$\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket
- An Agent Based Approach -

by

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This work is dedicated:

to my husband Serafín and my children;

and to my parents Svetla and Alexander.

Summary

The increasing use of the credit and debit cards has converted them into important players in the world scene. In this market, the card providers deal with two different groups of users: consumers, that demand goods and merchants, that sell them. In addition, the consumers and the merchants demand electronic payment methods for their commercial transactions.

The growing importance of these payment methods, explain the interest of economists and policy makers in understanding the different aspects of the market dynamics. The analytical models used for this purpose are built on representative players, whereas the large number of heterogeneous interactions among consumers, merchants and card providers is impossible to be taken into account.

Nevertheless, we strongly believe that studying the complex phenomena emerging from the direct and indirect interactions among the market participants will advance our understanding of the payment card market. For that reason this thesis introduces the first, to our knowledge, agent-based $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket ($\mathcal{APCM}$), which simulates the commercial transactions of consumers and merchants at the point of sale.

The agent-based model allows each electronic card provider to decide the price of the offered payment instrument. The price structure consists of variable net benefits[1] and fixed fees on both sides of the market and each component of the structure has a specific level. We also include marketing effort into our model to raise awareness among

[1]The variable net benefits are the difference between the variable fees and variable benefits and could be either positive or negative.

consumers and merchants for each instrument offered in the market.

We position consumers and merchants on a torus, representing their respective geographical locations and investigate the dynamics of the usage of payment cards through their commercial transactions. For that reason, we allow each consumer to know a small number of merchants. We investigate different scenarios of initial card holdings of consumers and acceptances of merchants, which are given exogenously. The decisions by the merchants are limited to the choice of payment cards they subscribe to. The consumers, on the other hand, have to make decisions affecting the subscription to payment cards, which merchant to choose for their purchase and which payment card to use in a transaction.

Then, in each time step each consumer chooses a merchant and, if some of the merchant's and the consumer's cards are common to both of them, the consumer faces the choice which card to use for his purchases. Otherwise, if there are no common cards, the transaction is settled by cash. After a certain number of time steps[2] the consumers as well as the merchants make the choice whether to drop the subscription to a card or subscribe to a card not currently subscribed to. These interactions are then repeated for a thousand time steps and the resulting market structure, allows us to assess the performance of the card issuers, given the variable benefits, the fixed fees and the marketing effort applied by them.

This artificial market environment has a descriptive aim. With this setting, we have tested several scenarios and we have observed that the emerging phenomenon resembles

[2]The number of time steps is different for each consumer and merchant and it is determined by an individual poisson distribution.

the real market. We have demonstrated that the artificial market allows us to estimate the demand for payment instruments, and measure its price sensitivity.

We have also demonstrated how machine learning can be used for strategy design. The aim of this study is of normative nature. To that end, we applied the Generalised Population Based Incremental Learning (GPBIL) algorithm to find combinations of price structure and marketing effort that achieve the card issuers' goals, such as maximization of profit, or market share, or both. Experimental results show that the prices found by the GPBIL are statistically more effective than randomly-generated prices.

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Glossary

Acquirers: Financial institutions that provide payment methods to merchants.

Agent: in the context of this thesis, agent refers to a software that contains data and behavioural methods representing an entity. This entity forms part of a computationally constructed world.

Agent-Based Computational Economics (ACE): is the computational study of economic processes modeled as dynamic systems of interacting agents.

Artificial Intelligence (AI): refers to intelligence as exhibited by an artificial (manufactured, non-natural) entity. AI is studied in overlapping fields of computer science, psychology, neuroscience and engineering. It deals with intelligent behavior, learning and adaptation.

Artificial payment card market: is a computational model aimed to reproduce the interactions at the point of sale among consumers and merchants in order to study the complex two-sided nature of the payment card market.

Card Issuers/Card Providers: banks that provide payment cards to consumers. In the context of this thesis the card issuers are also acquirers.

Consumer/Buyer: is the person who acquires a good or service in order to satisfy his needs.

Electronic payment instrument: is a non-cash payment instrument. A transaction, performed with this instrument is made on electronic related systems.

End-users: are the consumers and the merchants, who used the payment cards to perform their commercial transactions.

Evolutionary Computation: is a subfield of artificial intelligence (more particularly computational intelligence) dedicated to the study of complex phenomena, involving combinatorial optimization problems. It applies naturally inspired concepts such as populations, crossover, mutations and survival of the fittest to find solutions to the studied phenomena.

Generalised Population Based Incremental Learning (GPBIL): ia a method that applied the concept behind the PBIL algorithm to problems with real numbers variables domains.

Interchange fee: amount paid for each transaction with a card, usually by the bank of the merchants (acquirer) to the bank of the consumers (issuer).

Merchant/Seller: establishments that sell products and could eventually accept payment cards.

Point of Sale: a variable location where a transaction occurs, including or not software and hardware related systems.

Population Based Incremental Learning (PBIL): is a type of Estimation of Distribution Algorithm EDA. These evolutionary computation algorithms generate new population by sampling the probability distribution from a set of selected individuals of previous generations.

Two-sided platform: is a product that in order to be placed in the market, the provider needs to attract two different groups of consumers.

Related Publications

Biliana Alexandrova-Kabadjova, Edward Tsang, and Andreas Krause. Evolutionary learning of the optimal pricing strategy in an artificial payment card market. In Anthony Brabazon and Michael O'Neill, editors, Natural Computing in Computational Economics and Finance, Studies in Computational Intelligence, page forthcoming. Springer, 2007.

Alexandrova Kabadjova B., Krause A., Tsang E., "The Price Structure and the Demand Sensitivity in the Artificial Payment Card Market", 13th International Conference on Computing in Economics and Finance (CEF07), HEC Montral, Montral, Quebec, Canada

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Alexandrova Kabadjova B., Krause A. and Tsang E., "An Agent-Based Model of Interactions in the Payment Card Market", 10th Annual Workshop on Economic Heterogeneous Interacting Agents (WEHIA 2005) at University of Essex, United Kingdom

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Chapter 1

Introduction

1.1 Background

The markets, places where people for thousands of years have traded in order to exchange goods and services, enclose complex phenomena shaped by traditions, political regulations and social interactions. Nowadays, these conditions are the basis of the economical organization of the modern societies. For this reason, economists have been studying for many years the shape of processes emerging from individual decisions of the market participants and the consequent impact of those macro processes on personal decisions. However, understanding how this changes through time is a real intellectual challenge. Today, through the development of computational tools, scientists have been able to go further inside the complex phenomena emerging from market dynamics.

Among the different kinds of markets, there are markets formed over two-sided (or multi-sided) platform industries [1]. In these markets, a platform (or several platforms) allows the end-users to interact with each other. In order to establish the platform in the market, the provider needs to keep the two (or multiple) sides involved, and possibly charge them both an appropriate price. Video games, the yellow pages, software systems, the media and electronic payment cards are all good examples of two-sided platforms.

Recently, such markets have attracted the attention of academics and researchers [2, 3, 4, 5], as they have become a common business practice and the rules and prices governing those markets are different from the rules present in the traditional markets formed over one-sided platform industries.

Electronic payment cards are a typical example of two-sided platforms. In recent years these payment instruments (debit and credit cards) have become a widely accepted payment form throughout the world [6]. In 2002 alone, 1.8 billion payment cards have been used to spend $2.7 trillion in goods and services globally. Since then these numbers have shown a significant growth. The increasing acceptance/use of these electronic payment instruments has converted them into an important player in the world scene.

Behind these payment methods lies a complex industry, formed by the conjunction of business, law, economics, technology and public policy. In addition to this, the growing importance of payment cards [7], and the theoretical framework initialized by Baxter in [8] and later the models in [9] and [10], have given an impulse to a significant amount of research in this area.

The analytical models in [9] and [10] are an excellent starting point for understanding the fundamental relationship among the participants of the payment card market. In order to study the dynamics of the market, the authors focus their analysis on the most representative players and the interactions among them. In the next chapter we will explain some of the most significant analytical models in more detail; here we introduce some commonly used concepts in the payment card industry.

The different owners of credit and debit cards systems operate their businesses in either open or closed schemes. The open scheme, also known as the four parties scheme, is the way Visa and Mastercard organise the relationships among the financial institutions and the end-users involved in the industry. It is called a four parties scheme as there are four main participants: the consumers (the users of payment cards), the merchants (establishments that accept the payment cards), the issuers (the banks that provide card to the consumers) and the acquirers (the financial institutions that provide payment methods to the merchants). In the case of the closed scheme, also known as unitary or three-party scheme, the role of the issuer and the acquirer are represented by the same financial institution This is the way in which American Express operates.

In the four parties scheme, there exists a complex fees structure, that establishes the business relationships among the parties. For many years the focus in the literature has been on this fees structure, with the emphasis laid on the interchange fee [11]. The interchange fee is paid for each transaction with a card, usually by the bank of the merchants (acquirer) to the bank of the consumers (issuer). The extensive literature can generally be divided into models analysing the problems surrounding the use of a single card [10, 9, 12, 13, 14], and those that allow competition between payment methods as the models in [2, 15, 16].

Furthermore, in recent years the research regarding the mechanism governing the markets of the two-sided platforms [1], [17] has opened the opportunity to understand the industrial organization of those markets [18]. In particular, the literature has paid special attention to studying the existence of indirect externalities arising among the end-users of the two-sided platforms. For instance, taking the case of the payment card market, the benefits from holding a specific card, among other factors, depend on how often the card can be used. The more merchants accept the card, the larger the benefits to the consumer; similarly the more consumers hold the card the larger the benefits to the merchant. These indirect externalities are an important aspect of the competition among different payment card platforms.

1.2 Motivation

Despite the growing body of literature, given the economic significance of the industry and the interest of the policy makers [19, 20, 21, 22, 23] there are still many questions waiting to be answered, regarding the competition in the market of retail payment cards. In addition, many scientists accept that understanding the heterogeneous interactions and the indirect externalities among the end-users of the two-sided platform is intellectually challenging [11]. Given the complexity of the system, we strongly believe that a different approach is required in order to gain a better understanding of the market dynamics.

In this context, we consider that the Agent-based Computational Economics (ACE)[1] methodology [24] will give us more insight into the complex relationships among the main players of the market. This methodology allows us to reproduce the payment card market's dynamics at the micro level and observe the emerging processes at the macro level.

[1]Please refer to chapter 2 section 2.2 for a brief literature overview.

For that reason, in this thesis we propose the creation of the first, to our knowledge, multi agent-based model [25], [26], which artificially simulates the commercial transactions at the point of sale among consumers and merchants. Built over a three parties scheme, the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket ($\mathcal{APCM}$) explicitly represents the interaction among the end-users of the platform[2] and their inherent relationships with the payment card provider. The aim of the model is to simulate the competition among card issuers by explicitly reproducing the mutually constrained demands for payment instruments of consumers and merchants.

In order to achieve this, we incorporate into the model a set of factors, which to our understanding, impact the individual decisions of consumers and merchants to use/accept payment cards. Among those factors, we have considered a two-sided price structure and indirect externalities arising throughout the interactions among consumers and merchants. The aggregate form of these individual decisions represents the shape of the mutually constrained consumers' and merchants' demands.

Regarding the competition among card issuers, we have allowed them to charge variable and fixed fees to both sides of the market. Consequently, the emerging market dynamics is determined by the intersections of different competitors' prices and a complex shape of end-users' demand. In addition, the intersections between price and demand on one side are affected by the intersection of price and demand on the other side.

This complex setting of the $\mathcal{APCM}$ allows us to study important aspects of the payment card market, which the analytical models are not able to do. In particular, in this

[2] Consumers and merchants.

thesis we have implemented a global search, which explores the areas of intersections between the aggregated consumers' and merchants' demands and the price structure applied by the payment card providers. This search is aimed to find a price level and price structure that guarantee an average number of card transactions and the highest possible profit for the card issuers.

1.3 Overview of the Thesis

We organise the thesis in the following way:

In Chapter 2 we provide an overview of the two main fields of research related to the creation of the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket. We start our exposition with a discussion on the existing single card analytical models of the market, presented in section 2.1. At the end of the section, we give the relationship between the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket and the previous theoretical works. Following on from this, in section 2.2 we explain which factors have motivated the existence of Agent-based Computational Economics (ACE) methodology. We finish this section by explaining why we have decided to use the ACE framework to study the complexity of the payment card market.

In chapter 3 we explain the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket ($\mathcal{APCM}$) model. The aim of the model is to reproduce the interactions of the consumers and merchants at the point of sale (POS). We start with an introduction in section 3.1, then in section 3.2 we present formally the different types of agents. Following, in 3.3 we define the framework of interactions among consumers and merchants and in 3.4 we explain the structure of the payment card providers' strategy. Next, in section 3.5 we present in detail the algorithm that reproduces artificially the relationship between the payment card providers and the end-users of the two-sided platform, as well as the interactions among consumers and merchants at the point of sale.

In chapter 4, we present a simulation study of the $\mathcal{APCM}$ in order to evaluate to what degree we have reproduced the dynamics of the market. For this reason we have designed two experiments, explained in detail in sections 4.3 and 4.4 respectively. The first experiment is designed to test the sensibility of the end-users' demand for electronic instruments, measured in terms of consumers' and merchants' fixed price in a market, where all competitors price their cards equally. The second experiment is aimed at assessing the market share and the profit of the competitors in a dynamic environment, where each card issuer prices his cards independently.

Following this, in chapter 5, we propose the use of the Generalised Population Based Incremental Learning (GPBIL) algorithm [27] in order to design the payment card providers' strategies, under specific criteria. The algorithm performs a heuristic search over a complex multidimensional landscape. For that reason, in section 5.1 we explain formally the main features of the GPBIL. Next, in section 5.2 we present the set of procedures used to perform a search over the areas of intersection between the end users' demand and the price structure and price level set by the providers. The aim of the application is to find a strategy[3] that satisfies specific criteria, defined by the researcher.

Next, in chapter 6, we test the application of the algorithm experimentally. To that end, we apply the GPBIL to search for a strategy that guarantees an average number of

[3]Mainly price level and price structure.

card transactions in the market and the highest possible profit for the competitors. Subsequently, we evaluate the performance of the obtained strategies. In section 6.2 we explain in detail the experiment used to explore the areas of intersection between the complex shape of the consumers' and merchants' demand and the competitors' prices. We present the observations related to the results obtained by the search and give some conclusions. Following on, in section 6.3 we test the performance of the evolved profit-maximising strategy against the performance of randomly generated strategies. We support our conclusions with the results and observations related to the experiment.

Finally in chapter 7 we summarise our work and explain the main contributions of the thesis. In addition we discuss some limitations of the model and present an outlook on further research.

Chapter 2

The Payment Cards Market and Agent-Based Computational Economics: Overview of the Literature

In this chapter we present an overview of the two main research fields involved in the conception of this thesis. First we dedicate section 2.1 to the literature related to the payment card market. To give a general flavour of the theoretical works developed in this field, we present a discussion about the existing single card analytical models. Then in section 2.2 we explain the factors, which have given origin to the Agent-based Computational Economics(ACE). In addition, we present the initial work in this field, which has motivated a considerable amount of research, using this methodology.

2.1 The Payment Card Market

2.1.1 The Interchange Fees

During their short existence[1], payment cards have not only changed considerably the way we pay for goods and services, but have also changed the period of time over which we pay for them. Furthermore, credit cards have produced the so-called "silent revolution" [6], which has allowed consumers to have a convenient, secure and reliable means of payment [20].

