Stochastic Search

An element of randomness and statistics

Hill Climbing:

Min-conflict, Informed Backtrack, GSAT*

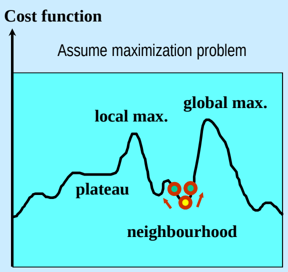
Metaheuristic Search: GENET, GLS*, GGA*

** Note: not in Tsang 1993*

Why Stochastic Search?

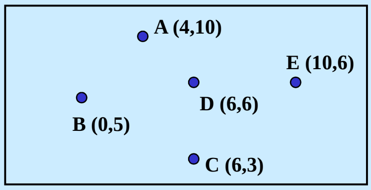
- Schedule 30 jobs to 10 machines:
  - Search space: 10^{30} leaf nodes
- Generously allow:
  - Explore one in every 10^{10} leaf nodes!
  - Examine 10^{10} nodes per second!
- Problem takes **300 years** to solve!!!
- How to contain *combinatorial explosion*?

Background: Local Search



- Ingredients:
  - Cost function
  - Neighbourhood function
  - [Optional] Strategy for visiting neighbours
    - e.g. *steepest ascent*
- Problems:
  - local optimum
  - Plateau
  - When to stop?
    - Ok with satisfiability
    - But not optimization

Example: The Travelling Salesman Problem (TSP)

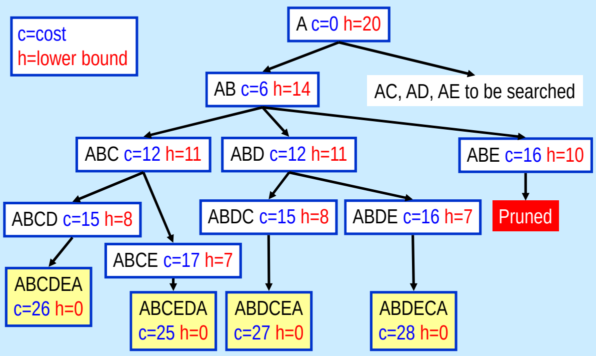


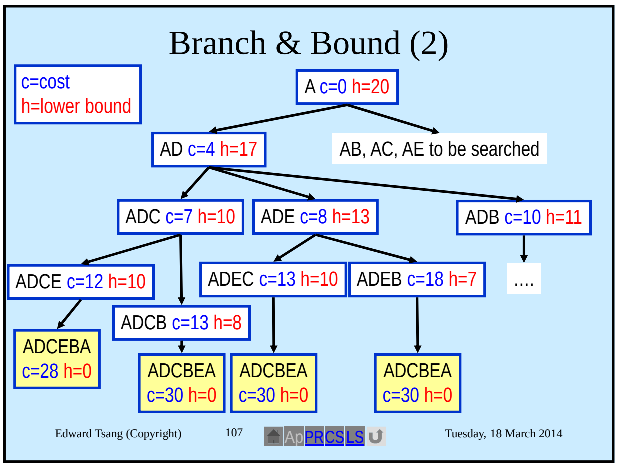
- Goal: to find *shortest route* through all cities
- Optimization involved: minimization

Distance Table for an example TSP

	A	B	C	D	E
A	--	6	7	4	7
B	6	--	6	6	10
C	7	6	--	3	5
D	4	6	3	--	4
E	7	10	5	4	--
Heuristic:	*4*	*6*	*3*	*3*	*4*

Branch & Bound (1)




