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Fuzzy logic, trading uncertainty and technical trading

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ABSTRACT

From the market microstructure perspective, technical analysis can be profitable when informed traders make systematic mistakes or when uninformed traders have predictable impacts on price. However, chartists face a considerable degree of trading uncertainty because technical indicators such as moving averages are essentially imperfect filters with a nonzero *phase shift*. Consequently, technical trading may result in erroneous trading recommendations and substantial losses. This paper presents an uncertainty reduction approach based on fuzzy logic that addresses two problems related to the uncertainty embedded in technical trading strategies: market timing and order size. The results of our high-frequency exercises show that 'fuzzy technical indicators' dominate standard moving average technical indicators and filter rules for the Euro-US dollar (EUR-USD) exchange rates, especially on high-volatility days.

1. Introduction

Technical trading models typically rely on technical indicators constructed from past price and volume information that generate discrete (buy or sell) trading recommendations. Such models are atheoretic and ignore fundamental information about the price; however, they have been shown to result in trading profitability. Specifically, the success of technical trading violates the weak form of the efficient market hypothesis, which states that past prices should not assist traders in earning unusually high returns. In general, the literature examines the practical value of two types of analysis: *charting*, which identifies geometric patterns in the history of prices, and *technical indicators approach*, which mechanically applies mathematical trading rules constructed from past and present prices. Studies that find charting profitable include Chang and Osler (1999), Lo et al. (2000) and Savin et al. (2007), whereas evidence for the profitability of technical indicators can be found in Neftci (1991), Levich and Thomas (1993), Brock et al. (1992), Neely et al. (1997), Allen and Karjalainen (1999) and Gençay (1992).

Some of the above contributions combine technical analysis with other statistical methodologies. For example, Lo et al. (2000) show that charting based on automatic pattern recognition with kernel regressions adds value to the investment process. In a related study, Savin et al. (2007) produce similar results for price patterns. Gençay (1992) uses technical indicators to feed an artificial neural network model and thereby demonstrates that trading signals produced by such a combination outperform a simple buy-and-hold strategy. Allen and Karjalainen (1999) apply genetic programming to search for ex-ante "optimal" trading rules. In summary, several papers present evidence that technical analysis can be informative for the price, although its profitability can vary over time, which is in line with the adaptive markets hypothesis (Lo, 2004).[1]

In light of the market microstructure theory, technical trading may be profitable when informed traders make systematic mistakes or when uninformed traders have a predictable impact on price (Harris, 2003). When technical traders reveal and trade on mistakes made by informed traders, they in turn themselves become informed traders. The trading of these so-called *information-oriented technical traders* corrects prices and improves market efficiency. Information-oriented technical trading is,

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[1] See Irwin and Park (2007) and Menkhoff and Taylor (2007) or, more recently, Neely and Weller (2012) for extensive surveys on the application of technical analysis in the foreign exchange and equity markets.

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however, quite difficult in practice because informed traders correct their mistakes and learn from their past actions. This process diminishes profitable information-oriented technical trading opportunities. In contrast to technical trading on the informed traders' actions, *sentiment-oriented technical traders* exploit predictable price patterns caused by uninformed traders. Such an order-anticipating approach attempts to front-run the uninformed traders and trade before they trade. Sentiment-oriented technical trading can be useful when it correctly anticipates the impacts that uninformed traders will have on prices. In this paper, we extend the activity of sentiment-oriented technical traders (or order anticipators) to 'uncertainty reduction', whereas the uninformed traders are considered to be pure technical traders who employ simple technical indicators.

The practice of mechanically applying technical indicators in investment management without any uncertainty considerations could potentially be dangerous. Uncertainties in foreign exchange (FX) and equity markets can originate from, for example, market regime shifts, the impact of large trades on price, short-sale restrictions, incomplete data or behavioral issues. Recently, Lo and Mueller (2010) have argued that the presence of inappropriately identified uncertainties in a quantitative investment strategy can adversely affect risk management efforts. They introduce five levels of uncertainty: perfect certainty (e.g., direct trading costs), risk (e.g., probability distributions of trading volumes), fully reducible uncertainty (e.g., statistical framework for time series analysis), partially reducible uncertainty (e.g., multiple market regimes) and irreducible uncertainty (e.g., tail risk). Each of these levels is to be addressed with an appropriate set of skills and methodologies. For instance, the application of time series or linear regression analysis to tail events rather than extreme value theory could be disastrous for an investment strategy. In the same vein, utilizing technical indicators in trading while neglecting the uncertainty aspect of such actions would probably result in a series of unnecessarily risky trading recommendations.

Our analysis addresses the issue of trading model uncertainty (or trading uncertainty) that belongs to the partially reducible uncertainty domain, as defined above. This situation involves the uncertainty in decision making that arises if there is insufficient knowledge regarding the appropriateness of the trading model. Trading uncertainty represents a refinement of the economic uncertainty concept also known as model uncertainty. Model uncertainty generally arises from the potential incorrectness of the choice of the model that is generating the data. In this context, Pesaran and Timmermann (2002) link model uncertainty to an investor facing many competing forecasting models.[2] This problem is the reason why predictability and profitable opportunities in financial markets are short-lived (Timmermann and Granger, 2004).[3]

This paper introduces fuzzy logic as a tool that can help traders to control for the trading uncertainty aspect of employing technical indicators. The information generated by technical indicators is possibly imprecise, incomplete or unreliable. Fuzzy logic by its very nature tolerates uncertainty by defining variables, i.e., technical indicators, as imprecise linguistic terms that cover a broad fuzzy variable range; for example, trading signals can be expressed in a more sophisticated fashion as a 'strong buy', 'hold' or 'very strong sell.' Furthermore, technical indicators such as moving averages are prone to indicating a turning point later than the it actually occurs, as they are essentially imperfect filters with a nonzero *phase shift* (Gençay et al., 2001). Management of uncertainty in such situations is of the utmost importance. Fuzzy logic involves more continuous and conservative decision making than buy or sell recommendations, and it thereby partially reduces trading uncertainty in volatile markets. In addition, fuzzy logic can reduce trading costs by controlling for the order size, whereas pure technical indicators commit all available funds to a trading position. By using fuzzy logic, we attempt to resolve two problems related to the uncertainty embedded in investment strategies based solely on technical trading rules: market timing ("when to trade") and order size ("how to trade").