The growing importance of the electronic payment instrument in the context of modern economics, is the reason why economists and policy makers have paid attention to this market. For instance, in 2000, cards accounted for 35% of consumer expenditure in the United Kingdom, 30% in Australia, and 25% in the United States of America [11]. A peculiar characteristic of this market is that it is built over a two-sided platform, in which for a successful transaction involving the electronic payment method, the consumers have to hold a card and the merchants have to accept it as a payment. Furthermore, Visa and Mastercard allow commercial banks to distribute electronic payment methods in their own name. For this reason, consumers and merchants need to have a contract with one of the banks associated with the card issuers, in order to be able to use the payment method for their commercial transactions.

The first to study the implications of the two-sided nature of the non-cash payment methods was [8], who demonstrated a fundamental economic consequence of the joint demand of consumers and merchants. He realised that in the four parties scheme, this joint demand is satisfied by two entities: the consumers' bank also known as issuer and

[1] Since 1950.

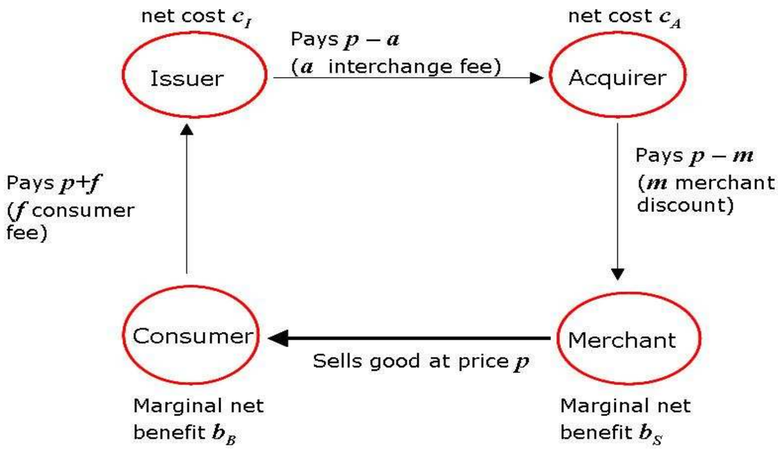


Figure 2-1: Analytical Model of the Payment Card Market

the merchants' bank known as acquirer. Therefore, he assumed among other factors that there exists a perfect competition between issuers and between acquirers and that consumers and merchants adopt the card if the technological benefits[2] are greater than the payments they have to make to the card providers.

In figure 2-1 we present the typical cost and benefits in a transaction with the four parties scheme. Here, we use similar notation to Baxter [8] in order to explain the interaction among the participants of the market. This differs from the notation used in our model formally presented in chapter 3.

Suppose a merchant is selling a good to the consumer at price p. In order for this transaction to be performed with a payment card, the consumer (the buyer) and the merchant (the seller) should have marginal net benefits b_B and b_S greater than zero. The total cost of the joint service is presented by the sum of the issuers' net cost c_I and the acquirers' net cost c_A. In order to cover this cost, the consumer is paying a fee f and the

[2]Mainly convenience, theft and fraud control.

merchant is having a discount m.

At the social optimum, the total cost $c_I + c_A$ should be equal to the total benefits of the marginal transaction $b_B + b_S$. Therefore f should be equal b_B and m should be equal to b_S. Nevertheless there is no reason to assume that $c_I = b_B$ and $c_A = b_S$ and as a consequence the money making bank should compensate the money losing bank by paying an interchange fee a. However, Baxter's theory says nothing about the sign of the socially optimum interchange fee, which could be defined as a positive or negative transfer from the acquirer to the issuer $a = b_S - c_A = c_I - b_B$.

After Baxter's model, during the nineties the interchange fee was a topic of interest mainly for industry insiders. Nevertheless, at the beginning of the century the payment card industry has become an important player in the scene of modern economies. It was at this time that academics, policymakers and competition authorities around the world started studying the implications of the existence of the interchange fee as well as the specific setting of it.

Among the first important extensions of Baxter's work was the model made by [9]. He dropped the assumption of perfect competition between issuers and acquirers in order to study the balancing role of the interchange fees. He developed a two-stage game, in which the interchange fee is determined to maximize the system private value in the first-stage. Schmalensee resolves the outcome of this game, given that the number of payment card transactions is a function of the issuers' and the acquirers' complementary efforts. Regarding the privately optimal interchange fee, he concluded that it depends on two factors. The first factor is determined by the difference in demand elasticities across consumers and merchants, whereas the second factor depends on the difference in costs across issuers and acquirers. Schmalensee argues that there is no economic basis to support public policy to reduce interchange fees to zero.

In an analysis complementary to [9], [10] were the first to endogenize consumers' and merchants' behaviour by modeling them as strategic players. They assume perfect competition among credit card acquirers and imperfect competition among issuers. Rochet and Tirole explicitly model the retail sector by assuming imperfect competition among identical merchants. In their study of the welfare implications of interchange setting and merchant surcharging[3], they obtain two key results.

In order to facilitate the welfare analysis, Rochet and Tirole set the focus in equilibrium, in which all merchants accept cards. They found firstly, under the assumption that surcharges are not allowed, that the privately optimal interchange fee is either equal to the socially optimal one, or is higher. In the case when the interchange fee is higher, there will be overprovision of card payment services in the market. Secondly, given that surcharges are allowed, the interchange fee becomes neutral. This will lead to underprovision of cards, and they argue that the impact on the social welfare is ambiguous.

Furthermore, an extension to the model presented in [10] model was made in [28], which considered different cases of competition among merchants. Wright analysed monopolistic pricing and perfect competition. He found two important issues. Firstly, in the case when the merchants have an intensive retail competition, the interchange fees cannot reallocate the costs and benefits between consumers and merchants as the theoretical analysis suggests. Secondly, in the case when merchants have significant market

[3]The merchants surcharge when they price the same product differently due to the different payment method used by the consumers.

power and the no-surcharge rule is applied, the interchange fee can be set in a way that appropriately reallocates the cost and benefits. If merchants are allowed to surcharge, they will do that excessively, which results in little holding and usage of cards in the market.

In parallel to the academic research conducted by [29, 30, 15, 11], the interest in the interchange fees of the open scheme payment system was also growing on the part of the regulators. The first to address the issue were Australia [31] and the United Kingdom [19]. In the United States of America, due to several antitrust cases against Mastercard and Visa, the authorities have also been studying this market [13, 32, 33]. Other countries in Europe and America [22] are starting to regulate the setting of interchange fees as well. Despite all these increasing bodies of research, many questions are still waiting for an answer.

2.1.2 Two-sided Markets

Small stores, located in the centre of an old town, are a typical example of two-sided markets. This fact by itself shows that those kind of markets have been present in economic life for a long time. Nevertheless, the concept of "two-sided market" and the theory related to their study, were created at the beginning of the twenty-first century. In the late Eighties and during the Nineties, some of the observed phenomena in those markets were explained partially with the theories of network externalities and the multi-price product. The reason for this is that the network products and the two-sided platforms exhibit non-internalised externalities among the end-users, whereas the industry of multi-price products used price structure in a similar way as the price structure of the two-sided markets. The idea behind the use of price structure is that it is less likely to be distorted by market power than the price levels.

In the presence of non-internalised externalities in the two-sided markets, many economists and public and private decision makers believe that price structure affects profit and economic efficiency [1]. Managers, looking for a better profit, spend considerable time figuring out which side to charge more without losing participation in the market. On the other hand, the policymakers in their belief that economic efficiency could improve if one side of the market pays less than the other, also spend time monitoring and studying the market's dynamics. The efforts of private and public decision makers are due to their beliefs that the price structure is not neutral. This means that changes in price on one side are not gradually adjusted and consequently one of the sides bears a higher proportion of the cost.

In this context, the theoretical understanding given by the literature so far is very important; yet the precise results of the analytical models are sensitive to assumptions about the economic relationships among market participants. Furthermore, those models derive their conclusions focusing on the optimum public and private setting of prices, saying nothing about other states observed in reality, which are outside these equilibria. However, making more realistic assumptions regarding the behaviour of the market participants could lead to analytically intractable models. In that sense, in [11] the authors recognise that understanding the determination of the interchange fees and their effect is "intellectually challenging".

For this reason, in our approach we will explicitly represent the behaviour of consumers and merchants and focus on the competition between cards issuers to attract subscribers and to promote transactions. We abstract from the role of banks in the market and implicitly assume that payment cards are directly issued by card providers. Additionally in the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket the merchants and the consumers make endogenous decisions[4] as they do in the model in [10]. In accordance with the findings in [9], we have explicitly simulated the different sensibility of demand between the two sides of the market. We aim to study the phenomenon emerging from the market interactions among consumers, merchants and card providers in a dynamic environment. We present our study in chapter 4.

2.2 Agent-based Computational Economics

Whereas the analytical models could be visualized at one of the extremes of the complex model spectrum, at the other end stand models, whose creation requires high computational skills. These complex models challenge the conventional way of social phenomena representation and try to expand the frontier of understanding reality. Among them is the approach we have followed to develop our model, called *Agent-based Computational Economics*(ACE). Before we explain the methodology of ACE, we briefly review the fundamental assumptions behind the analytical models.

The *rational choice paradigm* is the most commonly used form of modeling throughout the social sciences today, in particular within Game Theory [34]. In these analytical models researchers often assume that decision makers have all the capabilities and information needed to make an optimal, rational choice. This kind of ideal abstraction of the reality, could be very useful in an initial understanding of complex social phenomena.

[4]Please refer to chapter 3 section 3.3.

Nevertheless, the theoretical outcome predicted by these models often differs from what is observed in the real world. This contradiction has led many scientists to believe that creating a model with more realistic features will allow us to gain better understanding of the studied phenomena and as a consequence be more accurate in the prediction of the future.

Herbert Alexander Simon [35] was not necessarily the first, but by far the best known individual to claim that "decision-making" under uncertainty is not rational. His behaviour theory based on "bounded rationality" [36], [37] made revolutionary changes in microeconomics and he was one of the pioneers in the field of artificial intelligence. Later, [38] and [39] made an important contribution to the development of agents with bounded rationality using computational tools.

Other strong assumptions behind many economic theories, are that the participants of the model have *homogeneous preferences* and they *interact globally* [40]. In other words, the limited number of participants in the model exhibit the same preferences and all of them interact with each other. These agents are called "representative agents". Moreover the analysis is focused only at the point of equilibrium, and aspects such as asymmetric information, imperfect competition and network externalities among others are not considered.

One of the main purposes of *Agent-based Computational Economics*(ACE) is to handle these real-world issues, which has become possible due to the technological advances in computational tools. With the use of programming languages, the agent based approach allows us to represent explicitly agents with bounded rationality and heterogeneous preferences. Given a specific social structures, the simulation of the interaction among agents is the straight and the hard of the agent-based modeling (ABM). Even in its early stage of development, ABM is a promising area of research, which has opened the opportunity to social scientists to look for new insights in resolving relevant real-world issues. Considered "the third way of doing science" [34], modeling the behaviour of the autonomous decision-making entities allows researchers to analyse emergent phenomena in order to gain better understanding of the object of study [40]. In this sense *Agent-based Computational Economics*, defined as "the computational study of economic processes modeled as dynamic systems of interacting agents" [41], is a growing area inside the field of agent-based modeling.

These days, ACE research is developing very rapidly [42]. Among the several areas of research, is the so called bottom-up modeling of market processes. The idea behind this simulation is to explicitly represent the participants of the market processes, modeling them as software programs able to take autonomous decisions. Consequently, the interactions among the agents at the micro level give rise to regularities at the macro level (globally). The intention is to observe the emerging self-organizing process for a certain period of time, in order to study the presence of patterns or the lack of them. Currently the study of this self-organizing capability is one of the most active areas of ACE research.

One of the most crucial tasks in representing explicitly the market participants is the simulation of their autonomous decisions. Nowadays, advances in artificial intelligence have opened possibilities at tackling this issue. In particular, techniques such as Neural Networks, Genetic Algorithms (GA), Genetic Programming (GP) and other population based algorithms are widely used in the field [43, 44, 45].

The intuition behind the population based algorithms (also known as Evolutionary

Computation (EC) algorithms) is to copy the evolutionary mechanism observed in nature. More specifically, these kinds of algorithms[5] mimic the natural selection process and are usually applied to find near optimal solutions for difficult problems. In other words, given a specific *objective function* and defined *solution space* the Evolutionary Computation algorithm explores the space in order to find a solution that satisfies at its best the *objective function*.

Let us briefly explain exactly how this optimisation works. In order to find the best possible solution, the algorithm explores the *solution space* for several generations. In each generation a *population* of solutions selected from the space are evaluated according to the *objective function*. This function assigns a fitness to each solution. Similar to the natural selection mechanism, the best solutions are chosen either to be directly part of the next generation or through *crossover* between them, to create the new members of the next generation. This selection process is repeated until a near-optimal solution is found.

One of the first to use genetic algorithms[6] to study the competition among price-setting sellers in oligopolistic markets (markets with a small number of competitors) was Robert Marks [48]. Not long after his work in Santa Fe Institute using the framework of ACE, the first artificial stock market was created by [49]. This highly influential study[7] opens new lines of research in the understanding of the dynamics of the financial markets. Since then researchers have tried to penetrate complex social phenomena governing the financial markets [51, 52, 53, 54]. More recently, using genetic programming[8] artificial

[5]Including the GA, GP and Estimation of Distribution Algorithm (EDA).

[6]For more details in the topic please refer to [46], [47].

[7]For an introductory literature survey on this area please refer to [50].

[8]For more details on the topic please refer to [55], [56].

markets with sophisticated agents have been developed (for more details see [57] and [58]).

In other lines of research, different kinds of markets have caught the attention of academics. Among them we can mention the markets of electricity, labor, retail and business-to-business [42]. In this context the model of a decentralized market economy, developed by [59], is the first to study issues related to payment methods. The authors are interested in the emergence of a generally accepted payment (i.e. money).

In their model the agents follow a simple adaptive rule. In the simulation of the economy, there are specialised trading firms that coordinate the transactions and initially there are no institutions that support economic exchange. Peter Howitt and Robert Clower have shown that in the majority of cases, a fully developed market economy spontaneously emerges and one of the commodities traded becomes a universal medium of exchange.

Motivated by the promising results obtained by the researchers involved in *Agent-based Computational Economics*, we propose to create the first to our knowledge $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket model, that explicitly represents the interactions among consumers and merchants at the point of sale, in order to study the dynamics of competition among payment card providers. We believe that due to the importance of the market and the indirect externalities of the two-sided nature of the payment card product, this market is an excellent candidate to be modeled bottom-up using an *Agent-based Computational Economic* approach.

Chapter 3

$\mathcal{APCM}$: Agent-Based Computational Model of $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket

3.1 Introduction

In the previous chapter in section 2.1 we provided an overview of the main analytical models of the payment card market. In this chapter we formally introduce the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket model, aimed to reproduce the interactions at the point of sale among consumers and merchants in order to study the complex two-sided nature of the payment card market.

Similar to the approach of [10], in our model, consumers and merchants display endogenous behaviour. Additionally, we have assumed the bounded rationality of the agents and heterogenous interactions among them. The aim of the artificial market is descriptive [60], i.e. we want to simulate how consumers and merchants perform commercial transactions. For that reason, instead of designing the decision process as a complex machine learning mechanism, we have adopted a different approach, in which we calculate the probability that certain consumers' and merchants' decisions take place in the market.

We start by presenting the model elements in section 3.2; next in section 3.3 we explain the individual decisions of consumers and merchants, which allow them to interact. In the following section 3.4 the structure of the payment card providers' strategies is defined. Further in section 3.5 we explain in detail the algorithm of the payment card market simulation, and finally we present the summary of the chapter in section 3.6.

3.2 Elements of the Model

In the model we have reproduced the underlying demand for electronic payment instruments at the point of sale. In order to do so, we have created three key elements: merchants, consumers and payment cards. In this section we formally introduce these elements and their environment.

