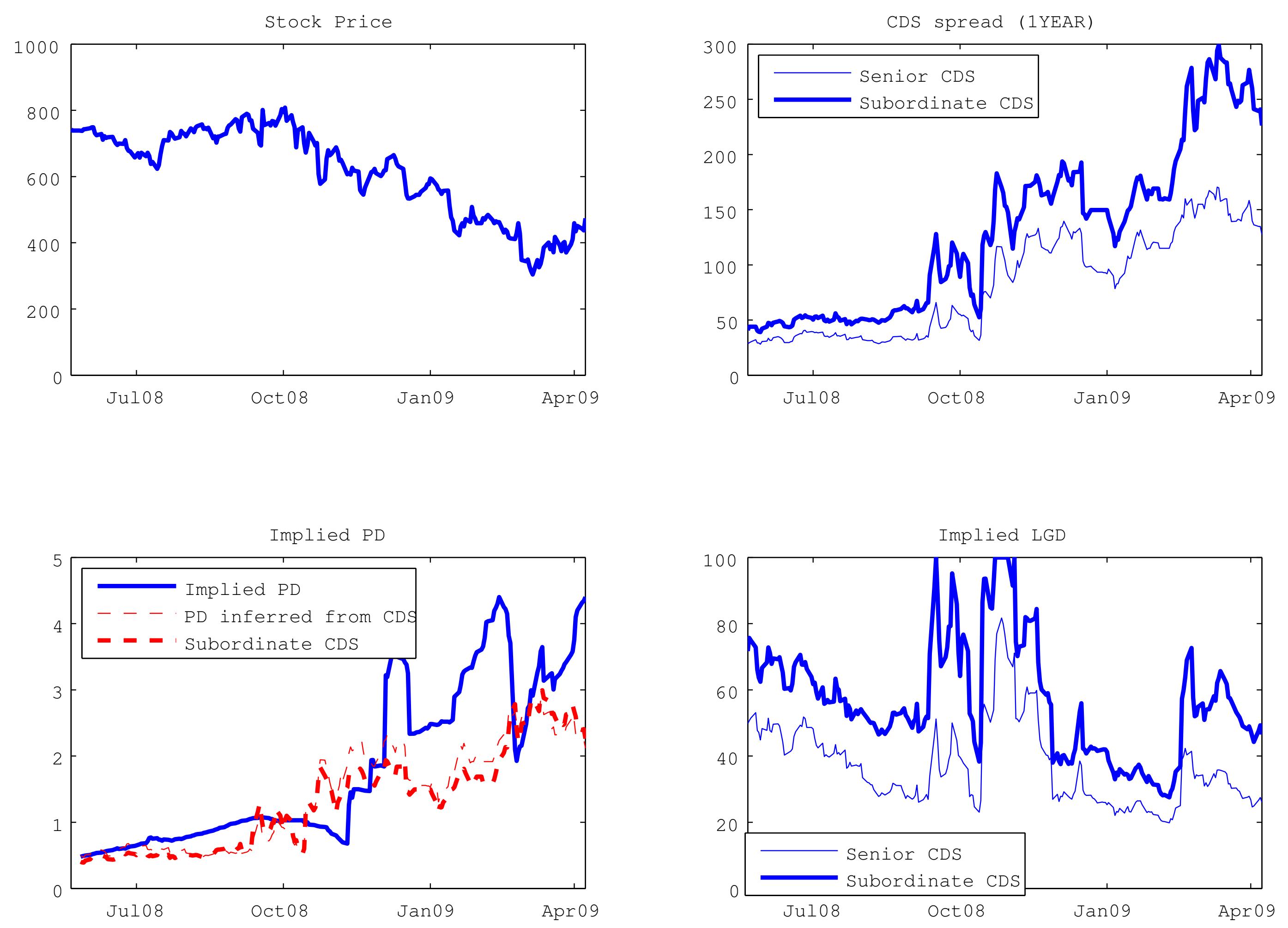
The implied PD of HSBC (non-bailed out bank) and non-finaicial sector companies have been stable and almost the same level as those calculated with the simple assumption(LGD=60%).

On the other hand the PD of deeply bailed out banks (RBS) has been much higer than those calculated with the simple assumption.