Studies on the application of fuzzy logic in financial economics have been scarce (Bekiros and Georgoutsos, 2007) and usually are considered mostly in tandem with other methodologies such as artificial neural networks (Gradojevic, 2007) or reinforced learning of agent-based systems (Bekiros, 2010; Tay and Linn, 2001). Additionally, Bojadziev and Bojadziev (1997) uses fuzzy logic to evaluate a client's risk tolerance based on the annual income and total net worth, whereas Serguieva and Hunter (2004) evaluate the risk associated with investing in 35 UK companies traded on the London Stock Exchange. With regard to technical trading, research efforts have centered on fuzzy logic-assisted charting (Zhou and Dong, 2004) but not on technical indicators. The fact that charting is primarily visual, whereas the technical indicators approach is essentially mathematical, suggests that the latter is more amenable to statistical methodologies such as fuzzy logic.

In this paper, our goal is to reduce the trading uncertainty of the standard technical indicators approach by utilizing fuzzy logic technical trading rules that are more robust with respect to errors in decision-making (trading). We directly compare the efficacy of standard technical indicators with that of fuzzy technical indicators for high-frequency (1-min) EUR-USD exchange rates in 2005. Furthermore, we develop five testable hypotheses that involve the relationship between high-frequency profitability and volatility (Hypotheses 1–3) and the ranking of the trading strategies (Hypotheses 4 and 5). We find that the extension of standard technical trading strategies with the fuzzy control methodology results in improved profitability. Our results show that fuzzy technical indicators are particularly useful for reducing trading losses on highly volatile days of the week. On such days, profits from pure technical trading strategies decrease and profits from fuzzy technical trading strategies increase. Overall, higher volatility leads to greater excess returns of fuzzy technical trading strategies relative to pure technical trading strategies. The documented gains in conditional mean returns are, in general, statistically significant over 50 weeks of 1-min EUR-USD exchange rates, whereas the buy-and-hold strategy performs poorly, irrespective of volatility. Finally, we link our results to market microstructure in the sense that the profitability of fuzzy logic-based technical trading might be used to explain the motivation for pursuing sentiment-oriented technical trading strategies. Such strategies are devised by order anticipators who front-run uninformed traders who apply simple technical trading rules. In accordance with Bekiros (2010), we argue that fuzzy control acts as a learning mechanism through which it is possible to better predict turning points and to thus react before uninformed technical traders trade. Hence, we conclude that the profitability of sentiment-oriented technical trading is particularly pronounced during high-volatility trading sessions.

In Section 2, we provide a brief overview of fuzzy logic, including an illustrative example for the S&P-500 Index. The data are described in Section 3. This section also develops testable economic and ranking hypotheses for the competing trading strategies. The construction of our fuzzy technical indicators and our results are reported in Section 4. We conclude and offer some potential future research avenues in Section 5.

[2] This type of uncertainty is also used to denote heteroskedasticity in the conditional variance of asset returns (Easley and O'Hara, 2010).

[3] Model averaging techniques are typically used to reduce model uncertainty. Such approaches can be found when model uncertainty is present in the shaping of monetary policy (Cogley et al., 2011), hedge fund pricing (Vrontos et al., 2008) and investment management (Avramov, 2002; Pesaran et al., 2009).

2. Fuzzy logic fundamentals

Fuzzy logic is built upon the notion of fuzzy sets (Zadeh, 1965). Unlike traditional sets (intervals), fuzzy sets allow for the concept of *partial membership*. This enables discrimination between elements that are relevant to the phenomenon of interest and those of borderline importance that involve imprecision and uncertainty. Information granules, such as "high speed", "significant risk" or "strong sell", can be processed using fuzzy logic whereby each linguistic term ("high", "significant" and "strong") describes a fuzzy set. A fuzzy set (A) defined in X is represented by its membership function as follows: $A{:}X \rightarrow [0,1]$; where $A(x)$ denotes a degree of membership of x in A. Membership functions can be of various types, including triangular, trapezoidal, Gaussian, sigmoidal and polynomial. It should be noted that larger values of a membership function indicate higher degrees of membership.[4]

Any fuzzy model has three main components: (1) a fuzzy "rule base" in the form of a set of "if-then" rules (expert knowledge about the model), (2) a fuzzification module that transforms the explanatory variables (inputs) into fuzzy variables and (3) a defuzzification module that converts the conclusion from the fuzzy domain into the dependent variable (output). To design the fuzzy model, information must be gathered on how to construct the rule base. Typically, this information is represented by the expert knowledge about the process or is compiled by studying the historical data. The rules can, for instance, state that "if the long moving average is ⟨LARGE⟩ and the short moving average is ⟨VERY SMALL⟩, then the technical trading signal is ⟨STRONG SELL⟩."[5]

In a fuzzy system, the process of generating the output (trading recommendation) begins with fuzzifying the inputs (components of technical indicators, such as moving averages or filter values) and then executing all of the active rules from the rule base. The process generates fuzzy conclusions about the output variable for each rule. The conclusions of the active rules used in decision making are then aggregated into a fuzzy conclusion about the output variable that captures the influence of the output membership functions associated with the rules. After defuzzification, a single value output (trading position) is generated.