3.2.1 Merchants

Suppose we have a set of merchants $\mathcal{M}$ with $|\mathcal{M}| = N_{\mathcal{M}}$, who are offering a homogeneous good at a common price and face marginal cost of production lower than this price. With the elimination of the price competition among merchants, we can concentrate on the competition among payment cards providers and how the card choice affects merchants. The merchants are located at random intersections of a $N \times N$ lattice, where $N^2 \gg N_{\mathcal{M}}$. Let the top and bottom edges as well as the right and left edges of this lattice be connected.

3.2.2 Consumers

Consumers occupy all the remaining intersections of the above lattice. The set of consumers is denoted $\mathcal{C}$ with $|\mathcal{C}| = N_{\mathcal{C}}$, where $N_{\mathcal{C}} \gg N_{\mathcal{M}}$ and $N^2 = N_{\mathcal{C}} + N_{\mathcal{M}}$. Each consumer has a budget constraint that allows him to buy exactly one unit of the good offered by the merchants in a single interaction. He will do so only by visiting a merchant. The utility gained from the consumption of this good is greater than its price. In order to obtain the good any consumer $c \in \mathcal{C}$ has to travel to a merchant $m \in \mathcal{M}$. The distance imposes travel costs on consumers, which reduces the attractiveness of visiting a merchant. We have explored three types of *network connections* among consumers and merchants $nc \in \mathcal{NC} = \{l, sw, r\}$, where l stands for local, sw for small world and r for random.

Interactions in a Local Network

In the case where the interactions among consumers and merchants are on a lattice with local connections (please refer to figure 3-1), the distance travelled by a consumer c to a merchant m, is measured by the "Manhattan distance" $d_{c,m}$ between the locations on the lattice; the longitude between two adjacent intersections of the lattice is normalized to unity. Let $\mathcal{M}_c$ denote the set of merchants a consumer considers going to, given that we restrict him to the nearest merchants.

Interactions in Small World Networks and Random Networks

Consider the consumer $c \in \mathcal{C}$ with a set of merchants to visit $\mathcal{M}_c$. In order to create a small world, we replace merchant $m \in \mathcal{M}_c$ with a new merchant $m' \in \mathcal{M}$ with probability $w = 0.01$. We assume that the consumer c will face the same travel cost $d_{c,m}$ to go to the

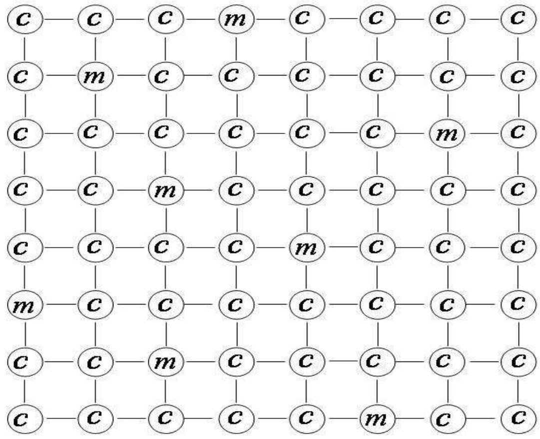


Figure 3-1: Lattice of Consumers and Merchants

new merchant m'. In the case of interactions on a lattice with random connections, we replace the merchant $m \in \mathcal{M}_c$ with a new merchant $m' \in \mathcal{M}$ with probability $w = 0.80$.

3.2.3 Payment Cards

There exists a set of payment cards $\mathcal{P}$ with $|\mathcal{P}| = N_{\mathcal{P}} + 1$ and $N_{\mathcal{P}} \ll N_{\mathcal{M}}$. The first payment method is the benchmark and can be interpreted as a cash payment, whereas all other payment forms are card payments. Cash is used by all consumers and is accepted by all merchants. On the other hand, in order for a card payment to occur, the consumer as well as the merchant must have a subscription to the card in question. We will show below that consumers prefer card payments over cash payments. A fixed subscription fee of $F_p \geq 0$ could be charged for each interaction to the consumer, whereas $\varGamma_p \geq 0$ could be charged for each interaction to the merchant. The domains of those fees, $\mathbb{D}_{F_p}$ and $\mathbb{D}_{\varGamma_p}$ are subsets of real numbers. Cash payments do not attract any fees.

For each unit of goods sold using a payment card $p \in \mathcal{P}$, a merchant $m \in \mathcal{M}$ receives net benefits of β_p. Such benefits may include reduced costs from cash handling, and could differ across payment methods. These are identical for all merchants for a given card. The domain $\mathbb{D}_{\beta_p}$ is a subset of real numbers. Note that the benefits β_p could have a negative value. This means that the variable fees paid by the merchant to the card issuer is bigger than the benefits he received from the same electronic payment method. Cash payments do not produce any benefits.

Consumers also receive net benefits from paying by card, b_p, but no benefits from cash payments. Here, the benefits may arise from delayed payment, insurance cover or cash-back options. The benefits are the same for all consumers, but could differ across card purveyors. The $\mathbb{D}_{b_p}$ is a subset of real numbers and could also include negative values as in the case of the merchants.

At this point we would like to make a special emphasis of the fact that the payment card market is a two-sided market. In other words, the payment card providers could establish different price levels at each side. In our case, the consumers' fixed fee F_p and the consumer benefits b_p are the price structure the card issuers apply to the consumers, while the merchant fixed fee Γ_p and the merchant benefits β_p are the price structure the payment card providers apply to the merchants. We assume that in the industry the fixed fees have a bigger impact over the behaviour of the consumers and the merchants compared to the impact that the benefits have.

In addition, the issuer of the payment method has to decide how much it should spend on publicity $l_p \in \mathbb{D}_{l_p}$, in order to increase the number of consumers and merchants using the payment card that he is providing. The publicity domain, $\mathbb{D}_{l_p}$, is a subset of real numbers.

Finally, the variables controlled by the card providers $F_p, \varGamma_p, \beta_p, b_p$ and l_p form its strategy. In order to assess the performance of the cards' strategies we have established several measurements, properly explained in subsections 4.4.2 and 4.3.3 in the following chapter.

3.3 Consumer and Merchant Decisions

The three elements of our model are in constant interaction. In order for the commercial transactions among consumers and merchants to take place, first the payment card providers have to determine their strategies, i.e. they have to assign a value to the variables under their control. We explain the process of how the competitors select their strategies in section 3.4, while in the current section we focus on the decisions corresponding to the consumers and merchants, which determine the interactions between them.
The consumers have to decide which merchant to choose for their purchase and which payment card to use in a transaction, which may have a subsequent impact on their decision to subscribe or to drop payment cards. On the other hand, the merchants' decisions are limited to the choice of payment cards they subscribe to.

3.3.1 Consumers' decisions

Choice of a Merchant

We assume that when deciding which merchant to visit the consumer has not yet decided which of the cards he holds will be used. Suppose $\mathcal{P}_{c,m}$ is the set of cards the consumers and merchants have in common. Given that $|\mathcal{P}_{c,m}| = N_{\mathcal{P}_{c,m}}$, we assume that the more payment cards the merchant m and the consumer c have in common, the more attractive a merchant becomes, as the consumer always carries all his cards with him. Additionally the smaller the distance $d_{c,m}$ between the consumer and the merchant, the higher the likelihood that this merchant will be chosen by the consumer. From these deliberations we propose to use a preference function for the consumer to visit the merchant:

$$\mathbf{v}_{c,m} = \frac{\frac{N_{\mathcal{P}_{c,m}}}{d_{c,m}}}{\sum_{m' \in \mathcal{M}_c} \frac{N_{\mathcal{P}_{c,m'}}}{d_{c,m'}}} \tag{3.1}$$

Each consumer $c \in \mathcal{C}$ chooses a merchant $m \in \mathcal{M}$ with probability $\mathbf{v}_{c,m}$ as defined in equation 3.1. The consumers will continuously update their knowledge on the number of common payments they share with a particular merchant, by observing the number of common payments of all shops they may visit.

Choice of a Payment Card

The consumer decides which payment card he wants to use with the merchant he has chosen. We assume a *preferred card choice* in which he chooses from the cards he holds and the merchants accepts, $\mathcal{P}_{c,m}$, the card with the higher benefits b_p. In the case that there is only one element in the set $\mathcal{P}_{c,m}$, the card in common is used; otherwise if the merchant does not accept any of the consumers' cards, the transaction is settled using cash payment.

Consumer Subscription

Initially the consumers have a certain number of cards, which are assigned according to the *initial cards* distribution $ic \in \mathcal{IC} = \{z, r, a\}$. This is user defined, and has three possible options. It could be z, which means that all consumers start with zero or no cards; it could be r, which means that the consumers start with a randomly assigned number of cards, which value could be between 1 and the maximum number of cards in the market, and finally it could be a, which means that the consumers start by having all the cards in their pockets.

Later, consumers have to decide whether to cancel a subscription to a card they hold and whether to subscribe to new cards. Every consumer $c \in \mathcal{C}$ keeps track of whether the cards he owns, $\mathcal{P}_c$, are accepted by the merchant or not. Given that $|\mathcal{P}_c| = N_{\mathcal{P}_c}$, if card $p \in \mathcal{P}_c$ is accepted by the merchant $m \in \mathcal{M}_c$ he is visiting, the consumer increases the score of the card $\omega^-_{c,p}$ by one. Here $\omega^-_{c,p}$ is an element of the vector specified as

$$\left(\omega^-_{c,1}, \ldots, \omega^-_{c,N_{\mathcal{P}_c}}\right).$$

Assume that he cancels his subscription with probability[1] defined in 3.2, given that the number of merchants visited is ω_c.

$$\pi^-_{c,p} = \frac{x^-_c k}{x^-_c k + exp\left(\frac{\omega^-_{c,p}}{\omega_c}\right)} \tag{3.2}$$

Here $x^-_c k$ accounts for the inertia of the consumer in changing cards;
$k = \left(1 + F_p + N_{\mathcal{P}_c} + \frac{\varepsilon}{\mathsf{k}+b_p}\right)$, whereas $\mathsf{k} = 1.1$, $\varepsilon = 1$ and $x^-_c = 0.05$ are constants.
On the other hand, let $\mathcal{P}^-_c$ with $|\mathcal{P}^-_c| = N_{\mathcal{P}^-_c}$ be the set of payment cards, to which the consumer does not have a subscription. Let us say the consumer c is visiting a merchant

[1]The probabilities defined in equations 3.2 and 3.3 are affected by the publicity applied by each payment card provider. The role of publicity is explained in subsection 3.4.

m. Suppose that they do not have any cards in common, i.e. $\mathcal{P}_{c,m}= \{\emptyset\}$, and the set of cards the merchant accepts $\mathcal{P}_m \neq \{\emptyset\}$. In that case the consumer increases the score $\omega^+_{c,p}$ by one $\forall p \in \mathcal{P}_m \subset \mathcal{P}^-_c$. Here $\omega^+_{c,p}$ is an element of the vector, which is defined as

$$\left(\omega^+_{c,1}, \ldots, \omega^+_{c,N_{\mathcal{P}^-_c}}\right).$$

Given that $x^+_c = 2$ is a constant and accounts for the willingness to adopt new cards, the probability of subscribing to these cards is then determined by

$$\pi^+_{c,p} = \frac{exp\left(\frac{\omega^+_{c,p}}{\omega_c}\right)}{x^+_c k + exp\left(\frac{\omega^+_{c,p}}{\omega_c}\right)} \tag{3.3}$$

3.3.2 Merchants' Decisions

We mentioned earlier that the decisions made by the merchant are limited to subscription to a payment card method or to its cancellation.

Merchant Subscription

Similar to the consumers, the merchants initially have a certain number of cards, which are also assigned according to the *initial cards* distribution $ic \in \mathcal{IC} = \{z, r, a\}$. The options are explained in subsection 3.3.1.

Merchants keep track of all cards presented to them by consumers. Every time a card $p \in \mathcal{P}$ is presented to the merchant $m \in \mathcal{M}$ and he has a subscription to this card, i.e. $p \in \mathcal{P}_m$, he increases the score of $\theta^-_{m,p}$ by one. Given that $|\mathcal{P}_m| = N_{\mathcal{P}_m}$, $\theta^-_{m,p}$ is an element of the vector defined as

$$\left(\theta^-_{m,1}, \ldots, \theta^-_{m,N_{\mathcal{P}_m}}\right).$$

On the other hand, if the merchant does not have a subscription to the card, i.e $p \in \mathcal{P}_m^-$, the score of $\theta_{m,p}^+$ is increased by one. Given that $|\mathcal{P}_{m^-}| = N_{\mathcal{P}_{m^-}}$, we define $\theta_{m,p}^+$ as an element of the vector:

$$\left(\theta_{m,1}^+, \ldots, \theta_{m,N_{\mathcal{P}_m^-}}^+\right).$$

He decides to cancel the subscription of a card with probability[2]

$$\pi_{m,p}^- = \frac{x_m^- q}{x_m^- q + exp\left(\frac{\theta_{m,p}^-}{\theta_m}\right)} \tag{3.4}$$

where θ_m denotes the number of cards presented. Similarly he decides to subscribe to a new card with probability

$$\pi_{m,p}^+ = \frac{exp\left(\frac{\theta_{m,p}^+}{\theta_m}\right)}{x_m^+ q + exp\left(\frac{\theta_{m,p}^+}{\theta_m}\right)} \tag{3.5}$$

where $x_m^- q$ and $x_m^+ q$ represent the inertia to changes as before;
$q = \left(1 + \mathit{\Gamma}_p + N_{\mathcal{P}_m} + \frac{\varepsilon}{\mathsf{k}+\beta_p}\right)$, whereas $x_m^- = 0.05$ and $x_m^+ = 9$ are constants.

3.4 Payment Cards Strategies

Revisiting the specifications of the payment card $p \in \mathcal{P}$ from subsection 3.2.3, the following information is available to every payment method:

F_p	Consumer fixed fee
$\mathbb{D}_{F_p}$	Consumer Fixed Fee Domain
$\mathit{\Gamma}_p$	Merchant fixed fee
$\mathbb{D}_{\mathit{\Gamma}_p}$	Merchant Fixed Fee Domain

[2]The probabilities defined in equations 3.4 and 3.5 are affected by the publicity applied by each payment card provider. The role of publicity is explained in subsection 3.4.

b_p	Consumer benefits
$\mathbb{D}_{b_p}$	Domain of the Consumers' Benefits
β_p	Merchant benefits
$\mathbb{D}_{\beta_p}$	Domain of the Merchants' Benefits
l_p	Publicity cost
$\mathbb{D}_{l_p}$	Publicity Cost's Domain
$N_{T_p^*}$	The total number of transactions, which partially represents the market share
Φ_p	The total profit[3] made by $p \in \mathcal{P}$

Here, we define the solution space of the payment card's strategy as

$$\mathbb{S} = \mathbb{D}_{F_p} \times \mathbb{D}_{\mathit{\Gamma}_p} \times \mathbb{D}_{b_p} \times \mathbb{D}_{\beta_p} \times \mathbb{D}_{l_p}$$

rewritten as

$$\begin{aligned} &\mathbb{S} = \mathbb{D}_1 \times \cdots \times \mathbb{D}_5 \\ &\text{with } \mathbb{D}_1 = \mathbb{D}_{F_p}, \mathbb{D}_2 = \mathbb{D}_{\mathit{\Gamma}_p}, \mathbb{D}_3 = \mathbb{D}_{b_p}, \mathbb{D}_4 = \mathbb{D}_{\beta_p}, \mathbb{D}_5 = \mathbb{D}_{l_p} \end{aligned} \tag{3.6}$$

Therefore, now we can describe a strategy $\mathbf{s}_p$ as a sample solution from $\mathbb{S}$ for a payment card $p \in \mathcal{P}$. Additionally, we denote $\mathbf{s}_{p_i}$ with $i = 1, \ldots, 5$ as the sample element of the strategy from the i^{th} domain $\mathbb{D}_i$.

