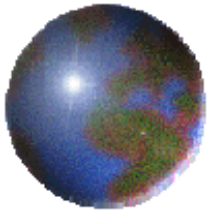

# Analysis of the Determinants of the iTraxx CDS Spreads using the Skewed Student's t AR-GARCH Model

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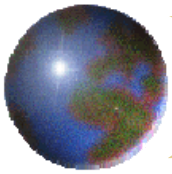


# Introduction

- Providing a benchmark for hedging or arbitrage decisions
- Structural form models
  - Firm's value
  - Equity volatility
  - Risk-free rate term structure
  - Bystrom (2005)
  - Alexander and Kaeck (2008)
- AR-GARCH with Skewed Student's t Marginal Density Function

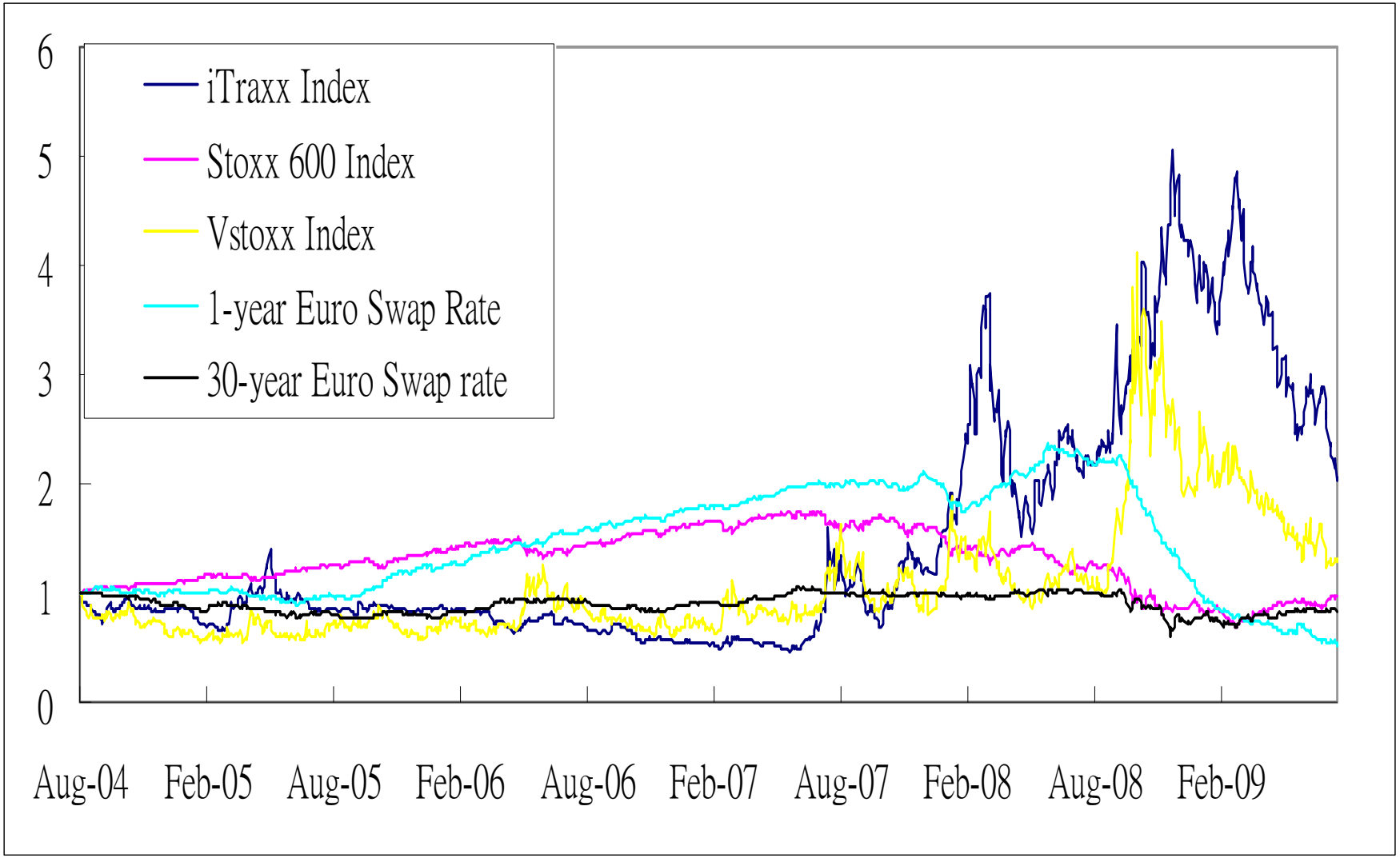
# Data

- The whole sample period: August 16 2004 to July 31 2009 (1254 daily observations)
  - The pre-crisis period: August 16 2004 to July 25 2007
  - The crisis period : July 26 2007 to July 31 2009
- CDS Index: the iTraxx 5-Year  Europe CDS index
- Firm's Value: the STOXX600 index
- Equity Volatility
  - Benkert (2004)
  - the VSTOXX 50 index
- Risk-free Rate
  - Houweling and Vorst (2002)
  - 1-year Euro Swap rate & 30-year Euro Swap rate

# Linear Regression Analysis

$$\Delta CDS_t = \beta_0 + \beta_1 \Delta STOXX_t + \beta_2 \Delta VSTOXX_t + \beta_3 \Delta CDS_{t-1} + \beta_4 \Delta SWAP(1)_t + \beta_5 \Delta SWAP(30)_t + \varepsilon_t$$

- Bystrom (2005): Positive first-order autocorrelation
  - Ljung_Box test
    - Q(20)=61.8043
    - Q(30)=78.883