The following example will illustrate fuzzy decision making ('fuzzy technical indicators') and compare it with the standard moving average technical indicators approach. Fig. 1 plots daily closing prices along with the 50-day moving average (MA (50)) for the S&P-500 Index from July 1, 2010 to September 30, 2010. We will concentrate on two occasions when the price penetrated MA (50) from below, thus indicating a buy signal: August 17, 2010 and September 2, 2010. On the first date, the S&P-500 Index closes at 1092.54, and the MA (50) is 1088.33. On the second date, the corresponding figures are 1090.10 and 1081.26, respectively. The standard moving average technical indicator generates a buy signal on both days and incurs a loss on the first signal because the price makes an unanticipated drop on August 19. However, the fuzzy moving average technical indicator accounts for the magnitude of discrepancy between the S&P-500 Index value and the MA (50) and generates a 'WEAK BUY' signal (i.e., invest roughly 40% of your current endowment) on August 17. Hence, in contrast to the standard moving average, the fuzzy logic approach does not commit all investment funds to the position. Furthermore, on September 2, 2010, fuzzy logic recognizes the larger discrepancy between the S&P-500 Index value and the MA (50) and generates a 'STRONG BUY' recommendation (i.e., invest roughly 92% of your current endowment). The same signal is generated by the standard MA (50) indicator, and as the S&P-500 Index continues to rise, both strategies generate a profit.

Our model has two inputs, i.e., MA (50) and daily closing price, which are both fuzzified on the interval [0,1] into the following five triangular fuzzy membership functions: "VERY SMALL," "SMALL," "MEDIUM," "LARGE" and "VERY LARGE". The output is a trading recommendation that is fuzzified on the interval $[-1,1]$ into five triangular fuzzy membership functions with the following labels: "STRONG SELL," "WEAK SELL," "HOLD," "WEAK BUY" and "STRONG BUY". The rule base contains $5^2 = 25$ rules that compare all possible combinations of the two inputs and produce the appropriate outputs. Of interest are the fuzzy trading recommendations on August 17, 2010 and September 2, 2010. For the values of the inputs on August 17, the fuzzy system output is 0.409, which corresponds to a 'WEAK BUY' signal. On September 2, the output generated by the fuzzy system is 0.918, which is a 'STRONG BUY' signal. Clearly, this example shows that by producing a more conservative trading signal on August 17, fuzzy control successfully avoided the extent of losses incurred by the standard moving average trading indicator.

In our example, MA (50) missed the turning point on August 19, 2010 because of the so-called *phase shift* of the moving average filter (Gençay et al., 2001). An ideal trading filter should retain lower frequencies with lesser weights towards higher frequencies. Such a filter preserves the temporal memory of the data while eliminating excessive higher frequency noise. The fuzzy Gaussian filter has such a capability. However, the MA (50) filter selectively concentrates near zero frequency with a compressed presence at lower frequencies. Such arbitrary frequency selection may omit the temporal memory necessary to identify local trends and turning points.

The fuzzy rule base generates a continuous decision surface in the form of mapping from the inputs to the output. It basically accounts for the distance between the inputs and produces a trading signal that identifies the exact fraction of the funds that are to be allocated to a position. Here, the distance between the inputs (i.e., between the price and the moving average or the filter) is viewed as a measure of trading uncertainty that increases as the distance decreases. As can be seen in the above example, this additional processing of pure technical indicator signals reduces the losses from missed turning points. Additionally, fuzzy logic cuts trading costs by not committing all available funds to a trading position. Therefore, although both strategies are subject to the same transaction cost, fuzzy control can adjust the trading volume.

The remainder of this section describes the relevant aspects of the fuzzy control design employed by this paper. As in Gradojevic (2007), the input membership functions are Gaussian and the output is represented by triangular membership functions. Furthermore, the inference mechanism is the so-called "Mamdani inference" (Mamdani and Assilian, 1975), whereas the defuzzification method is the "centroid of area". All (long-short) moving average differences and the differences between the filter value and price are normalized and fuzzified on the interval $[-1,1]$. Gaussian fuzzy membership functions are used, as they produce a relatively smooth input–output mapping. These functions are defined by a mean and standard deviation that are arbitrarily set to slice the variable domain into overlapping Gaussian functions that have the same shape and the highest degree of membership for the mean value. Each input is characterized by nine states, and the following fuzzy sets are assigned: "VERY NEGATIVE," "NEGATIVE," "MEDIUM NEGATIVE," "WEAK NEGATIVE," "STABLE," "WEAK POSITIVE," "MEDIUM POSITIVE," "POSITIVE" and "VERY POSITIVE". Similarly, the trading recommendation variable is assumed to have nine states represented by linear-shaped functions as follows: "VERY STRONG SELL," "STRONG SELL," "MEDIUM SELL," "WEAK

[4] Gradojevic (2007) and Cox (1992) provide a detailed treatment of fuzzy logic fundamentals. Essentially, fuzzy logic in the form of approximate reasoning has found applications in medicine, engineering, business, economics and meteorology.

[5] The design of the fuzzy logic model applied in this paper largely follows Gradojevic (2007).

