CHAPTER 7

Conclusions

7.1 Contributions

The main theme of this thesis has been to work towards the goal of a more systematic approach to the selection and modification of *ZDC* formulations. There has been relatively little work done previously in this area. However, the construction of an effective *ZDC* formulation is a vital part of the overall problem solving process and, as we have seen in our work, it can have dramatic effects on the eventual cost of solving. In the course of our work, we have built on work by Nadel and developed heuristic techniques aimed at improving the problem solving process. We have made several significant contributions. We have addressed elements of the complete *ZDC* formulation selection process, so demonstrating what needs to be done is this area of research and, significantly, how we can actually address those needs.

7.1.1 A Context for ZDC Formulation Comparison

In chapter 3 we described a context for the heuristics selection of *ZDC* formulations. We built on the idea of *ZDC* formulation selection being a heuristic process and outlined the task of finding a good *ZDC* formulation as being one of finding a minimum in the space of all *ZDC* possible formulations for a problem, in terms of search costs. Such a context has not previously existed and its development provides a clear focus for the many facets of the *ZDC* formulation process. Our context identifies several new concepts such as *ZDC* formulation suggestion heuristics, H_s, move

operators, T, and evaluation heuristics, H_e. The ideas presented provide us with the basic building blocks for constructing systems which aim to introduce a level of automation into the process of *ZDC* formulation selection.

7.1.2 Extension of Nadel's Theoretical Complexity Work

The work presented in chapter 4 revisits previous work on the theoretical complexity of constraint satisfaction problems by Nadel. Furthermore we make a major extension to that Nadel's ideas by extending its applicability to the important class of intelligent backjumping algorithms. Our extensions have been shown to be accurate empirically, through a range of experiments in chapters 4, 5 and 6. By demonstrating that Nadel's work can be extended in this way, we are presented with the possibility of further extensions to hybrid algorithms such as forward checking with back-jumping (Prosser 1993).

In addition, we performed a thorough evaluation of Nadel's theoretical complexity models. This had not previously been done and as a result, we have been able to identify the usefulness of the approach and restrictions on it. Our conclusions are that theoretical complexity models do have a major role to play in the *ZDC* formulation process, forming a significant part of any measure vector that forms the basis of *ZDC* formulation evaluation heuristics, H_e.

7.1.3 Evaluation Heuristics for Variable Aggregation

In chapter 5 we investigated the application of evaluation heuristics to the variable aggregation transformation. This transformation can result in large gains in search cost when applied to certain problem classes. However, it can result in large losses in search cost in other cases. Our contribution in this chapter was to develop new, modified versions of the theoretical complexity equations, such that we could use them as the basis of a set of evaluation heuristics for variable aggregation.

Our α heuristics were demonstrated to be effective in making the variable aggregation transformation more robust in selecting when to merge variable pairs and when not to. We saw gains in solving cost measuring several orders of magnitude in some cases. Our work therefore makes this important *ZDC* transformation applicable to a wider range of problem classes, while having no a priori knowledge of the nature of those classes.

7.1.4 Evaluation Heuristics for Redundant Composition Constraints

The work in chapter 6 illustrated several aspects of our context by using *ZDC* formulation evaluation heuristics to improve the usefulness of adding redundant constraints. The addition of redundant constraints is one of the most important tools that can be used when constructing *ZDC* formulations. However, as we demonstrated, some redundant constraints can prove beneficial, while others can prove detrimental to the eventual cost of solving.

We developed a set of new *ZDC* formulation evaluation heuristics, for use with the addition of redundant composition constraints. We have shown how this simple, but effective, *ZDC* transformation can be significantly improved. As with the work in chapter 5, our approach resulted in improvements in both robustness and magnitude of the savings generated when adding redundant composition constraints.

The work in chapters 5 and 6 demonstrates an important point, that it is often dangerous to make general conclusions about the suitability of a particular *ZDC* formulation transformation function because in some cases the effects can be detrimental to the cost of solving. We have shown that it is possible to make informed decisions within the transformation process such that these adverse effects are reduced. In many respects, our work represents the first attempt to systematically modify a given *ZDC* formulation using intelligent feedback.

7.2 Future Work

The work we have presented in this thesis leads us to many new areas for future work. One obvious candidate is to extend the applicability of the heuristics which we have developed to cover a wider range of problems. In particular, it would useful to extend them to constraint satisfaction problems which contain general constraints. This would then allow us to apply our techniques to a wider range of problem classes, including many real world applications. Nadel gave indications about how this might be achieved in (Nadel 1983b).

In addition, the combination of *ZDC* evaluation heuristics with other properties, or measures, such as those described in chapter 2, may help improve their effectiveness. One possible approach is to use a hierarchical measure resolution function as suggested in chapter 3, where the theoretical

complexity measures are at the top of the hierarchy. There is much work which could be done in this area.

There is clearly scope for the further development of many aspects of the context we have described. There are many *ZDC* transformation techniques that have been developed already. An area of work that would be of great benefit would be to extend the range of evaluation heuristics to cover these. Recent work has also seen an increase in interest in the idea of *ZDC* formulation manipulation (Freuder & Sabin 1997)(Weigel & Faltings 1997). The coupling of the work we have described in this thesis with the ideas presented in those papers could lead to yet more effect problem solving techniques.

Another area where the use of evaluation heuristics might be useful is through the modification of a problem's *ZDC* formulation during the actual search process. This idea of using adaptive techniques for changing the nature of algorithms during search has been proposed (Borrett et al 1996a)(Borrett et al 1996b)(Sakkout et al 1996). If heuristics exist to suggest and evaluate the transformation of *ZDC* formulations of sub-problems, then it may be possible to use this approach to adaptively improve the *ZDC* formulation and improve the effectiveness of search still further.