Publicity Impact on the Payment Cards

Before we give an explanation of how the payment card providers could decide on their strategy, let us in this subsection explain how we have modeled the impact of publicity. We assume that the expense in publicity, l_p, conducted by the card issuers for each interaction, has a direct impact on the consumer and merchant decisions to subscribe/cancel a card. The probabilities, π_c^+, π_c^- , π_m^+ , π_m^-, given in equations 3.2 to equation 3.5 respectively, are then adjusted according to the rule presented in the following equation

$$\boldsymbol{\Delta}\pi = \tau\pi\left(1 - \pi\right) \tag{3.7}$$

Here π substitutes any of the above probabilities, Δ represents the difference between the original value of π and the adjusted π, whereas $\tau = \alpha\left(\varphi - exp\left(-l_p\right)\right)$. The constants α and φ satisfy the constraints $\pi - \boldsymbol{\Delta}\pi \geq 0$ and $\pi + \Delta\pi \leq 1$. We model the publicity expenses as a non-linear impact on the consumers' and merchants' decisions to subscribe to a card. Furthermore, we explicitly assume that the amount spent in publicity will reduce the probabilities to cancel a card, π_c^-, π_m^-, and will increase the probabilities to subscribe to a new payment method, π_c^+, π_m^+.

Decisions of the Payment Card Providers

Let $\vec{s} = (\mathbf{s}_1, \ldots, \mathbf{s}_{N_\mathcal{P}})$ be the vector of sample strategies for all payment methods. The payment card providers' decisions consist of creating such a vector. The model of $\mathcal{APCM}$ reproduces the interactions among consumers and merchants at the point of sale, given a vector of sample strategies and a specific number of interactions I. The basic mechanism of sampling $\mathbf{s}_p$ from $\mathbb{S}$ is following a random process. We demonstrate in section 3.5 how we have applied this method in our model.

Additionally, the vector $(\mathbf{s}_1, \ldots, \mathbf{s}_{N_{\mathcal{P}}})$ could be an outcome of an extensive search over the strategy space, guided by particular criteria of interest. In chapter 5 we propose the use of Generalised Population Based Incremental Learning in order to find a joint probability distribution over this space, that allows the payment card provider to generate a predesigned strategy.

Moreover, the price level and the publicity cost could be user defined for each card in order to study a particular aspect of the market. In chapter 4 we give an example of how we can explore the features of the agent-based model by studying the impact of every component s_{p_i} over the performance of the selected strategy s_p.

In order to visualize how the payment card provider caters for both sides of the market with its strategy s_p, in figure 3-2 we have shown graphically how the elements of the strategy affect the particular consumer and merchant decisions to subscribe to or cancel a particular card. More specifically, we represent the strategy's elements as circles and the decisions of the consumers and the merchants as squares. In addition, in the figure we have shown how the profit and the market share[4] of the competitors have been determined.

[4]The market share could be determined either in terms of number of consumers, number of merchants and number of card transactions or only in terms of card transactions.

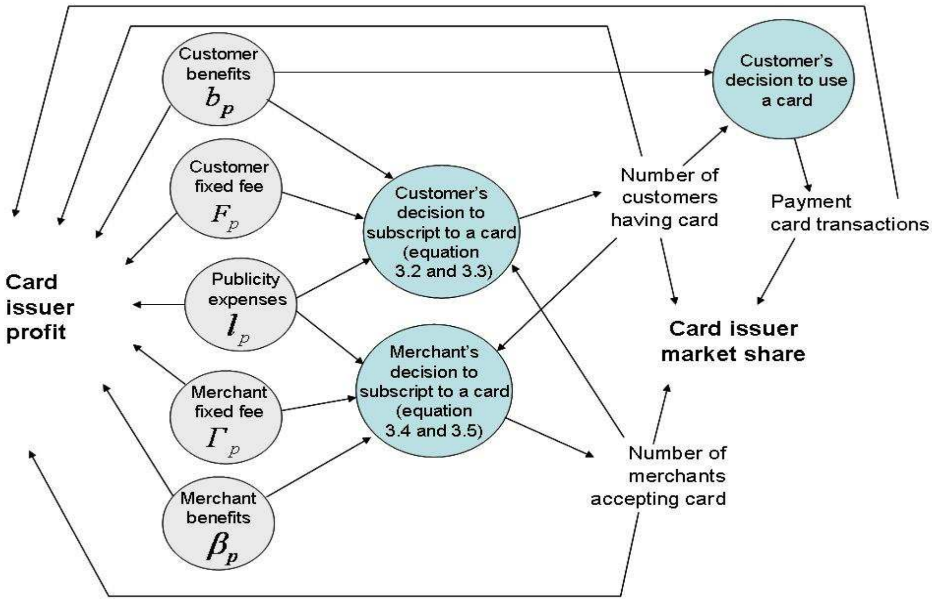


Figure 3-2: Dependency of Variables

This figure represents the dependency of the market structure on the strategy of the card issuers. The gray circles represent the strategy's elements, whereas the green circles represent the consumers' and merchants' decisions related to the card usage and the card subscriptions. The issuer's market share is measure in terms of number of consumers having card, number of merchants accepting card and the number of card transactions. The market structure is composed by the card issuer's market share and profit.

3.5 $\mathcal{MARKET}$ Simulation

In sections 3.3 and 3.4 we present the decisions of the three elements of the model. In this section we will explain the dynamics of the simulation. This simulation have been developed using Java language. We have organized the section as follows: we start with the main structure of the $\mathcal{MARKET}$ process, presented in subsection 3.5.1. In the remaining parts of the section we explain the individual functions, which are the building blocks of the simulation.

3.5.1 Procedure $\mathcal{MARKET}$

Before we explain in detail the code corresponding to the $\mathcal{MARKET}$ procedure, we introduce some auxiliary specifications with respect to the consumer and merchant decisions.

Auxiliary Specifications

Regarding the decisions of consumers and merchants to subscribe to a new card or cancel an existing subscription, let us recall from section 3.3 the information available to the agents with respect to how widespread is the use of each card.

Each consumer $c \in \mathcal{C}$ registers

ω_c	The number of merchants visited
$\left(\omega^-_{c,1}, \dots, \omega^-_{c,N_{\mathcal{P}_c}}\right)$	The number of merchants accepting the card p $\forall p \in \mathcal{P}_c$, where $\mathcal{P}_c$ is the set of cards the consumer is subscribed to.
$\left(\omega^+_{c,1}, \dots, \omega^+_{c,N_{\mathcal{P}^-_c}}\right)$	The number of merchants accepting the card p $\forall p \in \mathcal{P}^-_c$, where $\mathcal{P}^-_c$ is the set of cards the consumer is not subscribed to.

Each merchant $m \in \mathcal{M}$ registers

θ_m	The number of visits made to m
$\left(\theta^-_{m,1}, \dots, \theta^-_{m,N_{\mathcal{P}_m}}\right)$	The number of times the card p is used $\forall p \in \mathcal{P}_m$ where $\mathcal{P}_m$ is the set of cards the merchant accepts
$\left(\theta^+_{m,1}, \dots, \theta^+_{m,N_{\mathcal{P}^-_m}}\right)$	The number of times the card p is presented $\forall p \in \mathcal{P}^-_m$ where $\mathcal{P}^-_m$

is the set of cards the merchant does not accept

Given this information, we define the following vectors:

$$\vec{\omega}_c = \left(\omega_c, \left(\omega^-_{c,1}, \dots, \omega^-_{c,N_{\mathcal{P}_c}}\right), \left(\omega^+_{c,1}, \dots, \omega^+_{c,N_{\mathcal{P}^-_c}}\right)\right);$$

$$\vec{\theta}_m = \left(\theta_m, \left(\theta^-_{m,1}, \dots, \theta^-_{m,N_{\mathcal{P}_m}}\right), \left(\theta^+_{m,1}, \dots, \theta^+_{m,N_{\mathcal{P}^-_m}}\right)\right);$$

$$\vec{\omega} = \left(\vec{\omega}_1, \dots, \vec{\omega}_{\mathcal{C}}\right);$$

$$\vec{\theta} = \left(\vec{\theta}_1, \dots, \vec{\theta}_{\mathcal{M}}\right);$$

$$\eta_{c,m} = \left(\vec{\omega}_c, \vec{\theta}_m\right).$$

Here, $\eta_{c,m}$ represents the record $\vec{\omega}_c$ of the consumer c and the record $\vec{\theta}_m$ of merchant $m \in \mathcal{M}_c$, whereas c and m interact in a particular $t \in I$.

Additionally, regarding the frequency with which each consumer and merchant takes the decision to subscribe to a new card or cancel an existing subscription, we introduce the concept of *decision time*. This varies among individuals and means that every consumer/merchant in a particular interaction $t \in I$ decides which card $p \in \mathcal{P}_c / p \in \mathcal{P}_m$ could be cancelled and/or which card $p \in \mathcal{P}^-_c / p \in \mathcal{P}^-_m$ could be included in $\mathcal{P}_c/\mathcal{P}_m$. In addition the *decision time* for each agent is determined by the Poisson distribution with mean λ defined by the user, i.e. every agent follows their own Poisson distribution, having all the same λ.

Finally, with respect to the way the strategy is formed, we define two methods: selecting randomly the values from the strategy space, by applying the process *sampling*(); or the user defined strategy, by applying the process *readStrategy*(). We distinguish among these two options by the *forming strategy* condition $fs \in \mathcal{FS} = \{r, u\}$, where r is the option of the randomly defined strategy, whereas u is the option of the user defined strategy.

$\mathcal{MARKET}$ Process

Procedure $\mathcal{MARKET}$
1 $I = \mathfrak{I}; \mathbb{S} = \mathfrak{S}; N_{\mathcal{P}} = \mathfrak{N}_{\mathcal{P}};$
2 FOR $p = 1, \ldots, N_{\mathcal{P}}$ DO
3 $\quad \mathbf{s}_p = formingStrategy\left(\mathbb{S}\right)$
4 $\mathcal{APCM}\left(\left(\mathbf{s}_1, \ldots, \mathbf{s}_{N_{\mathcal{P}}}\right), I\right)$
5 END

Figure 3-3: Pseudo code of $\mathcal{MARKET}$ procedure

Figure 3-3 shows the basic structure of the $\mathcal{MARKET}$ procedure. This starts by receiving the user defined parameters of the number of interactions $\mathfrak{I}$, the strategy space $\mathfrak{S}$ and the number of payment card providers $\mathfrak{N}_{\mathcal{P}}$. In lines 2 and 3 a loop is performed to execute the function $formingStrategy()$ for all payment card providers. The function receives as a parameter the strategy space $\mathbb{S}$ and returns a sample solution $\mathbf{s}_p$. The detailed explanation of this function is presented in subsection 3.5.2.

Finally, the sample vector of the strategies assigned to all payment card providers $(\mathbf{s}_1, \ldots, \mathbf{s}_{N_{\mathcal{P}}})$ and the number of interactions I are passed as a parameter to the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket ($\mathcal{APCM}$) procedure, in which the simulation of the $\mathcal{I}$nteractions at the $\mathcal{P}$oint $\mathcal{O}$f $\mathcal{S}$ale ($\mathcal{IPOS}$) among consumers and merchants is taking place. In subsection 3.5.4 we explain the procedure in detail.

3.5.2 Function $formingStrategy()$

Function $formingStrategy\ (\mathbb{D}_{\mathbf{1}} \times \ldots \times \mathbb{D}_{\mathbf{5}})$
1 $fs = \mathfrak{fs}$
2 IF $isEqual(fs, r)$ THEN
3 $\quad \mathbf{s}_p = sampling\ (\mathbb{S})$
4 ELSE
5 $\quad \mathbf{s}_p = readStrategy()$
6 END

Figure 3-4: Pseudo code of $formingStrategy()$ function

The function $formingStrategy()$ begins by receiving from the experimenter the value of the *forming strategy* condition $\mathfrak{fs}$. This could have two possible options u, which means that it is user defined or r - randomly defined. Following this step, in line 2 the procedure verifies if the value of fs is equal r. In the case in which it is true, in line 3 the procedure generates a sample strategy. In subsection 3.5.3 we explain in more details the code for the *sampling*() function, which receives the strategy space as a parameter and returns a sample solution.

In line 5 an alternative way to create a strategy is presented. Here, the strategy is defined by the user and it is read from a text file.

3.5.3 Function $sampling()$

The *sampling*() function (line 3 figure 3-4) returns a strategy s_p. The components s_{p_i} are randomly selected from the domains $\mathbb{D}_i$, where $i = 1, \ldots, 5$. The function is presented as a code fragment in figure 3-5.

Function $sampling\left(\mathbb{D}_{\mathbf{1}} \times \ldots \times \mathbb{D}_{\mathbf{5}}\right)$
1 FOR $i = 1, \ldots, 5$ DO
2 $\quad s_{p_i} = random(\mathbb{D}_i)$
3 RETURN $(s_{p_1}, \ldots, s_{p_5})$

Figure 3-5: Pseudo code of *sampling*() function

3.5.4 Procedure $\mathcal{APCM}$

Procedure $\mathcal{APCM}\left(\left(\mathbf{s}_1, \ldots, \mathbf{s}_{N_{\mathcal{P}}}\right), I\right)$
1 $N_{\mathcal{M}} = \mathfrak{N}_{\mathcal{M}}$; $N_{\mathcal{C}} = \mathfrak{N}_{\mathcal{C}}$; $N_{\mathcal{M}_c} = \mathfrak{N}_{\mathcal{M}_c}$; $nc = \mathfrak{nc}$; $ic = \mathfrak{ic}$; $\lambda = \mathfrak{l}$
2 FOR $m = 1, \ldots, N_{\mathcal{M}}$ DO
3 $\quad \mathcal{P}_m = createMerchantCardSet\,(ic)$
4 $\quad \vec{\theta}_m = zeros()$
5 FOR $c = 1, \ldots, N_{\mathcal{C}}$ DO
6 $\quad \mathcal{M}_c = createNetConnectoins\,(\mathcal{M}, N_{\mathcal{M}_c}, nc)$
7 $\quad \mathcal{P}_c = createConsumerCardSet\,(ic)$
8 $\quad \vec{\omega}_c = zeros()$
9 FOR $t = 1, \ldots, I$ DO
10 $\quad \mathcal{IPOS}\left(\lambda, \vec{\omega}, \vec{\theta}, \vec{\mathcal{M}}_c, \vec{\mathcal{P}}_m, \vec{\mathcal{P}}_c, \vec{s}\right)$
11 END

Figure 3-6: Pseudo code of $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket procedure

In this subsection we explain the $\mathcal{APCM}$ procedure. In the first line the user assigns values to the parameters he defines. These parameters are listed bellow:

$\mathfrak{N}_{\mathcal{M}}$ the number of merchants

$\mathfrak{N}_{\mathcal{C}}$ the number of consumers

$\mathfrak{N}_{\mathcal{M}_c}$ the number of merchants visited by consumer c

$\mathfrak{nc}$ the kind of *network connection*, which could be:

- l local
- sw small world
- r random

$\mathfrak{ic}$ the *initial cards* distribution, which could be:

z	zero
r	random
a	all

$\mathfrak{l}$ the mean of the Poisson distribution

In line 2 a loop is performed over the number of merchants $N_{\mathcal{M}}$. Inside the loop in line 3, first the function $createMerchantCardSet()$ is executed. This function creates the card set for the merchant[5], receiving as a parameter the *initial cards* distribution ic. In the case where the initial number of cards is randomly decided, the merchant also selects randomly which card p is part of $\mathcal{P}_m$. The vector that contains the card set $\mathcal{P}_m$ $\forall m$ is defined as $\vec{\mathcal{P}}_m = (\mathcal{P}_1, \ldots, \mathcal{P}_{\mathcal{M}})$. In line 4 the merchant's vector $\vec{\theta}_m$, which keeps a record of the widespread use of the cards, is initialized to zero.