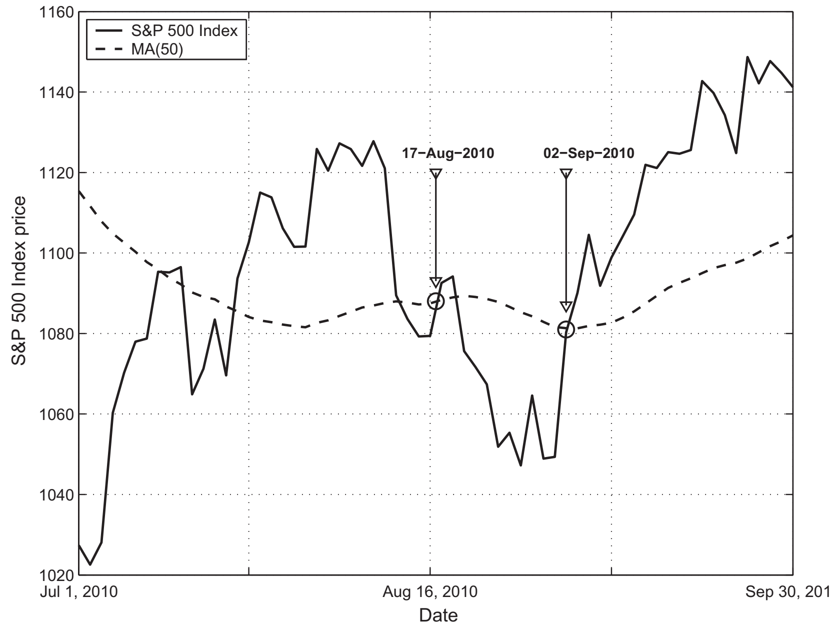

Fig. 1. Moving average technical indicators for the S&P-500 Index. The data cover the period from July 1, 2010 to September 30, 2010. The daily closing S&P-500 Index prices are plotted with a solid line, whereas the 50-day moving average (MA (50)) is plotted as a dashed line. We consider two dates when a buy signal was generated: August 17, 2010 and September 2, 2010 (marked with circles).

SELL," "HOLD," "WEAK BUY," "MEDIUM BUY," "STRONG BUY" and "VERY STRONG BUY". These states uniquely define the trading strategy in which positive signals are interpreted as long positions and negative signals as short positions.[6]

3. Data and testable hypotheses

3.1. Data characteristics

Our dataset is from the Electronic Broking Services (EBS) (level 1.5) and consists of tick-by-tick FX transaction prices for the EUR/USD exchange rates spanning January 10 through December 23, 2005 for a total of 50 weeks (250 days). EBS operates as an electronic limit order book and is used for global interdealer spot trading. EBS is dominant for the EUR-USD and USD-JPY currency trading, whereas the GBP-USD currency pair is traded primarily on Reuters. The average daily EUR-USD trading volume (in USD) on EBS in 2003 was between 50 and 70 billion dollars, which was well above that on the NYSE (40 billion dollars). To avoid extreme high-frequency noise and no-activity periods in very small time windows and also to introduce as much uncertainty as possible, we focus on the 1-min frequency. This gives us 1440 observations over each 24-h period for a total of 360,000 data points. The top panel of Fig. 2 displays the 1-min USD/EUR exchange rate, and the bottom panel presents the volatility as the squared 1-min returns. Clearly, the USD appreciated over the data span, from about 1.35 USD/EUR to 1.18 USD/EUR. This trend is followed by several volatility outbursts that are mostly located at support levels. Considering that the USD appreciation trend was strongly reversed in 2006, 2005 can be viewed as a relatively risky year for currency trading.

In terms of trading intensity, on average, for the EUR-USD market, there are roughly 8000 buy orders and 6000 sell orders on a given day. Fig. 3 plots the average number of 1-min buy (Panel A) and sell (Panel B) orders as well as the total number of trades (Panel C) on each day of the week. As EBS data level 1.5 do not reveal the EUR-USD trading volume, even though the shapes of intraday trading activity curves resemble those from the literature, they should be interpreted with caution. In this paper, we are interested in the day-of-the-week effects and will thus ignore any intraday activity. It appears that, on average, Monday is the day with the lowest trading activity, whereas Fridays exhibit the highest number of trades, which is driven by buy orders (see Panel A). We will show in the following section that our fuzzy control methodology will be the most useful on Fridays, when the price volatility is highest.

3.2. Economic and ranking hypotheses

The first research question of interest concerns the high-frequency relationship between the volatility of foreign exchange returns and technical trading returns. Kho (1996) finds that periods of higher (lower) technical trading returns correspond to high (low) risk premia and volatility. However, Reitz (2006) argues that the information content of a technical trading signal is low when high exchange rate volatility disturbs inference. This mixed evidence leads to the first testable hypothesis:

Hypothesis 1. Higher volatility is associated with lower profits from pure technical trading strategies.

Next, we are interested in whether fuzzy technical indicators are more useful than pure technical indicators in periods of high volatility. Therefore, our second testable hypothesis is:

Hypothesis 2. Higher volatility is associated with greater profits from fuzzy technical trading strategies.

The benchmark trading strategy for both technical indicator approaches is the buy-and-hold strategy. This passive strategy may be profitable over longer time periods with low volatility. However, in high frequency data, buying and holding could be risky regardless of volatility. This argument motivates our third testable hypothesis:

[6] Increasing the number of states to 11 or 13 or decreasing the number of states to 7 or 5 does not alter our main conclusions about the usefulness of fuzzy control in technical trading.