Branch & Bound (2)
c=cost
h=lower bound
A c=0 h=20
AD c=4 h=17
AB, AC, AE to be searched
ADC c=7 h=10
ADE c=8 h=13
ADB c=10 h=11
ADCE c=12 h=10
ADEC c=13 h=10
ADEB c=18 h=7
....
ADCEBA c=28 h=0
ADCB c=13 h=8
ADCBEA c=30 h=0
ADCBEA c=30 h=0
ADCBEA c=30 h=0
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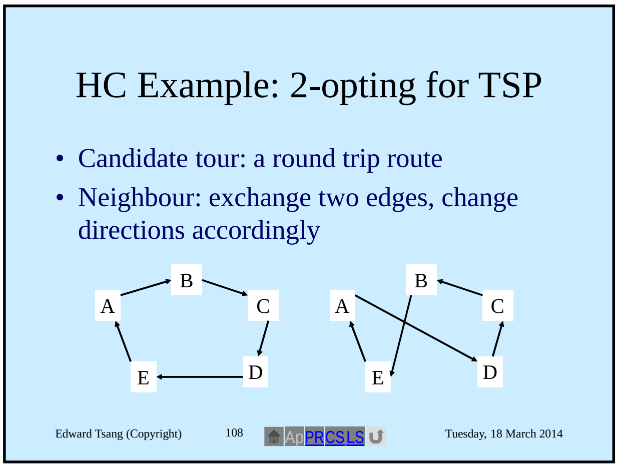
HC Example: 2-opting for TSP
• Candidate tour: a round trip route
• Neighbour: exchange two edges, change directions accordingly
A
B
C
D
E
A
B
C
D
E
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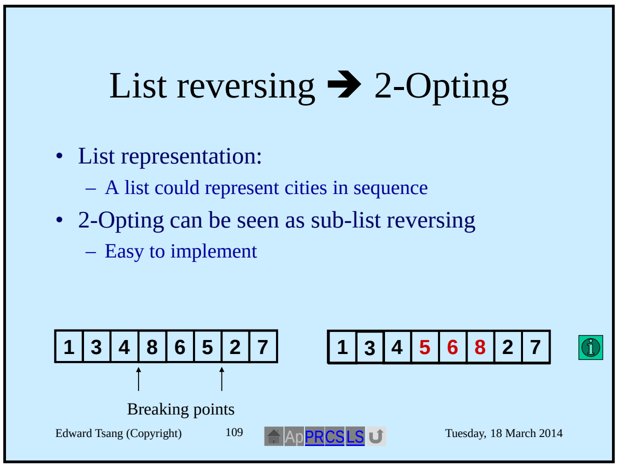
List reversing ➔ 2-Opting
• List representation:
– A list could represent cities in sequence
• 2-Opting can be seen as sub-list reversing
– Easy to implement
1 3 4 8 6 5 2 7
1 3 4 5 6 8 2 7
Breaking points
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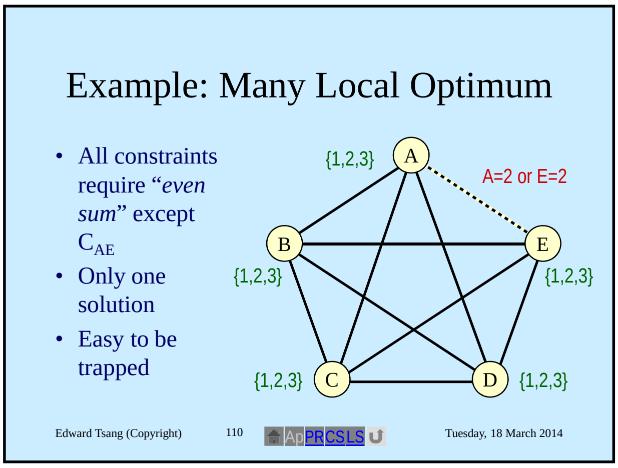
Example: Many Local Optimum
• All constraints require "even sum" except C_AE
• Only one solution
• Easy to be trapped
{1,2,3} A
A=2 or E=2
B {1,2,3}
E {1,2,3}
{1,2,3} C
D {1,2,3}
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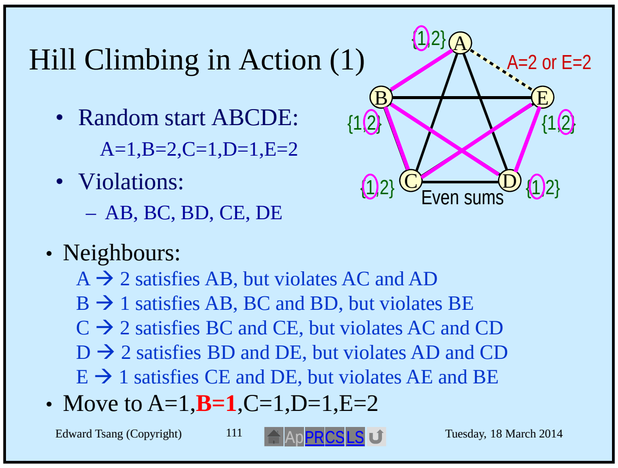