Starting in line 5, a loop for each one of the consumers is executed in a similar way. The consumer's network connections are formed by the execution of the $createNetConnections()$[6] in line 6. This function receives as parameters the set of merchants $\mathcal{M}$, the number of merchants connected to the consumers $N_{\mathcal{M}_c}$ and the kind of network connection nc defined by the user. After completing the task required, the function returns the set of merchants $\mathcal{M}_c$, which the consumer could visit in the process of $\mathcal{I}$nteractions at the $\mathcal{P}$oint $\mathcal{O}$f $\mathcal{S}$ale. The vector that contains the set of merchants $\mathcal{M}_c$ corresponding to all consumers, is defined as $\vec{\mathcal{M}}_c = (\mathcal{M}_1, \ldots, \mathcal{M}_{\mathcal{C}})$.

In line 7, the set of cards belonging to consumer $\mathcal{P}_c$ is created by performing the func-

[5]When the procedure $\mathcal{APCM}$ is executed inside a loop the value of ic is kept the same for the whole loop.

[6]When the procedure $\mathcal{APCM}$ is executed inside a loop the network connections are kept the same for the whole loop.

tion *createConsumerCardSet*(). This function receives the *initial cards* distribution *ic* as a parameter. In the case of a randomly assigned number of cards, the consumers decide randomly which p is part of $\mathcal{P}_c$. We defined the vector $\vec{\mathcal{P}}_c = (\mathcal{P}_1, \ldots, \mathcal{P}_{\mathcal{C}})$ as the one that contains the set of cards $\mathcal{P}_c$ of all consumers. In the last line of the loop the consumer's vector $\vec{\omega}_c$ is again initialized to zero.

Finally, in lines 9 and 10, a loop for each interaction $t \in I$ is performed executing the procedure $\mathcal{IPOS}$. The details of the procedure are presented in subsection 3.5.5.

3.5.5 Procedure $\mathcal{IPOS}$

Procedure $\mathcal{IPOS}\left(\lambda, \vec{\omega}, \vec{\theta}, \vec{\mathcal{M}}_c, \vec{\mathcal{P}}_m, \vec{\mathcal{P}}_c, \vec{s}\right)$
FOR $c = 1, \ldots, N_{\mathcal{C}}$ DO
    $m = \mathit{chooseMerchant}\,(\mathcal{M}_c)$ ***equation 3.1**
    $\eta_{c,m} = \mathit{shopping}\left(\vec{\omega}_c, \vec{\theta}_m, \mathcal{P}_m, \mathcal{P}_c\right)$
FOR $c = 1, \ldots, N_{\mathcal{C}}$ DO
    IF $\mathit{decisionTime}\,(c, \lambda)$ THEN
        $\mathcal{P}_c = \mathit{consumerCardSet}\,(\vec{\omega}_c, \vec{s}, \mathcal{P}_c)$
FOR $m = 1, \ldots, N_{\mathcal{M}}$ DO
    IF $\mathit{decisionTime}\,(m, \lambda)$ THEN
        $\mathcal{P}_m = \mathit{merchantCardSet}\left(\vec{\theta}_c, \vec{s}, \mathcal{P}_m\right)$
END

Figure 3-7: Pseudo code of $\mathcal{I}$nteractions at the $\mathcal{P}$oint $\mathcal{O}$f $\mathcal{S}$ale procedure

In this subsection we explain the procedure of $\mathcal{IPOS}$, which explicitly reproduces the $\mathcal{I}$nteraction at the $\mathcal{P}$oint $\mathcal{O}$f $\mathcal{S}$ale among consumers and merchants. The algorithm of the procedure is shown in figure 3-7. We start our presentation by listing the input parameters.

λ the mean of the poisson distribution;

$\vec{\omega}$	the vector of all consumers' record of how widely the cards are spread;
$\vec{\theta}$	the vector of all merchants' record of how widely the cards are spread;
$\vec{\mathcal{M}}_c$	the vector of all consumers' set of known merchants;
$\vec{\mathcal{P}}_m$	the vector of all merchants' set of cards;
$\vec{\mathcal{P}}_c$	the vector of all consumers' set of cards;
$\vec{s}$	the vector of strategies of the payment card providers.

The procedure consists of three consecutive loops. The first is performed over the number of consumers $N_{\mathcal{C}}$. Inside this loop each consumer c executes two functions: $chooseMerchant()$ in line 2 and $shopping()$ in line 3. The function $chooseMerchant()$ consists of applying the probability defined in equation 3.1 for each consumer, whereas the algorithm of the function $shopping()$ is presented in detail in subsection 3.5.6.

The second loop is executed after all consumers have made their shopping transactions. This loop is also performed over the number of consumers, but on this occasion has a different purpose. Inside the loop, for each consumer in line 5, the procedure validates if it is $decisionTime()$. The function $decisionTime()$ returns true if the number generated by the poisson distribution with mean λ is equal to the current number of interactions. If it is the case, in line 6 the function $consumerCardSet()$ is performed. The algorithm of this function is presented in subsection 3.5.7.

The third and final loop is performed over the number of merchants, $N_{\mathcal{M}}$. Inside the loops for each merchant, in line 8 the procedure validates if it is $decisionTime()$ in the same way as it does with the consumers: if the number generated by the poisson distribution with mean λ is equal to the current number of interactions. If the value returned by the function is equal to true, then in line 9 the function $merchantCardSet()$ is performed. This is the final task of the procedure. The algorithm of the last function is presented in subsection 3.5.8.

3.5.6 Function *shopping*()

Function *shopping* $\left(\vec{\omega}_c, \vec{\theta}_m, \mathcal{P}_m, \mathcal{P}_c\right)$

$\omega_c = \omega_c + 1$; $\theta_m = \theta_m + 1$
IF *isEmptySet* $(\mathcal{P}_{c,m})$ THEN
    FOR $p = 1, \ldots, N_{\mathcal{P}_m}$ DO $\omega^+_{c,p} = \omega^+_{c,p} + 1$
    FOR $p = 1, \ldots, N_{\mathcal{P}_c}$ DO $\theta^+_{m,p} = \theta^+_{m,p} + 1$
ELSE
    $p = selectCard(\mathcal{P}_{c,m})$ ***preferred card choice**
    $\omega^-_{c,p} = \omega^-_{c,p} + 1$
    $\theta^-_{m,p} = \theta^-_{m,p} + 1$
RETURN$\left(\vec{\omega}_c, \vec{\theta}_m\right)$

Figure 3-8: Pseudo code of *shopping*() function

In this subsection we explain the algorithm of the function *shopping*() presented in figure 3-8. Before we start, let us recall from subsection 3.5.1 the definition of the vectors $\vec{\omega}_c$ and $\vec{\theta}_m$.

$$\vec{\omega}_c = \left(\omega_c, \left(\omega^-_{c,1}, \ldots, \omega^-_{c,N_{\mathcal{P}_c}}\right), \left(\omega^+_{c,1}, \ldots, \omega^+_{c,N_{\mathcal{P}^-_c}}\right)\right);$$

$$\vec{\theta}_m = \left(\theta_m, \left(\theta^-_{m,1}, \ldots, \theta^-_{m,N_{\mathcal{P}_m}}\right), \left(\theta^+_{m,1}, \ldots, \theta^+_{m,N_{\mathcal{P}^-_m}}\right)\right);$$

These vectors represent the records kept by consumers and merchants respectively, regarding the use and acceptance of a particular card in the market.

The function receives the following list of input parameters:

$\vec{\omega}_c$	the vector of all cards' acceptance records, which belongs to the consumer c;
$\vec{\theta}_m$	the vector of all cards' usage record, which belongs to the merchant m;
$\mathcal{P}_m$	the set of merchant's cards;
$\mathcal{P}_c$	the set of consumer's cards.

The function *shopping*() determines the payment method used in a transaction between a particular consumer c and a particular merchant $m \in \mathcal{M}_c$. In line 1 the number of visited merchants ω_c by the consumer c and the number of visitors θ_m to the merchant m are increased by 1 respectively. Following, in line 2 the function validates if the set of common cards $\mathcal{P}_{c,m}$ *isEmptySet*(). In the possible event that the consumer and merchant do not have a card or cards in common, the instructions in lines 3 and 4 are performed, otherwise the tasks of lines 6, 7, and 8 are carried out.

When $\mathcal{P}_{c,m} = \emptyset$, the transaction between the consumer and the merchant is settled with cash. In line 3 a loop is executed over the number of the merchant's cards, $N_{\mathcal{P}_m}$, in order to update the consumer's record $\omega^+_{c,p}$ of the cards that are accepted by the merchant m, but c does not have a subscription to them. Similarly in line 4 a loop is performed over the number of the consumer's cards, $N_{\mathcal{P}_c}$. The aim of the loop is to update the merchant's record $\theta^+_{m,p}$ regarding the cards that are used by the consumer c, but are not accepted by m.

In the opposite case, when the consumer and merchant have cards in common, $\mathcal{P}_{c,m} \neq \emptyset$, the transaction is performed by one of the cards. The mechanism of selecting a particular card consists in applying a *preferred card choice* [7] and it is performed by function $selectCard()$ in line 6. Next in lines 7 and 8 the consumer's record $\omega^-_{c,p}$ and the merchant's record $\theta^-_{m,p}$ of the card p used in the transaction are updated respectively. Finally, in line 9 the function returns the new values of the vectors $\vec{\omega}_c$ and $\vec{\theta}_m$.

3.5.7 Function $consumerCardSet()$

In this subsection we explain the algorithm related to the function $consumerCardSet()$, which determines the cards a consumer would include in its set of cards $\mathcal{P}_c$ and which subscriptions to cards should be cancelled. In figure 3-9 we present the segment of pseudo code corresponding to this function. The input parameters are the following: the vector of the records regarding the acceptance of all cards kept by the consumer $\vec{\omega}_c$, the vector of all payment cards strategies $\vec{s}$ and the set of consumer's cards $\mathcal{P}_c$.

[7] It is defined in subsection 3.3.1.

Function $consumerCardSet\,(\vec{\omega}_c, \vec{s}, \mathcal{P}_c)$
1 FOR $p = 1, \ldots, N_{\mathcal{P}_c}$ DO
2 IF $cancelSubscription\,\left(\omega^-_{c,p}, \omega_c, \mathbf{s}_p\right)$ THEN
3 $\mathcal{P}_c = reduceCardSet(p)$ ***equation 3.2**
4 FOR $p = 1, \ldots, N_{\mathcal{P}^-_c}$ DO
5 IF $includeSubscription\,\left(\omega^+_{c,p}, \omega_c, \mathbf{s}_p\right)$ THEN
6 $\mathcal{P}_c = increaseCardSet(p)$ ***equation 3.3**
7 RETURN($\mathcal{P}_c$)

Figure 3-9: Pseudo code of $consumerCardSet()$ function

The main body of the function is divided into two loops. The first loop is performed over the number of cards the consumer has a subscription to, $N_{\mathcal{P}_c}$. Inside the loop in line 2 the boolean function $cancelSubscription()$ is executed. It calculates the consumer's probability of dropping a card determined by equation 3.2, and compares the obtained probability with the randomly generated number. If the number is lower than the predefined probability, then the function $cancelSubscription()$ returns the true value. In this case the 3rd line of the function $consumerCardSet()$ is performed and the card p is removed from the set of the consumer $\mathcal{P}_c$, by applying the function $reduceCardSet()$. Otherwise no action is taken.

The second loop is carried out over the number of cards the consumer does not have a subscription to, $N_{\mathcal{P}^-_c}$. Similarly to the previous loop, in line 5 the boolean function $includeSubscription()$ is performed. This function calculates the consumer's probability of subscribing to a new card and compares the obtained value with a randomly generated number. If this number is lower than the calculated probability, the boolean function returns true. In this case line 6 in figure 3-9 is executed and the card p is added to the set of the consumer's cards, $\mathcal{P}_c$. This task is carried out by the function $increaseCardSet()$. At the end the function $consumerCardSet()$ returns the new value of the consumer's set $\mathcal{P}_c$.

3.5.8 Function $merchantCardSet()$

In this subsection we explain the algorithm of the function $merchantCardSet()$, which is shown in figure 3-10. The function receives three input parameters: the vector of the records regarding the usage of all cards kept by the merchant $\vec{\theta}_c$, the vector of all payment card strategies $\vec{s}$ and the set of the merchant's cards $\mathcal{P}_m$.

Function $merchantCardSet\left(\vec{\theta}_c, \vec{s}, \mathcal{P}_m\right)$
1 FOR $p = 1, \ldots, N_{\mathcal{P}_m}$ DO
2   IF $cancelSubscription\left(\theta^{-}_{m,p}, \theta_m, \mathbf{s}_p\right)$ THEN
3         $\mathcal{P}_m = reduceCardSet(p)$ ***equation 3.4**
4 FOR $p = 1, \ldots, N_{\mathcal{P}^{-}_m}$ DO
5   IF $includeSubscription\left(\theta^{+}_{m,p}, \theta_m, \mathbf{s}_p\right)$ THEN
6         $\mathcal{P}_m = increaseCardSet(p)$ ***equation 3.5**
7 RETURN($\mathcal{P}_m$)

Figure 3-10: Pseudo code of $merchantCardSet()$ function

The aim of the function is to determine which card should be added to the merchant's set and which card should be dropped. It starts by performing a loop over the number of cards accepted by the merchant $N_{\mathcal{P}_m}$. Inside the loop, in line 2 the boolean function $cancelSubscription()$ is carried out. This function returns true, if the merchant's probability to drop a card determined by the equation 3.4 is greater than a randomly generated number. If it is the case, the function $reduceCardSet()$ in figure 3-10 in line 3 is executed and the card p is removed from the merchant's set $\mathcal{P}_m$. If the boolean function $cancelSubscription()$ returns false, no action is taken.

Another loop starts in line 4. This loop runs over the number of cards that are not accepted by the merchants, $\mathcal{P}^{-}_m$. In line 5 the boolean function $includeSubscriptiont()$ is performed. This function calculates the merchant's probability of subscribing to a new card, defined in equation 3.5 and compares the obtained value with a randomly generated number. It returns true, if the resulting probability is greater than the random number. In this case the instruction in line 6 is executed, i.e. the card p is added to the merchant's set of cards $\mathcal{P}_m$. This task is performed by the function $increaseCardSet()$. Finally in line 7 the algorithm returns the new set of merchant's cards.

3.6 Summary

In this section we summarise the chapter that present the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket. The aim of the model is to simulate the interactions among consumers and merchants at the point of sale in order to study the dynamics of the payment card markets.

In section 3.2 we formally introduced the sets of agents implemented in our model. We have listed the notation used to represent the sets and their respective cardinality in table 3.1, where the consumer's set of merchants is $\mathcal{M}_c \subset \mathcal{M}$. In addition in table 3.2 we show the notation used to define the different sets of cards together with the corresponding cardinality.