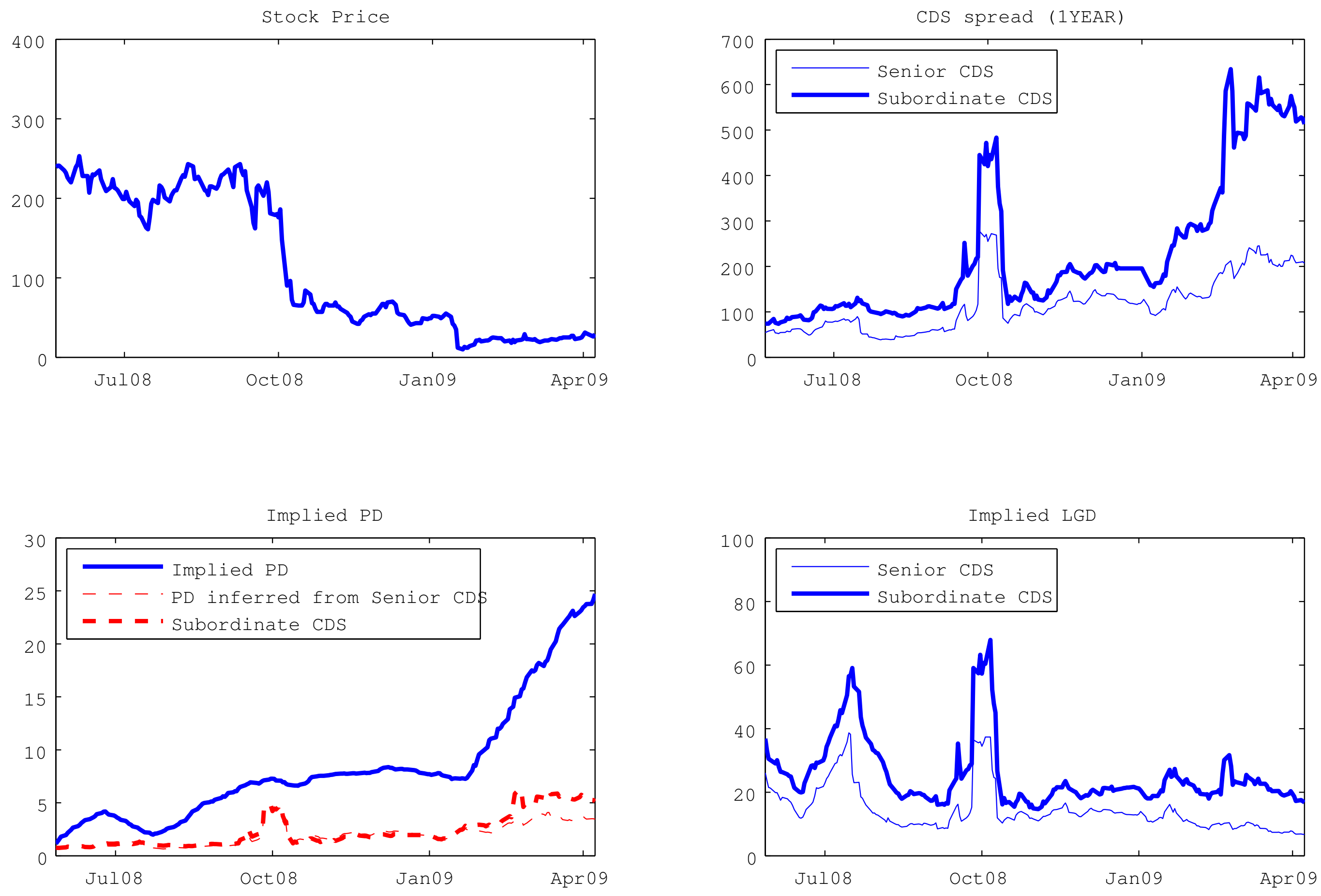
The implied LGDs indicate the simple assumption (LGD=60%) has been generally usuful approximation in HSBC and other non-financial sector compnies.

The implied LGD of beiled out banks especially RBS has been around 10-20% since the first capital injection by the UK government in October 2008.

Results (1) <HSBC>



Results (2) <RBS>



Concluding Remarks & Further Resaerch

- The results imply the variety and complexity of options when companies are in danger of bankruptcy.

  In the case of banks, options include not only default but government's bail out (nationalization etc).

- The analysis of the determinants of the implied PD and LGD.

  Which policy actions and events were crucial in the CDS spread?

- The relationship with loans and bonds.

  How the (il)liquidity of securities and derivatives influences the prices?