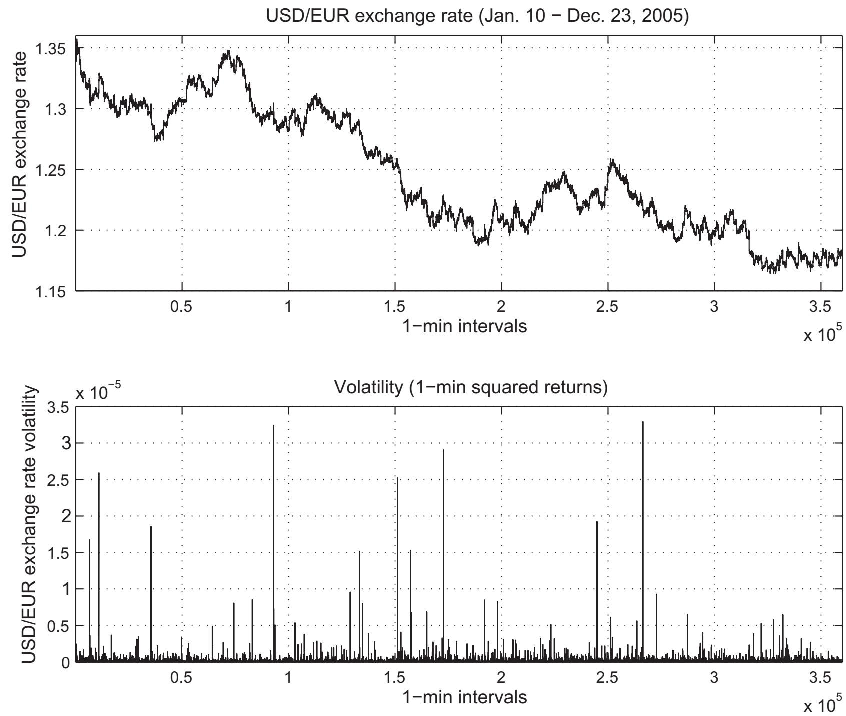

Fig. 2. Exchange rate and volatility. The top panel plots the 1-min USD/EUR exchange rate and illustrates the appreciation of the USD in 2005. The bottom panel plots the volatility of 1-min squared exchange rate returns.

Hypothesis 3. Volatility is not related to profits or losses from the buy-and-hold strategy.

In our high-frequency setting, it would also be important to establish ranking hypotheses with respect to the profitability of the technical trading-based and the buy-and-hold strategies. First, we would like to compare the two technical trading strategies as follows:

Hypothesis 4. Fuzzy technical indicators dominate pure technical indicators, whereas higher volatility leads to greater excess returns.

This hypothesis combines Hypotheses 1 and 2 and also generalizes the ranking relationship between the fuzzy and pure technical trading strategies. Finally, our goal is to explore the relative high-frequency profitability of the buy-and-hold strategy. Thus, we conjecture that:

Hypothesis 5. Fuzzy technical indicators and pure technical indicators dominate the buy-and-hold strategy, irrespective of volatility.

4. Results

4.1. Basic setting

This paper uses common moving average and filter technical trading rules. Moving average rules compare the short and the long moving averages:

$$m_t^l = \frac{1}{l}\sum_{i=0}^{l-1} P_{t-i}, \tag{1}$$

where P_t is the price at time t, and l is the length of the moving average. The buy and sell signals are calculated as

$$s_t^{l_1,l_2} = m_t^{l_1} - m_t^{l_2}, \tag{2}$$

where l_1 and l_2 are the lengths of the short and the long moving averages, respectively. Many possible combinations of moving averages can be used, but this paper will concentrate on $(l_1,l_2) = [(1,50), (1,200), (5,200), (2,200), (1,150)]$, where l_1 and l_2 are 1-min intervals. In a straightforward application of technical indicators, buy signals are generated when $s_t^{l_1,l_2} \geqslant 0$ and sell signals are initiated when $s_t^{l_1,l_2} < 0$. We will fuzzify these trading signals to obtain a smoother decision surface and then calculate the return from period t to $t+1$.[7]

Filter rules are applied in a slightly different fashion. Our trading exercise tracks 1-min prices and finds the percent difference between the current price and the current local minimum (and maximum) price. If the price increases by f percent above the local minimum, this increase represents a buy signal. A sell signal is generated when the price falls f percent below the local maximum. Hence, the filter rule depends on the filter size (f) and the data window over which the local minimum and maximum values are calculated. To remain consistent with moving average rules, we not only utilize the most recent 50, 150 and 200 observations as the basis for calculating local minima and maxima but also experiment with up to 500 observations. The filter sizes applied are the ones most commonly used in the literature: $f \in \{1\%, 2\%, 5\%\}$. For this rule, we fuzzify the difference between f and the percent difference between the price and the local minimum or maximum. As described above, trading positions are closed at $t+1$, and one-period returns are recorded.

To demonstrate the robustness of our methodology, we first estimate conditional mean returns over the first 25 weeks of the sample and then over weeks 26 through 50. We present the mean returns of plain vanilla technical trading strategies as well as the

[7] Notice that when trading signals are generated simultaneously by both strategies, fuzzy controlled trading always has lower or equal costs relative to standard technical trading.