Description	Symbol	Cardinality
Set of Merchants	$\mathcal{M}$	$N_{\mathcal{M}}$
Set of Consumers	$\mathcal{C}$	$N_{\mathcal{C}}$
Set of Payment Cards	$\mathcal{P}$	$N_{\mathcal{P}}$
Consumer's Set of Merchants	$\mathcal{M}_c$	$N_{\mathcal{M}_c}$

Table 3.1: Sets of the agents

Description	Symbol	Cardinality
Set of cards that belong to consumer c	$\mathcal{P}_c$	$N_{\mathcal{P}_c}$
Set of cards that belong to merchant m	$\mathcal{P}_m$	$N_{\mathcal{P}_m}$
Set of common cards between consumer c and merchant m	$\mathcal{P}_{c,m}$	$N_{\mathcal{P}_{c,m}}$
Set of cards that does no belong to consumer c	$\mathcal{P}_{c^-}$	$N_{\mathcal{P}_{c^-}}$
Set of cards that does no belong to merchant m	$\mathcal{P}_{m^-}$	$N_{\mathcal{P}_{m^-}}$

Table 3.2: Different sets of cards

Following this, in section 3.3 we defined the decisions of consumers and merchants, which establish the basis for the interactions among them. The symbols used to specify each decision are presented in table 3.3. Additionally, those decisions are shaped by variables, emerging from the simulation of the interactions. We present these variables in table 3.4.

Symbol	Description
$\mathbf{v}_{c,m}$	Consumer's chances of visiting a particular merchant
π_c^-	Consumer's chances of dropping a card
π_c^+	Consumer's chances of adding a new card
π_m^-	Merchant's probability of dropping a card
π_m^+	Merchant's probability of adding a new card

Table 3.3: Consumers and Merchants Decisions

Symbol	Description
$d_{c,m}$	Travel Cost
ω_c^-	Number of Merchants accepting a card that the consumer has
ω_c^+	Number of Merchants accepting a card that the consumer does not have
ω_c	Number of Merchants visited by the consumer
θ_m^-	Number of Consumers using a card that the merchant has
θ_m^+	Number of Consumers wanting to use a card that the merchant does not have
θ_m	Number of Consumers that have visited the merchant

Table 3.4: Variables emerging from the simulation

Next in section 3.4 we defined the strategy of the payment card providers. We list the elements of this strategy in table 3.5, whereas in table 3.6 we present the symbols used to name the domains of those elements.

Symbol	Description
F_p	Consumer Fixed Fee
$\mathit{\Gamma}_p$	Merchant Fixed Fee
b_p	Benefits of the Consumers
β_p	Benefits of the Merchants
l_p	Publicity Cost

Table 3.5: Payment Method's Strategy

Symbol	Domains
$\mathbb{D}_{F_p}$	Consumer Fixed Fee Domain
$\mathbb{D}_{\mathit{\Gamma}_p}$	Merchant Fixed Fee Domain
$\mathbb{D}_{b_p}$	Domain of the Consumers' Benefits
$\mathbb{D}_{\beta_p}$	Domain of the Merchants' Benefits
$\mathbb{D}_{l_p}$	Publicity Costs Domain

Table 3.6: Strategy's Domains

Finally in section 3.5 we explained the whole set of algorithms used to implement the simulation of the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket and we established the relationship among the different functions and procedures.

Chapter 4

Competition in the $\mathcal{APCM}$: An Empirical Study

4.1 Introduction

The notation of the model was formally introduced and described in chapter 3. The aim of this chapter is to test experimentally the model of the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket ($\mathcal{APCM}$). The model can generate a considerable quantity of data. The analysis of such data involves many aspects of the market dynamics, far beyond the scope of this chapter. Here, in order to explore the possibilities that the agent-based modelling gives us as a tool for analysis, we have designed two experiments.

Before we proceed with the study of the economic relationships presented in the data series produced by the model, in section 4.2 we perform a stationary test to the data. Thus, if the series turn out to be stationary, we will formulate regression models in order to explain the market structure in terms of the price, applied by the competitors.

The first experiment simulates a market, in which all competitors price equally the payment methods they offer. It is designed to investigate the shape of the demand of consumers and merchants and how these demands constrain each other. For that reason, based on the data obtained, we estimate the demand curve and we measure its sensitivity on both sides with respect to the fixed price. The experiment is performed in three scenarios with different numbers of competitors in the market. This study is presented in section 4.3.

The aim of the second experiment is to validate to what degree we have reproduced the dynamics of competition among payment card providers in a market, where each card has different price. In this study we use the parameters' setting of the previous experiment regarding the consumers' and merchants' demands. We test the market structure dependency on the competitors' strategy, given different degrees of *initial cards* distribution in the market. We present our findings in section 4.4.

In section 4.5 we provide a summary of the main findings and discuss some of the characteristics of the computational model.

4.2 Stationarity

In studying the time series data, in order to give a reasonable explanation of the analysed phenomena, the economists look for relationships between the economic variables. Nevertheless, to formulate a robust regression model that studies the behaviour of the data, we need first to look at the specification of the dynamic structure of the time series. In other words, we need to test the stochastic properties of the series, in order to ensure that the data is stationary. We say that the data series are stationary if the mean, the variance and the covariance with respect to other values of the same time series do not depend on the time.

The formal tests of stationarity are the so called tests for unit root. Nowadays, a wide variety of unit root tests exist. In our case, we have used the Augmented Dickey-Fuller test. It is applied to the data series generated from one execution of the model, given the setting of parameters, presented in table 4.1. The simulation is carried out with number of interactions $I = 20000$. The poisson distribution, used to determine how often consumers and merchants decide to drop or add a new card, has a mean $\lambda = 20$.

Symbol	Description or Domain	Value
$N_{\mathcal{M}}$	Number of Merchants	125
$N_{\mathcal{C}}$	Number of Consumers	1100
$N_{\mathcal{P}}$	Number of Payment Cards Provider	9
$N_{\mathcal{M}_c}$	Number of Merchants visited by the Consumer	5
x_c^-	accounting for the consumers' inertia to drop cards	0.05
x_c^+	account for the consumers' inertia to add new cards	2
x_m^-	account for the merchants' inertia to drop cards	0.05
x_m^+	account for the merchants' inertia to add new cards	9
α	account for the impact of the publicity cost	0.1
φ	account for the impact of the publicity cost	5
$\mathbb{D}_{F_p}$	Consumer Fixed Fee Domain	$[0, 10]$
$\mathbb{D}_{\Gamma_p}$	Merchant Fixed Fee Domain	$[0, 10]$
$\mathbb{D}_{b_p}$	Domain of the Consumers' Benefits	$[-1, 1]$
$\mathbb{D}_{\beta_p}$	Domain of the Merchants' Benefits	$[-1, 1]$
$\mathbb{D}_{l_p}$	Publicity Cost's Domain	$[0, \infty]$

Table 4.1: The sets of parameters

For each card issuer $p \in \mathcal{P}$, the model produces three time series: the number of consumers per p, the number of merchants per p and the number of transactions performed with p. Before we carry out the test, we have applied a natural logarithm to the time series. The null hypothesis tested is that the series has a unit root. In table 4.2 we report the critical values, which are the same for all tested time series. In addition in table 4.3 we presented the t-Statistic obtained for the nine card issuers. The probability of rejecting the null hypothesis is calculated based on the MacKinnon one side p-value [61]. The complete set of statistics produced by the test for all time series are included in appendix C of the thesis.

Confidence interval	Test critical values
1% level	-3.430505
5% level	-2.861493
10% level	-2.566786

Table 4.2: The test critical values

Issuer	Consumers per card	Merchants per card	Transactions per card
1	-12.83642	-10.86818	-12.82327
2	-14.03561	-11.11079	-12.24813
3	-13.5644	-8.662949	-10.12976
4	-12.92276	-11.39592	-11.95469
5	-11.58816	-8.308004	-8.767829
6	-14.3889	-8.893099	-12.13232
7	-14.22093	-11.3291	-13.18048
8	-11.55958	-9.574812	-13.18048
9	-10.99369	-10.12067	-12.52632

Table 4.3: t-Statistics of the time series produced by the model

Given the test critical values and t-Statistics presented in tables 4.2 and 4.3 respectively, the null hypothesis was rejected for all time series. In other word, the series produced by the model, are stationary processes. Thus, the estimated regression models, formulated for the experiments presented below are statistically meaningful.

4.3 Fixed Price Sensitivity of Demand

Electronic payment cards are known as a two-sided platform. This means that the electronic payment provider, in order to place his product in the market, needs to attract two different groups of users: consumers and merchants. The literature studying these markets is relatively new and the precise results from the existing analytical models depend on the assumptions regarding the relationship of the market participants [18]. Nevertheless, it seems to be widely accepted that the prices on both sides of the platform, depend on a complex way on two main factors [18]:

- the price elasticities of the demand on both sides;
- the indirect externalities between each side arising from the use of the product at the other side.

In the classical market model, the demand is modelled as a downwards sloping relationship between price and quantities. Nevertheless, given the complex price structure in the market of a two-sided platform [2] and the inherent externalities between the end-users, the demand for electronic payment instruments has a complex shape and the analytical methods are not able to model it explicitly. Nevertheless, in our artificial market, the use of agent-based methodology allows us to measure the aggregate consumers' and merchants' demands, emerging from the interactions at the point of sale.

In this section, we start with a brief analysis of the factors affecting the demand for payment instruments presented in subsection 4.3.1. Next, given that the competitors in the market have priced equally the payment methods that they are offering, we study the effects of different levels of consumers' fixed fees on both sides of the market. Based on the data obtained from the experiment, we estimate the demand of the $\mathcal{A}$rtificial $\mathcal{P}$ayment

$\mathcal{C}$ard $\mathcal{M}$arket. The test is carried out with three scenarios having different numbers of competitors. The details of the experiment are explained in subsection 4.3.2, whereas the notation of the variables used to evaluate the outcome of the study is listed in subsection 4.3.3. Further, in subsection 4.3.4 we present and discuss the data obtained for the different levels of consumers' fixed fees. In subsection 4.3.5 we close our exposition with conclusions regarding the findings of this experiment.

4.3.1 The Modelled Demand

Recall from section 3.2 that the consumers' and merchants' decisions to subscribe or cancel a card[1] on the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket are affected mainly by a number of factors.

The merchants' decisions are affected by:

- The price structure[2];
- The effect of the indirect externalities on the merchants' side, arising from the consumers' use of the payment instruments;
- The number of multiple cards held at the same time;
- The merchants' inertia to subscribe to a new card;
- The merchants' inertia to cancel a subscription.

While the consumers' decisions are affected by:

- The price structure;
- The effect of the indirect externalities on the consumers' side, arising from the merchants' acceptance of the electronic payment;

[1] Please refer to subsections 3.3.1 and 3.3.2.
[2] Fixed and variable fees.

- The number of multiple cards held at the same time;
- The consumers' inertia to subscribe to a new card;
- The consumers' inertia to cancel a subscription;

Consequently, throughout the simulation the aggregated form of these individual decisions represents a complex two-sided demand for electronic payment methods. In this subsection we briefly explain how the factors listed above are considered in the current experiment.

The price structure and the two-sidedness of the market

In the artificial market, each competitor has a complex price structure[3], which addresses independently both sides of the market[4]. According to the two-sided theory [1], the price in those markets is not neutral. In the present experiment, in order to study how a change in the price affects the dynamics of the $\mathcal{APCM}$, we test separately the impact of different price levels of consumers' and merchants' fixed fees[5]. We expect to observe a different level of quantity demanded not only on the consumers' side, but on the merchants' side as well. In other words, a price change on one side will modify the number of consumers and merchants having a card.

For that reason, we propose to measure the demand in terms of the number of end-users having a specific card at the end of the simulation. Let $N_{\mathcal{C}_p^*}$ be the average number of consumers per card from the ten executions performed in the experiment and $N_{\mathcal{M}_p^*}$ is the average number of merchants per card from the same ten executions. The quantity

[3] Fixed fees and variable benefits.

[4] Please refer to 3.4.

[5] Here we present the data obtained from different levels of the consumers' fixed fees and the estimated demand related to changes in the consumers' and merchants' fixed fees. The data of the complementary study of changes in the merchant fixed fees is presented in the second appendix.

demanded on the consumer side is denoted $N^{log}_{\mathcal{C}^*_p}$ and represents the logarithm of the average number of consumers per card, whereas the quantity demanded on the merchant side is $N^{log}_{\mathcal{M}^*_p}$ and represents the logarithm of the average number of merchants per card.

Given the way we have conducted the experiment, $N^{log}_{\mathcal{M}^*_p}$ represents the estimated demand on the merchants' side in terms of the consumers' or merchants' fixed fees, and $N^{log}_{\mathcal{C}^*_p}$ denotes the estimated demand on the consumers' side in terms of the fixed price of both sides.

Multiple cards held at the same time

Following with the analysis of the factors affecting the demand, in order to test the impact of multiple cards held at the same time, in the present experiment we simulate the dynamics of the artificial market with three different numbers of competitors $N_{\mathcal{P}} \in [2, 5, 9]$. In other words, the possible total number of multiple cards held will vary in accordance with the number of competitors.

Consumers' and Merchants' inertia

Finally, note that different values of the consumers' and merchants' inertia[6] to add or to drop cards, will represent a different shape of the end-users' demand. For that reason, in the experiment of this section the values of those constants are kept without changes for all examples[7]. In that way the form of the consumers' and merchants' demand will remain the same despite its complex shape.

[6]The inertia of the consumer and the merchant are modelled as constants in the formula of the individual decisions.

[7]The values of the constants are presented in table 4.7.

4.3.2 The demand and its sensitivity in $\mathcal{APCM}$

In this subsection, we explain the experiment we have conducted, in order to estimate the demand and its sensitivity on both sides resulting from different levels of consumer fixed fees in the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket. The experiment is carried out with different numbers of competitors in the market.

We start by recalling from the previous chapter the list of symbols used to represent the elements of the strategy. Those are presented in table 4.4. Next, we present a definition of demand sensitivity with respect to the consumer fixed fees in the context of the artificial market. Following on, we explain how we have conducted the test and finally we present the setting of the parameters.

Symbol	Description
F_p	Consumer Fixed Fee
Γ_p	Merchant Fixed Fee
b_p	Benefits to the Consumers
β_p	Benefits to the Merchants
l_p	Publicity Cost

Table 4.4: Payment Method's Strategy

In the context of the $\mathcal{APCM}$, given the two-sided nature of the market,the demand could be measured either on the consumers' or on the merchants' sides. Additionally, in each side, it could be calculated in terms of different levels of consumers' or merchants' fixed fees. In order to gain more insight regarding the demand shape, we measure also its sensitivity in terms of the fixed price of the opposite side. We estimate the curve of the demand, by using the data obtained from the experiment.

The study consists of three scenarios with different number of competitors $N_{\mathcal{P}} \in [2, 5, 9]$. In each scenario the initial quantity of payment methods are randomly assigned to consumers and merchants. For each scenario we have executed five cases. In each case the payment card issuers are using the same price. We distinguish from one case to another, because the consumers' fixed fees have a different price level[8]. The *starting case* uses the strategy, presented in table 4.5. In each one of the remaining four cases, the consumers' fixed fees take one of the following values $[0, 2, 6, 8]$. In other words, the consumers' fixed fees are decreased by 50% and 100% and consequently increased by 50% and 100%. The rest of the strategy elements remain without changes.

Additionally, each case is executed ten times with a different random seed. The random seed is used to ensure that the values of the simulation parameters are kept the same and the only change in the model is the change in the level of the consumer fixed fees and the number of competitors. For our analysis we report the average number of consumers per card and the average number of merchants per card. Given the procedure we have followed, for each scenario these two figures[9] represent the number of consumers' per card and the number of merchants' per card given the different level of consumers' fixed fees.