Hill Climbing in Action (1)
• Random start ABCDE:
A=1,B=2,C=1,D=1,E=2
• Violations:
– AB, BC, BD, CE, DE
{1,2} A
A=2 or E=2
{1,2} B
E {1,2}
{1,2} C
D {1,2}
Even sums
• Neighbours:
A → 2 satisfies AB, but violates AC and AD
B → 1 satisfies AB, BC and BD, but violates BE
C → 2 satisfies BC and CE, but violates AC and CD
D → 2 satisfies BD and DE, but violates AD and CD
E → 1 satisfies CE and DE, but violates AE and BE
• Move to A=1,B=1,C=1,D=1,E=2
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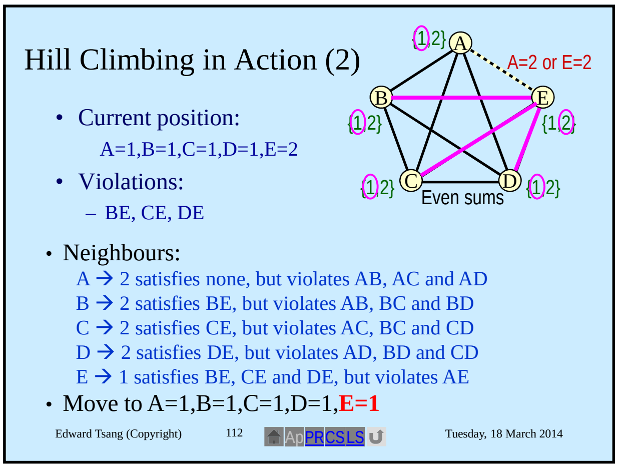
Hill Climbing in Action (2)
• Current position:
A=1,B=1,C=1,D=1,E=2
• Violations:
– BE, CE, DE
{1,2} A
A=2 or E=2
{1,2} B
E {1,2}
{1,2} C
D {1,2}
Even sums
• Neighbours:
A → 2 satisfies none, but violates AB, AC and AD
B → 2 satisfies BE, but violates AB, BC and BD
C → 2 satisfies CE, but violates AC, BC and CD
D → 2 satisfies DE, but violates AD, BD and CD
E → 1 satisfies BE, CE and DE, but violates AE
• Move to A=1,B=1,C=1,D=1,E=1
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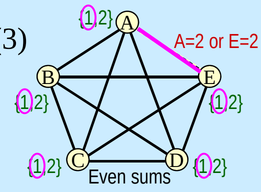
Hill Climbing in Action (3)