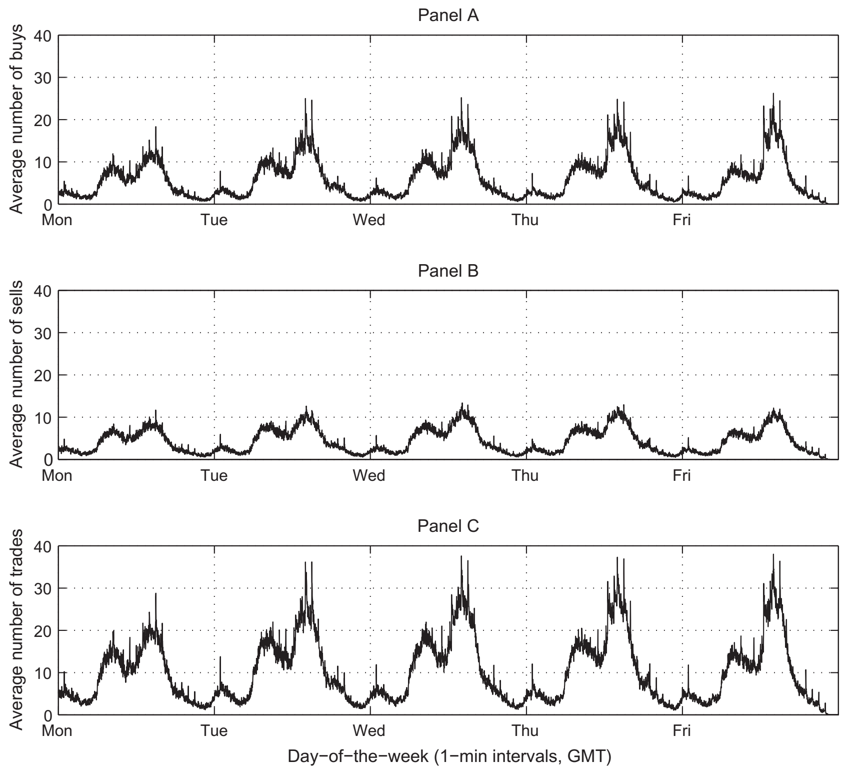

Fig. 3. Trading intensity. Top panel: The average number of buy orders over 1-min intervals for each day of the week. Middle panel: The average number of sell orders over 1-min intervals for each day of the week. Bottom panel: The average total number of buy and sell orders over 1-min intervals for each day of the week.

corresponding fuzzy technical indicators and those of the buy-and-hold strategy. After the 1-min trading signals are produced for each 24-h period, the returns are aggregated, and the total returns are calculated. This aggregation gives 125 values of total daily returns for both of the 25-week periods. The daily returns of each technical indicator variant are compared with the returns of the fuzzy technical indicators. As specified above, there are five variants of moving average technical indicators and three variants of filter rules that are employed in this work. Recall that our main goal is to study the day-of-the-week trading effects. Therefore, we compare the performance of all trading indicators on individual days (Monday through Friday) and then attempt to attribute the nature of the results to the volatility on such days. All tables include two basis point transaction costs for a one-way trade, which is realistic for large transactions (Neely et al., 1997).[8]

Table 1 presents estimates of the conditional mean returns over the two subsamples (25 weeks each) for the moving average technical trading indicators. The results are organized by the day of the week (columns), and all five variants of the moving average are reported jointly, thus constituting 125 total returns for each weekday. We are primarily interested in whether there exists a statistically significant difference between the aggregate mean returns of pure technical trading strategies (row 'Pure') and the returns of the fuzzy technical indicators (row 'Fuzzy'). Fuzzy technical indicators clearly dominate pure technical indicators, especially on volatile days, when the difference in mean returns is statistically significant.[9] Of special interest are Fridays (and the Thursdays of the second subsample) where, similar to Lyons (1998), we document the highest volatility figures. Friday trading rounds appear to involve substantial trading uncertainty that can be addressed by introducing fuzzy control into the traders' decision making. Typically, high volatility on such days originates from the currency traders necessity to end the week with a zero net position. The buy-and-hold strategy exhibits poor performance, earning mostly negative returns. Consequently, we conclude that the weak form of market efficiency is violated for our high-frequency exchange rate data and that fuzzy control brings additional benefits to technical trading.

Next, we evaluate the performance of filter rules. Table 2 reports the estimates of the conditional mean returns over the subsamples. As in Table 1, the columns represent weekdays, and all three variants of filter rules (1%, 2%, 5%) are reported jointly (from 75 total returns for each day). The most striking result is that the 'Fuzzy' figures are consistently positive across all days. From the first subsample, it is apparent that the fuzzy approach significantly improves upon the standard filter rules. This finding is confirmed over the second subsample, where again fuzzy control statistically adds value to pure filter rules. Overall, for filter rules, fuzzy control again performs better on high-volatility days, whereas pure filter rules generate negative returns.[10]

Tables 1 and 2 both demonstrate the validity of Hypotheses 1–3. These patterns are consistent across all time periods and for both types of technical indicators. To test Hypothesis 4, from Table 1 we calculate the excess returns for fuzzy technical indicators relative to the returns of pure technical indicators: Sample 1 (0.04, 7.06, 17.76, 15.89 and 21.47) and Sample 2 (0.05, 10.95, 2.47, 16.52 and 18.35). In Sample 1, the excess returns closely track the move-

[8] When we ignored transaction costs, we obtained similar results, but the dominance of 'fuzzy technical indicators' was not as pronounced as it was when trading costs were included.

[9] The simple *t*-test of the equality of means statistics should be interpreted with care because, based on the Shapiro–Wilk and Kolmogorov–Smirnov tests, the returns are not normally distributed. However, as the distributions are symmetrical, we can apply the Wilcoxon's signed-rank test.

[10] Recently, Kozyra and Lento (2011) have shown that the standard (trending) filter rules are not profitable, which is in line with our results. They also construct a contrarian filter rule and demonstrate its profitability over both the trending filter and the buy-and-hold strategy.

Table 1
Conditional mean returns for the moving average rules.

Sample (date)	Statistic	Mon	Tue	Wed	Thu	Fri
Jan 10–July 8, 2005	Fuzzy	6.11	12.67	13.37	14.90	16.65
	Pure	6.07	5.61	−4.39	−0.99	−4.82
	Buy and Hold	−8.52	−5.88	1.17	−21.45	−0.27
	(*t*-stat)	(0.01)	(2.22)	(3.74)	(5.47)	(3.14)
	[W]	[0.62]	[0.03]	[0.00]	[0.00]	[0.00]
	Volatility	4.88	6.43	7.36	7.13	8.31
July 11-Dec 23, 2005	Fuzzy	4.52	10.04	6.74	10.73	13.98
	Pure	4.47	−0.91	4.27	−5.79	−4.37
	Buy and Hold	−2.12	−0.99	12.54	7.87	−17.52
	(*t*-stat)	(0.89)	(2.67)	(0.86)	(2.87)	(3.15)
	[W]	[0.36]	[0.00]	[0.35]	[0.00]	[0.00]
	Volatility	6.01	7.70	7.64	8.25	7.95

Conditional mean returns are calculated for pure technical trading strategies (Pure), fuzzy technical indicators (Fuzzy) and the buy-and-hold strategy (Buy and Hold). Total returns are calculated for each trading day, and the corresponding days are compared in terms of aggregate total returns. The conditional return figures have been multiplied by 10^4. The numbers in parentheses are *t*-statistics (*t*-stat) for the difference of the means of the fuzzy and pure strategies, and the numbers in square brackets are the *p*-values for the Wilcoxon's signed-rank test (*W*). The average 1-min volatility (squared returns) on weekdays is shown in the last row (Volatility) for each subsample, and the figures have been multiplied by 10^4.