F_p	Γ_p	b_p	β_p	l_p
4	2	0.3	-0.3	7

Table 4.5: Unique Payment Cards' Strategy

The number of interactions is set to $\mathcal{I} = 3000$. The decision period of consumers and merchants is determined by a poisson distribution with $\lambda = 20$. The rest of the parameters are presented in two tables: table 4.6 lists the parameters set for the agents, whereas table 4.7 shows the constants[10].

[8]In the second appendix we have included a complementary study with different levels of merchants' fixed fees.

[9]The average number of consumers per card and the average number of merchants per card.

[10]We use exactly the same consumers' and merchants' inertia as in section 4.4.

Symbol	Description	Cardinality	Value
$\mathcal{M}$	Set of Merchants	$N_{\mathcal{M}}$	125
$\mathcal{C}$	Set of Consumers	$N_{\mathcal{C}}$	1100
$\mathcal{M}_c$	Consumers' Set of Merchants	$N_{\mathcal{M}_c}$	5

Table 4.6: The sets of the agents

Symbol	Description of the Constants	Value
ε	common constant for the inertia to change	1
x_c^-	the consumers' inertia to drop cards	0.05
x_c^+	the consumers' inertia to add new cards	2
x_m^-	the merchants' inertia to drop cards	0.05
x_m^+	account for the merchants' inertia to add new cards	9
α	account for the impact of the publicity cost	0.1
φ	account for the impact of the publicity cost	5

Table 4.7: Constants of the model

4.3.3 Measuring the outcome of the experiment

In this subsection we present the notation of the variables used to evaluate the outcome of the experiment.

- ΔF_p - The percentage of changes in the fixed price;
- $N_{\mathcal{M}_p^*}$ - The average number of merchants per card, calculated over ten executions;
- $N_{\mathcal{C}_p^*}$ - The average number of consumers per card, calculated over ten executions;
- $N_{\mathcal{M}_p^*}^{log}$ - The average number of merchants per card, on a logarithmic scale;
- $N_{\mathcal{C}_p^*}^{log}$ - The average number of consumers per card, on a logarithmic scale;
- $N_{\mathcal{M}_p^*}^{N_{\mathcal{P}}}$ - The estimated over $N_{\mathcal{M}_p^*}^{log}$ average number of merchants per card, where $N_{\mathcal{P}}$ is the number of competitors in the market;
- $N_{\mathcal{C}_p^*}^{N_{\mathcal{P}}}$ - The estimated over $N_{\mathcal{C}_p^*}^{log}$ average number of consumers per card;
- E_{F_p} - The demand sensitivity on the consumers' side;
- $\mathcal{E}_{F_p}$ - The demand sensitivity on the merchants' side arising from changes in the consumers' fixed fees;

4.3.4 The effect of changing the fixed price

In this subsection we present the results obtained in the five cases of the three scenarios, previously explained. The data are presented in table 4.8 in the following order, considered per column:

- The number of competitors $N_{\mathcal{P}}$ for each scenario;
- The values of the consumers' fee F_p according to each case;
- The percentage of changes in the fixed price ΔF_p;
- The average number of consumers per card $N_{\mathcal{C}_p^*}$;
- The average number of consumers per card $N_{\mathcal{C}_p^*}^{log}$, on a logarithmic scale;
- The estimated average number of consumers per card $N_{\mathcal{C}_p^*}^{N_{\mathcal{P}}}$ according to equation 4.1;
- The average number of merchants per card $N_{\mathcal{M}_p^*}$;
- The average number of merchants per card $N_{\mathcal{M}_p^*}^{log}$ on a logarithmic scale;
- The estimated average number of merchants per card $N_{\mathcal{M}_p^*}^{N_{\mathcal{P}}}$ according to equation 4.2;

The *starting case* is presented in bold font. The rest of the strategy's elements, presented in table 4.5 remain without changes.

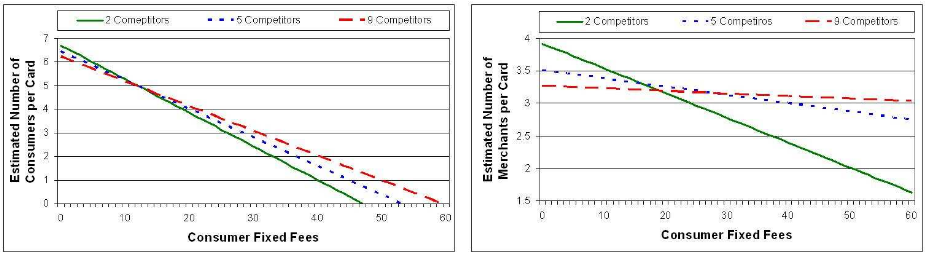
In addition, we have estimated in figure 4-1(a) the curves of consumers' demand on a semi logarithmic scale based on the data of the fifth column of table 4.8 and the price presented in the second column. The fifth column represents the logarithmic value of the average number of consumers per card, whereas the second column represents the real value of the price. In equation 4.1 we present the estimated linear regression of this demand $N_{\mathcal{C}_p}^{log,N_{\mathcal{P}}}$, where $N_{\mathcal{P}}$ is the number of competitors in the market. In the second appendix we have included a table which compares the least square errors obtained from

$N_{\mathcal{P}}$	F_p	ΔF_p	$N_{\mathcal{C}_p^*}$	$N_{\mathcal{C}_p^*}^{log}$	$N_{\mathcal{C}_p^*}^{log,N_{\mathcal{P}}}$	$N_{\mathcal{M}_p^*}$	$N_{\mathcal{M}_p^*}^{log}$	$N_{\mathcal{M}_p^*}^{log,N_{\mathcal{P}}}$
2	**0**	-100%	823	6.712956	6.704429	50	3.912023	3.921927
	2	-50%	613	6.418365	6.419255	47	3.8501476	3.845579
	4	—	**456**	**6.122493**	**6.134081**	**44**	**3.784190**	**3.769231**
	6	50%	344	5.840642	5.848908	40	3.688880	3,692884
	8	100%	264	5.575949	5.563734	37	3.610918	3,616536
5	**0**	-100%	634	6.45205	6.446507	33	3.496508	3.509415
	2	-50%	496	6.20658	6.20311	33	3.496508	3.484101
	4	—	**383**	**5.948035**	**5.959713**	**32**	**3.465736**	**3.458787**
	6	50%	301	5.7071103	5.716316	31	3.433987	3.433473
	8	100%	241	5.4847969	5.472919	30	3.401197	3.408159
9	**0**	-100%	505	6.224558	6.223202	26	3.258097	3.265941
	2	-50%	410	6.016157	6.013214	26	3.258097	3,258097
	4	—	**330**	**5.799093**	5.803226	**26**	**3.258097**	**3.250253**
	6	50%	267	5.587249	5.593237	26	3.258097	3.242408
	8	100%	219	5.389072	5.383249	25	3.218876	3.234564

Table 4.8: The effects of changing the consumers' fixed fee F_p

polynomial approximation of the data of first, second and third degrees in the case of 2 competitors. In addition, the *log* stands for the logarithmic estimation of the number of consumers per card and this notation is used to estimate the number of merchants per card. The data in the sixth column in table 4.8 is calculated, using these formulae. The fifth column is the real data obtained by the simulation, where the sixth column is the estimated data. For this estimated model, the coefficient of determination R^2, which represents the proportion of the variation in the number of consumers, explained by the linear regressor (the consumer fixed fee) is 0.9995 in the case of two competitors[11]. Additionally, the coefficient that multiplies the consumer fixed fees F_p is the demand sensitivity. In our analysis we take the sensitivity in absolute values, in a way that the lower the demand sensitivity the slower the reaction of consumers or merchants to a different level of fixed fees. This factor determines the slope of the line.

[11]The coefficient of determination for the other two cases is similar.



(a) Estimated Number of Consumers per card

(b) Estimated Number of Merchants per card

Figure 4-1: The Consumers' Fixed Fee F_p

$$
\begin{aligned}
N_{\mathcal{C}_p}^{log,2} &= -0.142587F_p + 6.704429 \\
N_{\mathcal{C}_p}^{log,5} &= -0.121699F_p + 6.446507 \\
N_{\mathcal{C}_p}^{log,9} &= -0.104994F_p + 6.223202
\end{aligned}
\tag{4.1}
$$

Observation CFF-1: In figure 4-1(a) we observe that the curves of the consumers' demand have different slopes for each scenario of a different number of competitors. Consequently, in equation 4.1 we note that the demand sensitivity of each scenario increases in comparison with the previous one: the higher the number of the competitors the lower the sensitivity in absolute terms.

Further, from the data presented in column eight of table 4.8 in figure 4-1(b) we have plotted the estimated curves of the merchants' demand in terms of consumer fixed fees. The precise formulae of the estimated linear function of the curves $N_{\mathcal{M}_p}^{log,N_{\mathcal{P}}}$ are shown in equation 4.2. In this estimation model, the coefficient of determination, R^2 for the case of two competitors is 0.9911.

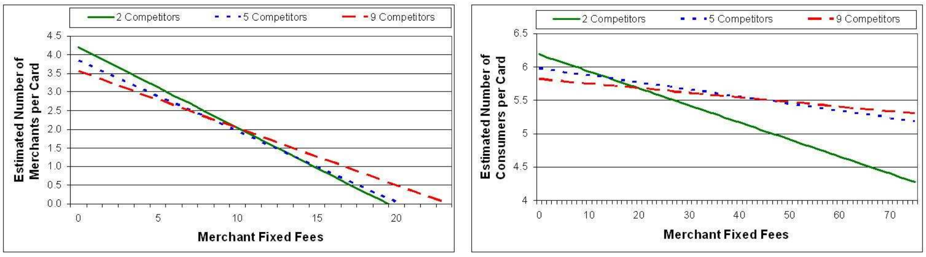
$$
\begin{aligned}
N_{\mathcal{M}_p}^{log,2} &= -0.038174F_p + 3.921927 \\
N_{\mathcal{M}_p}^{log,5} &= -0.012657F_p + 3.509415 \\
N_{\mathcal{M}_p}^{log,9} &= -0.003922F_p + 3.265941
\end{aligned}
\tag{4.2}
$$

Observation CFF-2: We observe in 4-1(b) that the three curves of the merchants' demand resulting from the changes in the consumer fixed fees have very different slopes. In addition, we notice in equation 4.2 that the curve with the higher sensitivity corresponds to the case with 2 competitors, whereas in the case of 9 card issuers in the market, the slope of the curve is getting close to zero.

Continuing, we present figures 4-2(a) and 4-2(b) and their corresponding equations, which represent the estimated curves of consumers' and merchants' demands measured in terms of the merchant fixed fees. Those figures are created from the results obtained by a complementary study regarding the different levels of merchant fixed fees, included in the Appendix B. More specifically, in the Appendix A we have included an Excel file with the results obtained from a test with different levels of merchants' fixed fees, consumers' benefits and merchants' benefits. In the Appendix B we have presented the data obtained in the case of the merchant fixed fees.

$$
\begin{aligned}
N_{\mathcal{M}_p}^{log,2} &= -0.215992\mathit{\Gamma}_p + 4.201329 \\
N_{\mathcal{M}_p}^{log,5} &= -0.18977\mathit{\Gamma}_p + 3.840062 \\
N_{\mathcal{M}_p}^{log,9} &= -0.153197\mathit{\Gamma}_p + 3.556419
\end{aligned}
\tag{4.3}
$$

Observation MFF-1: In figure 4-2(a) we observe that the curves of the merchants' demand have different slopes for each scenario with a different number of competitors. In addition, in equation 4.3 we note that the decrease among demand



(a) Estimated Number of Merchants per card

(b) Estimated Number of Consumers per card

Figure 4-2: The Merchants' Fixed Fee $\mathit{\Gamma}_p$

sensitivities has a similar rate: the higher the number of the competitors, the lower the sensitivity.

$$\begin{aligned} N^{log,2}_{\mathcal{C}_p} &= -0.025478\mathit{\Gamma}_p + 6.187096 \\ N^{log,5}_{\mathcal{C}_p} &= -0.010580\mathit{\Gamma}_p + 5.975831 \\ N^{log,9}_{\mathcal{C}_p} &= -0.006927\mathit{\Gamma}_p + 5.816523 \end{aligned} \tag{4.4}$$

Observation MFF-2: We observe in figure 4-2(b) that the curves of the consumers' demand resulting from the changes in the merchant fixed fees have very different slopes. In addition, we notice in equation 4.4 that the curve with the higher sensitivity corresponds to the case with 2 competitors, whereas in the case of 9 card issuers in the market, the slope of the curve is getting close to zero.

4.3.5 Analysis of the experiment

In this subsection, based on the results obtained and taking into account that the observations related to the consumers' and merchants' sides are qualitatively similar, we present some general conclusions.

Conclusion 1: From *Observation CFF-1* and *Observation MFF-2* it follows that we have reproduced explicitly the consumers' demand for electronic payment instruments. In addition we have shown that we are able to estimate its shape and calculate its sensitivity in terms of consumers' and merchants' fixed fees under different scenarios. Consequently, given the downward slope of the function, we can argue that the modelled demand is similar to the shape commonly used in the theoretical models.

Conclusion 2: Similarly, from *Observation CFF-2* and *Observation MFF-1* it follows that the agent-based artificial market allows us to explicitly reproduce the merchants' demand and calculate its sensitivity in terms of consumers' and merchants' fixed fees given different conditions in the market.

In general, we observe from the results that the sensitivity of the merchants' demand in terms of merchant fixed fees is higher than the sensitivity of the consumers' demand related to the consumer fixed fees. In other words, having the current setting of parameters of the artificial market (in particular, please refer to table 4.7), the consumers are less sensitive to changes in the consumers' fixed fees than the merchants are to changes in the merchants' fixed fees. In addition, we notice that the higher the number of competitors in the market the lower the sensitivity of the consumers and the merchants, corresponding to the fixed price applied to their side.

Further, we notice that in a market with equally priced payment instruments, the increase of the number of competitors reduces the influence of the consumers' fixed price on the merchants' demand, and vise versa. For instance, suppose we have two competitors in the market and the merchants' fixed fees are getting higher. This increase of the price causes a reduction in the average number of merchants per card and as a consequence a reduction of the average number of consumers is also observed. Nevertheless, in the case we have nine competitors, when the merchants' fixed fees are getting higher and as a consequence the average number of merchants per card is reduced, the average number of consumers per card changes very slowly. The same pattern is observed with the average number of merchants per card, when the consumers' fixed fees are increased.

4.4 Modelling the Emerging Competition

In the previous section we explored the shape of the consumers' and merchants' demand. Here, in a more dynamic environment we study the emerging competition reproduced by the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket ($\mathcal{APCM}$) model. The experiment designed to this end, is explained in subsection 4.4.1. For this experiment we use exactly the same consumers' and merchants' demand[12]. Given the initial setting of the model, we expect that each payment card provider achieves a stable market share, corresponding to the areas of intersections between the prices he applies and the end-users' demand. The measurements used to evaluate the performance of the competitors are listed in subsection 4.4.2.

The final section 4.4.3 is dedicated to present the results, the observations and the conclusions of the experiment. In this subsection we study how the total profit and the market share achieved are affected by the prices of the competitors and the degree of *initial cards* distribution in the market.

[12]We use the same values for the consumers' and merchants' inertia presented in table 4.7.

4.4.1 The Emerging Competition

In this section we describe the experiment designed to test the simulated dynamics in $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket. The resulting market share of the competitors and the total profit achieved by them allow us to evaluate how realistically our model reproduces the interactions in the market. Furthermore, as it is observed in reality, we want to test how the price level and the price structure applied by the card issuers affect the competition among payment card providers. The study is performed under scenarios with different degrees of *initial cards* distribution in the market. In addition, given that the price level and the shape of the end-users' demand do not change during the simulation, we expect that the market share achieved at a certain price level does not change statistically either.