- Current position:
  A=1,B=1,C=1,D=1,E=1
- Violations:
  - AE
- Neighbours:
  A → 2 satisfies AE, but violates AB, AC and AD
  B → 2 satisfies nothing, but violates AB, BC, BD and BE
  C → 2 satisfies nothing, but violates AC, BC, CD and CE
  D → 2 satisfies nothing, but violates AD, BD, CD and DE
  E → 2 satisfies AE, but violates BE, CE and DE
- No profitable repair possible → **local optimum** found

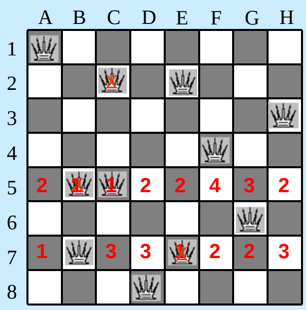
Guided Local Search →

Min-conflict Heuristic Repair

- Start with a random complete assignment
  - May initialise with min-conflict heuristic
- Repeat until all constraints are satisfied or run out of resources:
  - Randomly pick a variable *x* that is in conflict
  - Pick value *v* in domain of *x* such that
    - <*x*, *v*> violates the least number of constraints
    - break ties randomly

MC Heuristic Repair, Example

Solution found



- Start with random assignments
- C2 attacks G6
  D8 attacks E7
- Randomly pick one, say, E7, to repair

Count number of conflicts in each square

Randomly pick a square with least attacks, say, B7

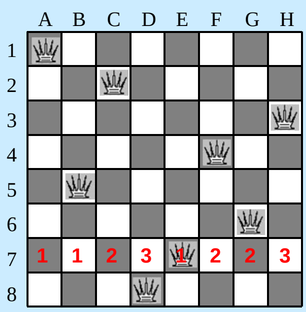
Repeat repair

Informed Backtrack
(complete algorithm)

- VarsLeft = random complete assignment
  - May initialize with min-conflict heuristic
- VarsDone = empty set
- Do until all variables violate no constraints:
  - Remove from VarsLeft variable *x* in conflict
  - Assign min-conflict value *v* to *x*, but
    - only accept *v* if <*x*,*v*> is consistent with VarsDone
  - Add <*x*,*v*> to VarsDone
  - Backtrack when necessary

Informed Backtrack, Example

- Step 1:
  VarsLeft = {1A, 2C, 3H, 4F, 5B, 6G, 7E, 8D}
  VarsDone = {}
  * Illegal variable picked: **7E**
  * Try a value for row 7, say, **7B**
  * Backtrack if needed
  explore 7B, 7A, 7E, …
- Step 2:
  VarsLeft = {1A, 2C, 3H, 4F, 5B, 6G, 8D}
  VarsDone = {**7B**}
  * Illegal variable picked: 5B
  * Pick a value for row 5, but not any value attacking 7B



Informed Backtrack, Analysis

- *Complete search* in nature
  - Basically *ordering variables* dynamically, guided by constraint violation
  - *Ordering values* by number of conflicts involved
  - All values are explored if needed
- Benefit of *hill-climbing*
  - Changing one label at a time
  - Chance to hit a solution by chance early
- Perhaps it deserves more research

The Satisfiability Problem (SAT)

- Boolean variables (true or false)
  - i.e. all domains are {0, 1}
- Constraints: in *Conjunctive Normal Form*:
  $(X_1 \lor \neg X_2 \lor X_3) \land (X_2 \lor X_4 \lor \neg X_5) \land \ldots$
- All CSPs can be translated to CNF
  - Each label <*x*, *v*> becomes one variable *XV*
  - If *XV* = 1, then *x* takes value *v*
  - Add clauses to ensure *x* takes one value only
- Note: given CSP with **n** variables, **m** values each:
  - The CSP has $\mathbf{m^n}$ leave nodes to explore
  - The equivalent SAT problem will have $\mathbf{2^{mn}}$ leaves

Satisfiability Problem, Example

- Suppose we say:
  A → B
  B → C
  C → ¬A
  which together refutes A
- 2-SAT problems are tractable
- Constraints may be represented by matrices

- Boolean variables
  A, B, C
- Constraints:
  ¬A ∨ B
  ¬B ∨ C
  ¬A ∨ ¬C
- Possible solutions:
  (<A,0><B,1><C,1>)
  (<A,0><B,0><C,1>)
  (<A,0><B,0><C,0>)