Table 2
Conditional mean returns for filter rules.

Sample (date)	Statistic	Mon	Tue	Wed	Thu	Fri
Jan 10–July 8, 2005	Fuzzy	0.34	3.40	7.52	7.13	10.57
	Pure	−1.17	−1.84	−3.19	−5.45	−6.62
	Buy& Hold	−10.19	−5.74	−3.23	−21.61	−4.18
	(t-stat)	(1.64)	(1.15)	(2.45)	(5.88)	(7.56)
	[W]	[0.42]	[0.07]	[0.00]	[0.00]	[0.00]
	Volatility	4.88	6.43	7.36	7.13	8.31
July 11–December 23, 2005	Fuzzy	4.25	5.36	8.01	9.61	8.13
	Pure	−3.71	−12.13	−6.08	−10.09	−4.91
	Buy and Hold	−3.51	−1.06	7.79	6.89	−18.09
	(*t*-stat)	(2.23)	(3.09)	(8.50)	(8.30)	(2.43)
	[W]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]
	Volatility	6.01	7.70	7.64	8.25	7.95

Conditional mean returns are calculated for pure technical trading strategies (Pure), fuzzy technical indicators (Fuzzy) and the buy-and-hold strategy (Buy and Hold). Total returns are calculated for each trading day, and the corresponding days are compared in terms of aggregate total returns. The conditional return figures have been multiplied by 10^4. The numbers in parentheses are *t*-statistics (*t*-stat) for the difference of the means of the fuzzy and pure strategies, and the numbers in square brackets are the *p*-values for the Wilcoxon's signed-rank test (*W*). The average 1-min volatility (squared returns) on weekdays is shown in the last row (Volatility) for each subsample, and the figures have been multiplied by 10^4.

Table 3
Conditional mean returns for the moving average rules and Gaussian shapes.

Sample (date)	Statistic	Mon	Tue	Wed	Thu	Fri
Jan 10–July 8, 2005	Fuzzy	8.87	4.68	13.00	12.00	10.00
	Pure	6.07	5.61	−4.39	−0.99	−4.82
	Buy and Hold	−8.52	−5.88	1.17	−21.45	−0.27
	(*t*-stat)	(0.65)	(−0.38)	(6.12)	(5.44)	(5.83)
	[*W*]	[0.21]	[0.98]	[0.00]	[0.00]	[0.00]
	Volatility	4.88	6.43	7.36	7.13	8.31
July 11–December 23, 2005	Fuzzy	3.79	5.79	9.12	11.00	7.39
	Pure	4.47	−0.91	4.27	−5.79	−4.37
	Buy and Hold	−2.12	−0.99	12.54	7.87	−17.52
	(*t*-stat)	(−0.79)	(1.78)	(1.66)	(4.86)	(3.29)
	[*W*]	[0.65]	[0.04]	[0.05]	[0.00]	[0.00]
	Volatility	6.01	7.70	7.64	8.25	7.95

The trading recommendation variable (output) is assumed to have nine states represented by Gaussian-shaped functions. Conditional mean returns are calculated for pure technical trading strategies (Pure), fuzzy technical indicators (Fuzzy) and the buy-and-hold strategy (Buy and Hold). Total returns are calculated for each trading day, and the corresponding days are compared in terms of aggregate total returns. The conditional return figures have been multiplied by 10^4. The numbers in parentheses are *t*-statistics (*t*-stat) for the difference of the means of the fuzzy and pure strategies, and the numbers in square brackets are the *p*-values for the Wilcoxon's signed-rank test (*W*). The average 1-min volatility (squared returns) on weekdays is shown in the last row (Volatility) for each subsample, and the figures have been multiplied by 10^4.

ments in volatility, and with the exception of Thursdays and Fridays, such a pattern is maintained in Sample 2. Nonetheless, the average excess returns in Sample 2 are the largest on Thursdays and Fridays, which represent the days with the highest volatility. As such volatility effects are also roughly exhibited in Table 2, we conclude that the findings support Hypothesis 4. Finally, both tables support our Hypothesis 5. The absence of a clear pattern in the buy-and-hold returns is somewhat puzzling, but this finding demonstrates that the profitability of this strategy in a high frequency setting is elusive.

Our results can also be explained from a market microstructure perspective. The fact that fuzzy control is not subject to a nonzero phase shift provides an advantage in making more timely trading decisions in comparison to pure technical indicators. In this sense, pure technical traders can be viewed as noise traders or even as "late-coming bandwagonists" that can artificially prolong the exchange rate trend and thereby "conceal" a turning point (Schulmeister, 2006). The timing inability of pure technical traders is further exaggerated by their mechanical and potentially predictable usage of technical indicators. The timing superiority of fuzzy technical traders is equivalent to the behavior of sentiment-oriented technical traders (Harris, 2003). These traders are capable of predicting the trades that uninformed traders (i.e., pure technical traders) will decide to make. Then, fuzzy traders trade before the uninformed traders do and thus profit when they correctly anticipate the impact that uninformed traders will have on the exchange rate. Hence, fuzzy control facilitates the successful anticipation of the future trades of the uninformed traders and the future turning points. Based on these notions, we argue that fuzzy reasoning might be able to explain the behavior of sentiment-oriented technical traders.