6
5
4
3
2
1
0
iTraxx Index
Stoxx 600 Index
Vstoxx Index
1-year Euro Swap Rate
30-year Euro Swap rate
Aug-04
Feb-05
Aug-05
Feb-06
Aug-06
Feb-07
Aug-07
Feb-08
Aug-08
Feb-09

# Regression Results

| | Whole Period | Pre-crisis Period | Crisis Period |
|---|---|---|---|
| **Const.** | 0.0003 | 0.0007 | -0.0002 |
| | (0.3511) | (0.9452 ) | (-0.1280) |
| **STOXX600** | -1.1441 | -0.6844 | -1.0342 |
| | (-11.6977 ) | (-4.2315) | (-6.6069 |
| **VSTOXX50** | 0.0966 | 0.0744 | 0.1860) |
| | (4.1254 ) | (3.1669) | (4.0181) |
| **Yesterday's Change** | 0.1561 | 0.2946 | 0.1199 |
| | (6.7866) | (9.2624) | (3.4020) |
| **1-Year Euro Swap Rate** | -0.3170 | -0.3920 | **-0.2565** |
| | (-3.0275) | (-2.6795) | **(-1.5390)** |
| **30-year Euro Swap Rate** | -0.1548 | -0.1447 | **-0.1447** |
| | (-1.8729) | (-1.2093) | **(-1.1430)** |
| $R^2$ | **0.3431** | **0.2541** | **0.3858** |
| **Adj.** $R^2$ | **0.3404** | **0.2490** | **0.3797** |

# Comparison with Alexander and Kaeck (2008)

| | Whole Period | Pre-crisis Period | Crisis Period |
|---|---|---|---|
| ***Using the STOXX600 Index as the Proxy of the Firm's Value*** | | | |
| $R^2$ | 0.3431 | 0.2541 | 0.3858 |
| Adj. $R^2$ | 0.3404 | 0.249 | 0.3797 |
| ***Using the Portfolio as the Proxy of the Firm's Value*** | | | |
| $R^2$ | 0.3113 | 0.2487 | 0.3644 |
| Adj $R^2$ | 0.3086 | 0.2436 | 0.3581 |

# AR-GARCH Model

- The conditional mean equation of a univariate time series $y_t$

$$y_t = E\left(y_t \mid \Omega_{t-1}\right) + \varepsilon_t$$

$$\varepsilon_t = \sigma_t z_t$$

- The conditional variance equation

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2,$$

where and are no-negative integers

$\alpha_0 > 0, \quad \alpha_i \geq 0, \quad i = 1, \cdots, q$

$\beta_j \geq 0, \quad j = 1, \cdots, p.$

# The skewed Student's t density function of Hansen (1994)

$$g(t \mid \nu, \lambda) = \begin{cases} bc\left(1 + \frac{1}{\nu - 2}\left(\frac{bt + a}{1 - \lambda}\right)^2\right)^{\frac{-(\nu+1)}{2}} & t < -\frac{a}{b} \\ bc\left(1 + \frac{1}{\nu - 2}\left(\frac{bt + a}{1 + \lambda}\right)^2\right)^{\frac{-(\nu+1)}{2}} & t \geq -\frac{a}{b} \end{cases}$$

- where , $2 < \nu \leq \infty$ , and , $-1 < \lambda < 1$

$$a = 4\lambda c\left(\frac{\nu - 2}{\nu - 1}\right),$$

$$b^2 = 1 + 3\lambda^2 - a^2,$$

$$c = \frac{\Gamma\left(\frac{\nu + 1}{2}\right)}{\sqrt{\pi(\nu - 2)}\,\Gamma\left(\frac{\nu}{2}\right)},$$

# Estimation Results of AR-GARCH Model

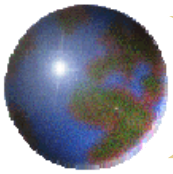
| | Normal | Student's t | Skewed Student's t |
|---|---|---|---|
| Const. | 0.0000 | 0.0007 | 0.000 |
| | (139565.08) | (4502.36) | (629697.4) |
| ARCH (1) | 0.2148 | 0.1327 | 0.1371 |
| | (59.6232) | (182.3046) | (138.0629) |
| GARCH (1) | 0.7711 | 0.8671 | 0.8597 |
| | (332.698) | (637.9965) | (1052.7893) |
| $\nu$ | $\infty$ | 5.6349 | 5.2808 |
| | | (7.8504) | (10.602) |
| $\lambda$ | 0 | 0 | 0.0731 |
| | | | (47.4667) |

| | Normal | Student's t | Skewed Student's t |
|---|---|---|---|
| **GOF** | 32.9425 | 47.6901 | 5.6463 |
| | 0.000 | 0.000 | 0.0594 |
| **ARCH-LM Test** | 0.059 | 0.2681 | 0.0007 |
| | 0.8081 | 0.6046 | 0.9796 |
| **Ljung Box Q-Test** | 19.4083 | 16.8717 | 16.8423 |
| | 0.931 | 0.9741 | 0.9744 |

| | Jarque-Bera Test | KS (Symmetric t) | KS(Skwed t) |
|---|---|---|---|
| **Statistic** | 8015.4678 | 0.0536 | 0.0193 |
| **p-Value** | 0.001 | 0.0007 | 0.6799 |

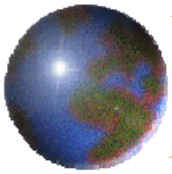
# Conclusion

- Risk-free rates has no statistically significant impact on the iTraxx CDS market during the crisis period
- The Skewed t AR-GARCH model is more suitable to capture the volatility clustering features in the iTraxx CDS market
- The sum of the coefficients on the lagged squared error and lagged conditional variance are very close to unity (i.e 0.9968)



# Further Work

- Out-of-sample forecasting
- Extending this study to investigate single-name CDSs



*Thank You!*