The GSAT Algorithm

- Parameters: *max_tries* & *max_flips*
- Do max_tries
  - Do max_flips
    - Pick an unsatisfied clause
    - Flip a variable that results in min. constraint violation
- Many many variations, including:
  - Adding random walks
  - Adding weights
  - Adding "*taboo lists*"

GSAT Example

- Boolean variables
  A, B, C
- Constraints:
  (a) ¬A ∨ B
  (b) ¬B ∨ C
  (c) ¬A ∨ ¬C
  (d) A ∨ B ∨ ¬C
- Random starting point
  (A=1, B=0, C=1)
- Solution found in step 3:
  (A=0, B=0, C=0)

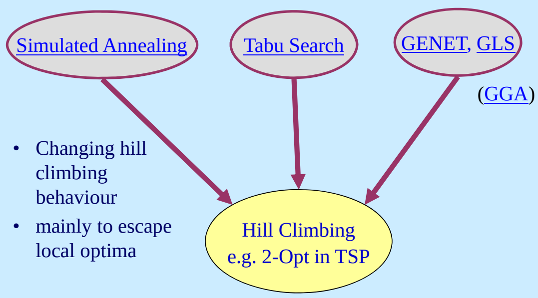
	A	B	C	Violation
1.	A=1	B=0	C=1	(a), (c)
neighbours	Violated constraint picked: (c)			
	A=0	B=0	C=1	(d)
	A=1	B=0	C=0	(a)
2.	A=1	B=0	C=0	(a)
Neigh-bours	A=0	B=0	C=0	--
	A=1	B=1	C=0	(b)
3.	A=0	B=0	C=0	--

GSAT in Local Optimal

- Constraints:
  (a) A ∨ ¬ B
  (b) ¬ A ∨ B
  (c) B ∨ ¬ C
  (d) ¬B ∨ C
  (e) ¬A ∨ C
  (f) A ∨ ¬C
  (g) ¬ A ∨ ¬ B ∨ ¬C
- Local optimal:
  (A=1, B=1, C=1)
  Solution missed:
  (A=0, B=0, C=0)

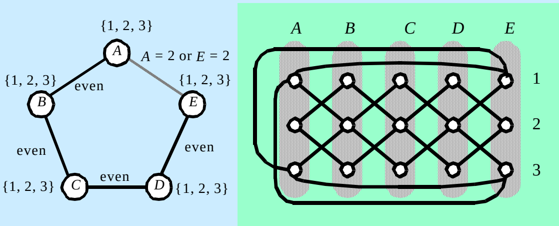
	A	B	C	Violation
	A=1	B=1	C=1	(g)
neighbours	A=0	B=1	C=1	(a), (f)
	A=1	B=0	C=1	(b), (c)
	A=1	B=1	C=0	(e), (e)
All moves are inferior to current position; unfortunately, the current position is not a solution				

Meta-heuristics



- Changing hill climbing behaviour
- mainly to escape local optima

GENET: Network Representation



- Build inhibitory connections
- Let the network converge to solutions

Guided Local Search

- Meta-heuristic Search
  - To sit on top of hill-climbing algorithms
    - can even sit on top of GAs
- Aims:
  - Escape from local minima
  - Introduce memory in the search process
  - Rationally distribute the search efforts

GLS: Augmented Cost Function

- Identifying solution ***features***, e.g. edges used
- Associate ***costs*** and ***penalties*** to features
- Augmented Cost Function

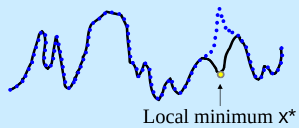
$$H(s) = G(s) + \lambda \cdot \Sigma p_i \cdot I_i(s)$$

  - where G is original cost function
  - λ is a parameter to GLS
  - p_i is penalty assoc. to feature i, initialized to 0
  - $I_i(s)$ = 1 if s exhibits feature i; 0 otherwise