4.2. Robustness analysis

The robustness with respect to fuzzy model implementation is of utmost importance.[11] By conducting robustness analysis it will be safe to exclude the possibility that the empirical outcome could be a result of data-mining. As mentioned previously, the paper borrows the fuzzy logic setting from Gradojevic (2007) who uses Gaussian membership functions for the input variables and triangular shapes for the output variable. He also shows that alternative shapes for the input space such as triangular and trapezoidal do not provide major profitability improvements. However, Gradojevic (2007) does not test alternative shapes for the output variable and, in this part of the paper, we provide the results of this robustness check. It is worth noting that Gaussian shapes for the output variable might reflect the issue of "uncertainty perception" better.

Table 3 provides the results for the moving average rules and the Gaussian trading recommendation variable (output). The trading recommendation variable is assumed to have nine states represented by Gaussian-shaped fuzzy membership functions as follows: "VERY STRONG SELL," "STRONG SELL," "MEDIUM SELL," "WEAK SELL," "HOLD," "WEAK BUY," "MEDIUM BUY," "STRONG BUY" and "VERY STRONG BUY". The membership functions of input variables (differences between long and short moving averages) are as specified in Section 2, i.e., Gaussian with nine states. The results in general confirm our Hypotheses 1–5 and are in line with those from Table 1. We conclude that triangular and Gaussian output membership functions yield similar results. This exercise demonstrates the robustness of the fuzzy technical trading model.[12]

5. Conclusions

The purpose of this paper is to provide an alternative panel of technical trading indicators that are more tolerant of uncertainties present in financial markets. Our trading uncertainty reduction approach concentrates on two choices that traders face: market timing and order size. In general, our goal is to capture the uncertainty in decision making that results from a lack of knowledge about the appropriate trading model. We recognize that incorrect trading signals can be generated by the assumed technical trading model, and we employ fuzzy logic to guide this decision-making process.

Standard moving average and filter strategies are complemented by the fuzzy control methodology, and this combination results in improved profitability. Our results for high-frequency EUR-USD exchange rates show that fuzzy technical indicators are particularly useful for improving trading profitability on highly volatile days. The documented gains in conditional mean returns are, overall, statistically significant over 50 weeks of 1-min exchange rates in 2005. We conclude that the success of our fuzzy uncertainty reduction technique stems from its two important properties: (1) smooth decision surface and (2) reduction in trading costs. Fuzzy rules generate a continuous decision surface that accounts for the distance between the inputs and produces a trading signal that identifies the exact fraction of the funds that should be allocated to a position. Meanwhile, pure technical indicators are subject to the phase shift and always invest all available funds in a position. Our conclusions complement and extend the findings of Kozhan and Salmon (2009), which is, to the best of our knowledge, the only other paper that links technical trading and uncertainty in FX markets. The authors find that chartists are mainly uncertainty-averse, which further reinforces the usefulness of our approach.

The profitability of moving average fuzzy technical indicators suggests some form of dependency on price volatility. Specifically, we find an increased superiority of the fuzzy approach on more volatile days of the week (i.e., Fridays and Thursdays), but not as much on Mondays, which are the least volatile days in the data. The findings for filter rules are more striking and show that pure filter trading signals appear to be less useful for trading purposes, which provides greater possibility for improvements to the fuzzy controller. An interesting pattern that we uncover is a robust direct relationship between the volatility level and the excess returns of the fuzzy technical indicators over the pure technical indicators. In addition, the buy-and-hold strategy is found to be ineffective and inferior to technical trading approaches.

Our results prove that even when pure technical trading is not profitable, the information content of a technical trading signal can be useful. Consequently, fuzzy technical traders can potentially be viewed as sentiment-oriented technical traders who can learn from the predictable technical trading strategies employed by uninformed, pure technical traders. In turn, sentiment-oriented technical traders themselves become informed. It is worth noting that sentiment-oriented technical trading can also be risky because it involves a successful front-running of uninformed traders. The problem that may arise here is that uninformed traders often push prices away from their fundamental values, which attracts value traders to trade. Value traders attempt to trade on the other side of the market and drive prices back to their fundamental values. In this scenario, if sentiment-oriented technical traders do not close their positions in a timely manner, they will lose their profits to value traders. Hence, fuzzy or sentiment-oriented technical trading is recommended for hard-to-value instruments (e.g., emerging market stocks or derivatives, non-publicly quoted firms and internet companies) and in a high-frequency environment where news does not emerge frequently enough while fundamentals are relatively static.

The methodology and findings presented here could be extended in several worthwhile directions. One would be to fuzzify alternative technical indicators such as the trading range breakout rules, momentum rules or rules based on volume. Each technical indicator would be subject to the same scrutiny in terms of analyzing the performance of several of its variants over subsamples in a high-frequency setting. In addition, to verify our results, high-frequency stock prices, derivative prices or other exchange rates could be considered over longer time periods. As some trading

[11] We wish to thank the anonymous Referee for this and other valuable suggestions.

[12] The results for filter rules are also robust in regards to the choice of the output fuzzy membership functional shapes. These results can be available upon request.

indicators, such as filter rules, are not sufficiently informative, a combination of trading indicators with a richer fuzzy rule base might be employed. In the same vein, we would like to better understand the role of volatility. Based on the study by Bekiros (2010), we plan to present the volatility input to the fuzzy controller and expand the rule base with more complex fuzzy rules of the following nature: "if the long moving average is ⟨LARGE⟩, and the short moving average is ⟨VERY SMALL⟩ and the volatility is ⟨VERY SMALL⟩, then the technical trading signal is ⟨SELL⟩." Finally, our future goal is to investigate intraday fuzzy technical trading profitability patterns and their relation to volatility. Thus, much remains to be learned regarding fuzzy control and its usefulness in financial risk management.

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