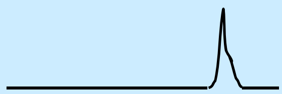
GLS & Filled Function Method

Augmented function to minimize, h' = h + f

Minimize (augmented) function h



At local minimum, add filled function f (penalty)



GLS Pseudo Code

- Iterative local search
- In a local minimum
  - Select Features
    - exhibited by the local minimum (search info.)
    - incur high costs (problem info.)
    - penalized fewer times in the past (GLS info.)
  - Increase penalties (strengthen constraints)
- Restart Local Search from Local Minimum

The GLS Algorithm

$H(s) = G(s) + \lambda * \Sigma p_i * I_i(s)$ Meta-heuristic Search

- Iterative local search
- In a local minimum
  - Select Features
    - Maximize *utility*

$$I_i(s^*) \times \frac{c_i}{1 + p_i}$$

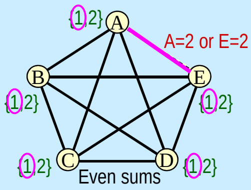
  - Increase penalties (strengthen constraints)
- Resume Local Search from Local Mi[nimum]

$I_i(s^*)$ = 1 if s* exhibits feature i; 0 otherwise

c_i = cost of feature i

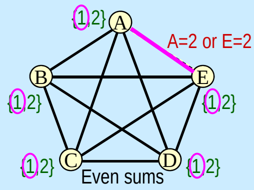
p_i = penalty of feature I (init. to 0)

GLS in Action (1)



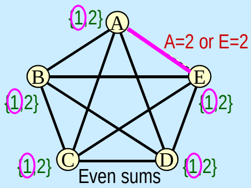
- Local optimum:
  - A=1,B=1,C=1,D=1,E=1
- Violations:
  - AE
- Neighbours:
  - A → 2 satisfies AE, but violates AB, AC and AD
  - B → 2 satisfies nothing, but violates AB, BC, BD and BE
  - C → 2 satisfies nothing, but violates AC, BC, CD and CE
  - D → 2 satisfies nothing, but violates AD, BD, CD and DE
  - E → 2 satisfies AE, but violates BE, CE and DE
- No profitable repair possible → **local optimum** found

GLS in Action (2)



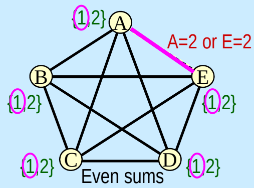
- Local optimum:
  - A=1,B=1,C=1,D=1,E=1
- Violations:
  - AE
- Features: I_{XY} = 1 means constraint on XY is violated
  - I_{AE} = true in this case
- Let cost for all features c_{XY} be 1
- Let λ = 1
- Penalty p_{AE} is incremented (from 0) to 1
  - Since only AE is violated

GLS in Action (3)



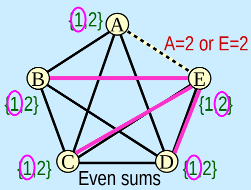
- Current position:
  - A=1,B=1,C=1,D=1,E=1
- Violations: AE
- $H(s) = G(s) + \lambda \cdot \Sigma\, p_i \cdot I_i(s)$
  - $= 1 + 1 \cdot 1 = 2$
- Neighbours:
  - A → 2 satisfies AE (2), but violates AB, AC and AD (3)
  - B → 2 satisfies nothing, but violates AB, BC, BD and BE
  - C → 2 satisfies nothing, but violates AC, BC, CD and CE
  - D → 2 satisfies nothing, but violates AD, BD, CD and DE
  - E → 2 satisfies AE (2), but violates BE, CE and DE (3)
- No profitable repair possible; penalise again

GLS in Action (4)



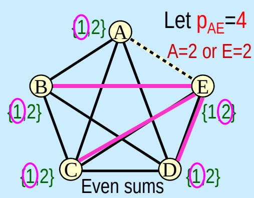
- Current position:
  - A=1,B=1,C=1,D=1,E=1
- Violations: AE
- $H(s) = G(s) + \lambda \cdot \Sigma\, p_i \cdot I_i(s)$
  - $= 1 + 1 \cdot 2 = 3$
- Neighbours:
  - A → 2 satisfies AE (3), but violates AB, AC and AD (3)
  - B → 2 satisfies nothing, but violates AB, BC, BD and BE
  - C → 2 satisfies nothing, but violates AC, BC, CD and CE
  - D → 2 satisfies nothing, but violates AD, BD, CD and DE
  - E → 2 satisfies AE (3), but violates BE, CE and DE (3)
- May make A=2 or E=2, if **sideways moves** allowed

GLS in Action (5)



- Current position:
  - A=1,B=1,C=1,D=1,E=2
- Violations: BE, CE, DE
- $H(s) = G(s) + \lambda \cdot \Sigma\, p_i \cdot I_i(s)$
  - $= 3 + 3 \cdot 0 = 3$
- Neighbours:
  - A → 2 satisfies nothing, but violates AB, AC and AD (3),
  - B → 2 satisfies BE (1), but violates AB, BC and BD (3)
  - C → 2 satisfies CE (1), but violates AC, BC and CD (3)
  - D → 2 satisfies DE (1), but violates AD, BD and CD (3)
  - E → 1 satisfies BE, CE and DE (3), but violates AE (3)
- May make E=1

GLS in Action (6)



- Current position:
  - A=1,B=1,C=1,D=1,E=2
- Violations: AE
- $H(s) = G(s) + \lambda \cdot \Sigma\, p_i \cdot I_i(s)$
  - $= 3 + 4 \cdot 0 = 3$
- Neighbours:
  - A → 2 satisfies nothing, but violates AB, AC and AD (3),
  - B → 2 satisfies BE (1), but violates AB, BC and BD (3)
  - C → 2 satisfies CE (1), but violates AC, BC and CD (3)
  - D → 2 satisfies DE (1), but violates AD, BD and CD (3)
  - E → 1 satisfies BE, CE and DE (3), but violates AE (4)
- Local optima reached, change p_{BE}, p_{CE} or p_{DE} to 1

GLS on TSP

Features:
- n^2 Features
- cost = distance given
- e.g. tour [1,5,3,4,6,2]

- Local search: 2-opting
- ➢ $\lambda = a \times g(t^*) / n$

a = parameter to tune, within (0, 1]

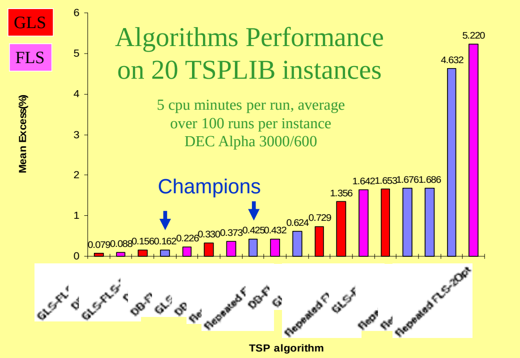
n = # of cities

t* = first local minimum produced by local search; g(t*) = cost of t*

	1	2	3	4	5	6
1					X	
2	X					
3				X		
4						X
5			X			
6		X				

Fast Local Search (FLS)

- For speeding up local search
  - through reduced neighbourhood
- Method:
  - **associate activation bit to problem features**
  - **Only active features examined for hill climbing**
- Cost for speed-up: lost of solution quality
- Rescue: solution quality compensated by GLS



Remarks: parameters in GLS

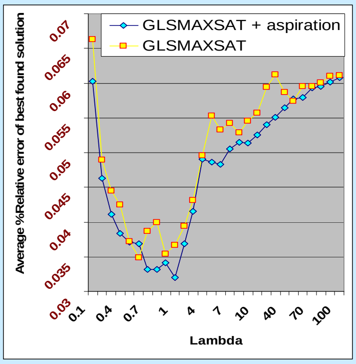
- Local search strategy
  - Needed in HC, SA, Tabu Search
- Features, costs
  - Sometimes come naturally from problem spec.
- Main parameter λ
  - Experimental results sometimes sensitive to λ
- *Less tuning to do than GA, NN, Tabu (& SA)*

Patrick Mills: GLS+

- **Aspiration**: if **G**(*s*) is better than best so far, then move to *s* even if **H(*s*)** is inferior
  - Work for MaxSAT and QAP but not SAT
  - Result generally improved at high λ value
- **Random moves:**
  - With probability *Pr* make random move
  - Results improved in QAP at low λ value
  - No effect on GLS → SAT / MAX SAT
- Combining Aspiration and Randomness:
  - GLS performance less sensitive to λ in QAP, [Max]Sat
- Where else / when will it work?

GLS + Aspiration

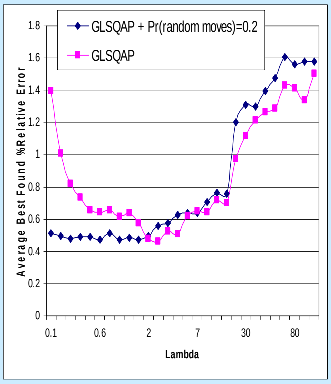
- Aspiration: if **G**(*s*) is better than best so far, then move to *s* even if **H(s)** is inferior
- Work for MaxSAT and QAP but not SAT
- Result generally improved at high λ value



G: Original Cost Function **H: Augmented Cost Function**

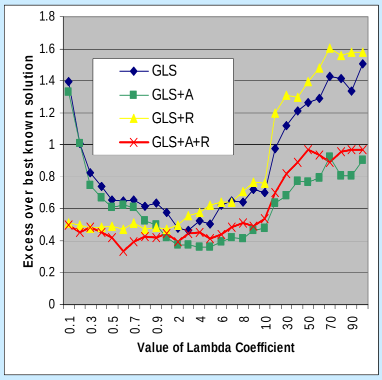
GLS + Randomness

- With probability *Pr* make random move
- Results improved in QAP at low λ value
- No effect on GLS → SAT / MAX SAT
- Randomness: when is it useful?



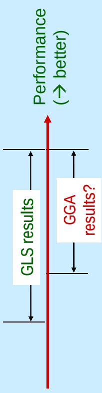
GLS + Aspiration + Randomness

- Result: performance is less sensitive to λ value
- Aspiration should become a standard feature of GLS
- Randomness sometimes helps
- Where/when will they succeed?



Guided Genetic Algorithm
Overview

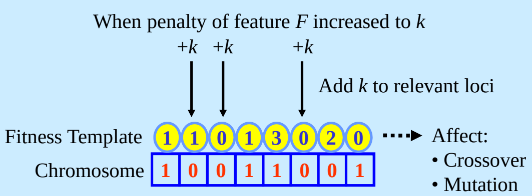
- Guided GA: Hybrid GLS + GA
- Using *Guided Local Search* as *meta-heuristic* for Genetic Algorithms
- Aims:
  - To extend the domain of GLS
  - To improve efficiency & effectiveness of GAs
  - To improve robustness of GLS



Guided Genetic Algorithm

- Initialize population
- Repeat
  - Run GA till best fitness remains unchanged for *n* generations (*n* is parameter)
  - Pick the best chromosome *X*
  - Penalize features of *X* according to GLS
  - Augment cost function
- Until Termination Condition

Using GLS Penalties in GGA



- High value in fitness template ⇒ instability

GGA Applications

- Royal Road Function
  - More effective than both GA and GLS
- Processors Configuration
- Radio Length Frequency Assignment
  - Gained robustness over GLS
- General Assignment Problem