We have divided our study in three parts. The first part verifies how the market share and the total profit are determined by the price level and the price structure of the individual competitors. In the same part, we test how sensitive the achieved market share and total profit are to the different degree of *initial cards* distribution. Next, we verify the correlation between the size of the market share and the interactions, and finally we evaluate in general the degree of complexity of the model.

For this experiment we use the definition of a strategy[13] proposed in the previous chapter in section 3.4. The strategy space $\mathbb{S}$ is defined as follows:

$$\mathbb{S} = \mathbb{D}_{F_p} \times \mathbb{D}_{\Gamma_p} \times \mathbb{D}_{b_p} \times \mathbb{D}_{\beta_p} \times \mathbb{D}_{l_p}$$

[13]The elements of the strategy $\mathbf{s}_p$ for a payment card $p \in \mathcal{P}$ are listed in table 4.4.

In particular, we focus our analysis on the elements of the strategy corresponding to the price structure, i.e. the consumers' fixed fees F_p, the consumers' benefits b_p, the merchants' fixed fees Γ_p and the merchants' benefits β_p. Each competitor uses an exogenously given random strategy selected from the strategy space defined above.

Regarding the varying degree of *initial cards* distribution in the market, we have established three scenarios of study. The different conditions of each scenario are related to the starting point of the simulation and are defined as follows:

Case All: The consumers and the merchants have *all* existing cards;

Case Random: The consumers and the merchants have a *randomly* assigned number of cards;

Case Zero: The consumers and the merchants have cash only.

The card providers select their strategies at the beginning of the simulation. More specifically, they choose randomly a particular price structure[14], establish a specific price level and decide how much they are going to spend in publicity. After that, the consumers $\mathcal{C}$ and the merchants $\mathcal{M}$, carry out commercial transactions with each other for a certain number of interactions I. In each interaction the consumers decide which merchant to visit and which instrument to use for his payment transaction. In addition, the merchants and the consumers periodically decide which card to keep, which card to drop and which new card to subscribe to. Those decisions are based on the price of the payment methods, the number of cards currently held and the consumers'/merchants' knowledge of the acceptance/use of an electronic payment method. At the end of the simulation we

[14] For instance the competitors could decide to charge or not to charge fixed fees to either side of the market, in addition, they could give variable benefits or to charge variable fees (negative benefits).

measure the profit and the market share achieved by each card provider.

Given that the card issuers generate randomly their strategies, in order to make a comparison among the three scenarios of different *initial cards* distributions and the price level and structure applied by the card providers, we have executed the process $\mathcal{MARKET}$ using the same random seed for each one of the three cases. In other words, we have repeated three times the same execution with the same competitors' strategies, changing only the condition of the *initial cards* distribution. At the end, we compare the individual performance of each payment card in each scenario in order to estimate the dependency of the competitors' market share and the total profit on the price level and the price structure, given the different degree of card distribution.

In the final part, which is about the complexity of the model, we present the general conclusions from the whole experiment.

Symbol	Description	Cardinality	Value
$\mathcal{M}$	Set of Merchants	$N_{\mathcal{M}}$	125
$\mathcal{C}$	Set of Consumers	$N_{\mathcal{C}}$	1100
$\mathcal{P}$	Set of Payment Cards	$N_{\mathcal{P}}$	9
$\mathcal{M}_c$	Consumers' Set of Merchants	$N_{\mathcal{M}_c}$	5

Table 4.9: The sets of the agents

The simulation is carried out with number of interactions $I = 20000$. The poisson distribution, used to determine how often consumers and merchant decide to drop or add a new card, has a mean $\lambda = 20$. The rest of the user defined parameters are divided in three groups. Firstly in table 4.9 the values of the different sets of agents are presented. Secondly, we have listed the setting of the constants, which have a direct impact on the decision making process of consumers, merchants and payment card providers (table 4.10). Finally, in table 4.11 the domains of each element of the strategy space are represented.

Symbol	Description of the Constants	Value
ε	common constant for the inertia to changes	1
x_c^-	the consumers' inertia to drop cards	0.05
x_c^+	the consumers' inertia to add new cards	2
x_m^-	the merchants' inertia to drop cards	0.05
x_m^+	the merchants' inertia to add new cards	9
α	the impact of the publicity cost	0.1
φ	the impact of the publicity cost	5

Table 4.10: Constants used in the agents' decisions

The values of the constants presented in table 4.10, regarding the consumers' and merchants' inertia to add and drop cards, affect the shape of the demand on both sides of the market. It is important to say that if we assign different values to those constants, the demand of the consumers and the merchants will have different trajectory. In the current experiment, we keep the value of the constants in table 4.10 fixed. Additionally, the consumers and the merchants have different inertia to add new cards.

The experiment was run for 10 independent executions. For illustration purposes, we select one of the ten examples. From the data obtained from the simulations, we eliminate the first 200 interactions, as part of the adaptation of consumers and merchants to the market conditions. We consider for our analysis the data series obtained in the remaining 19800 interactions.

Symbol	Domains	Value
$\mathbb{D}_{F_p}$	Consumer Fixed Fee Domain	$[0, 10]$
$\mathbb{D}_{\Gamma_p}$	Merchant Fixed Fee Domain	$[0, 10]$
$\mathbb{D}_{b_p}$	Domain of the Consumers' Benefits	$[-1, 1]$
$\mathbb{D}_{\beta_p}$	Domain of the Merchants' Benefits	$[-1, 1]$
$\mathbb{D}_{l_p}$	Publicity Cost's Domain	$[0, \infty]$

Table 4.11: Strategy's Domains

4.4.2 The performance of the competitors

In order to assess the performance of the payment card providers, we quantify their participation in the market. To this end, in this section we employ two categories of measurements: the market share and the total profit. The market share achieved by each competitor is calculated in terms of the percentage of merchants accepting the card, the percentage of consumers accepting the card and the percentage of transactions performed by the card. In addition, each one of these dimensions could be measured either for each interaction $t \in I$ or for an interval of interactions. To this end, we use the following notation.

For each interaction $t \in I$:

- $N_{t,\mathcal{M}_p}$ is the percentage of merchants accepting card $p \in \mathcal{P}$;
- $N_{t,\mathcal{C}_p}$ is the percentage of consumers having card $p \in \mathcal{P}$;
- N_{t,I_p} is the percentage of transactions made with card $p \in \mathcal{P}$;

For each interval of 100 continuous interactions I^{100}, where $I^{100} \subset I$:

- $N^{100}_{\mathcal{M}_p}$ is the percentage of merchants accepting card $p \in \mathcal{P}$;
- $N^{100}_{\mathcal{C}_p}$ is the percentage of consumers having card $p \in \mathcal{P}$;
- $N^{100}_{I_p}$ is the percentage of transactions made with card $p \in \mathcal{P}$;

At the end of the simulation:

- Φ_p is the total profit of card $p \in \mathcal{P}$.

The total profit Φ_p of the card issuer is calculated applying the following equation:

$$\Phi_p = \Phi_{\mathcal{C}_p} + \Phi_{\mathcal{M}_p} - \mathcal{L}_p \tag{4.5}$$

Here $\Phi_{\mathcal{C}_p}$ is the profit received from the consumers and $\Phi_{\mathcal{M}_p}$ is the profit received from the merchants. Those are calculated as

$$\Phi_{\mathcal{C}_p} = \left(\sum_{t=1}^{I} N_{t,\mathcal{C}_p} \cdot F_p - \sum_{t=1}^{I} N_{t,I_p} \cdot b_p \right)$$

$$\Phi_{\mathcal{M}_p} = \left(\sum_{t=1}^{I} N_{t,\mathcal{M}_p} \cdot \mathit{\Gamma}_p - \sum_{t=1}^{I} N_{t,I_p} \cdot \beta_p \right)$$

The sum of all publicity cost is denoted $\mathcal{L}_p$ and is calculated as

$$\mathcal{L}_p = \sum_{t=1}^{I} l_p$$

In this section, we have described the experimental setting and the performance measurements; in the following subsections we present the results of the experiment with their corresponding observations and conclusions.

4.4.3 The Dependency on Price and *initial cards* Distribution

In this section we present the results obtained from the test of emerging competition, related to the part dedicated to verify how the price level, the price structure and the *initial cards* distribution determine the market share and the total profit of the payment card providers.

p	F_p	Γ_p	b_p	β_p	l_p
1	3.16	3.95	-0.66	-0.88	0.69
2	6.91	7.36	-0.44	-0.41	8.67
3	6.85	7.61	-0.03	0.23	5.85
4	2.87	7.35	-0.13	0.50	14.98
5	8.00	5.63	0.03	-0.34	1.39
6	2.09	4.22	-0.79	-0.62	13.32
7	2.16	2.59	-0.27	0.15	8.99
8	7.02	6.45	0.47	-0.51	12.04
9	2.42	1.77	0.55	0.41	12.72

Table 4.12: Payment Cards' Strategy

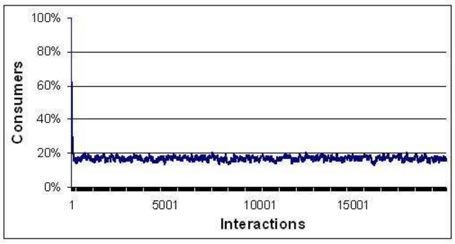
We have organised the subsection as follows. First, in table 4.12 we list the strategies[15] corresponding to each card issuer[16]. Secondly, in order to build the grounding for our analysis, we present the results, the estimations and the observations related to the market share achieved in the three cases of study. Next we list the results and the observations corresponding to the total profit obtained. At the end we state the conclusions.

Market share

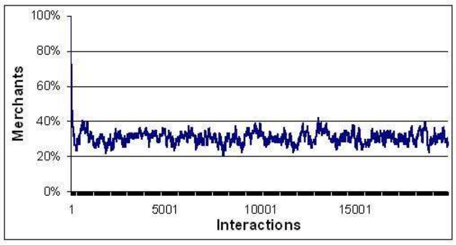
The market share is measured in terms of the percentage number of consumers $N_{t,\mathcal{C}_p}$, merchants $N_{t,\mathcal{M}_p}$ and transactions, N_{t,I_p} achieved with a card $p \in \mathcal{P}$. In figure 4-3 we have included an example, in which the x axis shows the number of interactions among consumers and merchants and the y axis presents the percentages of each dimension. In table 4.13, we present for each scenario the mean and the standard deviation reported

[15]The symbols of the elements of the strategy are listed in table 4.4.

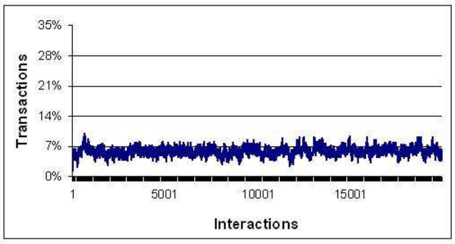
[16]The negative net benefits are interpreted as fees that the consumers or the merchants need to pay in each transaction.



(a) Consumers



(b) Merchants



(c) Transactions

Figure 4-3: Typical market share achieved by one of the card issuers

for $N_{t,\mathcal{C}_p}$, $N_{t,\mathcal{M}_p}$ and N_{t,I_p} of the competitors. Nevertheless not all payment transactions are performed by cards, some are carried out with cash. Regarding the market share of those transactions, knowing that the cash is a universal payment method, i.e. all consumers and merchant have it, we report only the percentage of average cash transactions as follows.

Percentage of cash transactions: *Case All:* The consumers and merchants start with all cards

Mean 43.68%

Standard Deviation 2.50%

Case Random: The participants start with randomly assigned cards

Mean 46.64%

Standard Deviation 2.47%

Case Zero: The participants start with no cards

Mean 50.76%

Standard Deviation 2.81%

		$N_{t,\mathcal{C}_p}$			$N_{t,\mathcal{M}_p}$			N_{t,I_p}		
p		*All*	*Rand.*	*Zero*	*All*	*Rand.*	*Zero*	*All*	*Rand.*	*Zero*
1	μ	35.29	32.92	31.89	37.03	27.66	14.92	10.80	9.66	6.64
	σ	1.05	1.20	1.38	3.26	2.82	3.02	1.35	1.28	1.54
2	μ	16.23	17.75	19.64	5.62	6.34	7.40	0.97	1.27	2.01
	σ	1.13	1.14	1.19	2.12	2.21	2.33	0.47	0.57	0.76
3	μ	19.44	19.59	19.72	6.80	6.91	7.03	1.53	1.57	1.85
	σ	1.10	1.11	1.14	2.18	2.17	2.18	0.60	0.59	0.76
4	μ	33.64	34.38	35.23	8.39	8.52	8.62	3.24	3.33	4.01
	σ	1.42	1.48	1.40	2.52	2.56	2.60	1.14	1.12	1.35
5	μ	16.82	16.91	16.95	7.60	8.50	9.50	1.50	1.78	2.05
	σ	1.11	1.12	1.10	2.46	2.44	2.55	0.62	0.65	0.68
6	μ	49.72	45.79	40.37	19.48	19.87	17.03	8.61	8.73	8.32
	σ	1.03	1.11	1.48	2.11	2.91	3.40	1.25	1.53	1.83
7	μ	36.23	37.97	40.16	22.68	22.95	24.89	8.63	9.13	11.01
	σ	1.45	1.53	1.55	3.89	3.93	3.95	1.71	1.72	1.93
8	μ	20.23	19.93	19.54	5.42	6.96	8.57	1.43	1.74	1.79
	σ	1.22	1.20	1.21	1.93	2.32	2.39	0.65	0.70	0.68
9	μ	42.22	40.53	38.94	35.95	33.37	31.30	19.59	16.15	11.55
	σ	1.75	1.40	1.45	4.28	4.24	4.22	2.67	2.19	1.86

Table 4.13: Mean and Standard Deviation of $N_{t,\mathcal{C}_p}$, $N_{t,\mathcal{M}_p}$ and N_{t,I_p} reported for the three cases for each card

Furthermore, based on the data presented in table 4.13 we have formulated several estimated univariate and multivariate models in semi-logarithmic scale, in order to find which of the price elements are the factors that explain in the best way the behaviour of the competitors' market share. We have studied the market share in terms of number of consumers per card, number of merchants per card and number of transactions per card. In our regression models the null hypothesis is that the independent variables do not explain the variable, which we want to estimate[17]. For that reason, in table 4.14 we present for the three models used the coefficient of determination R^2 and $P > |t|$ the probability that the t-Statistic of the independent variable is outside of the 95% confidence interval, in which the null hypothesis is true. In appendix D we have added the whole set of statistics produced for the regression models, which include the t values of the 95% confidence interval.

Estimated Variable	$N_{t,\mathcal{C}_p}$			
Independent Variable	F_p			
	$P > \|t\|$	R^2		
All	0.00	0.9353		
Random	0.00	0.9721		
Zero	0.00	0.9946		
Estimated Variable	$N_{t,\mathcal{M}_p}$			
Independent Variable	F_p		Γ_p	
	$P > \|t\|$	R^2	$P > \|t\|$	R^2
All	0.153	0.8682	0.015	0.8682
Random	0.051	0.9412	0.002	0.9412
Zero	0.027	0.9849	0.000	0.9849
Estimated Variable	N_{t,I_p}			
Independent Variable	F_p		Γ_p	
	$P > \|t\|$	R^2	$P > \|t\|$	R^2
All	0.006	0.9459	0.007	0.9459
Random	0.001	0.9703	0.001	0.9703
Zero	0.000	0.9922	0.000	0.9922

Table 4.14: The statistically significant factors of the market share

First, we present the estimated average number of consumers per card on semi-logarithmic scale, $N_{\mathcal{C}_p}^{log,case}$, where *log* stands for the logarithmic value of the estimated variable and *case* represents the particular case of initial card distribution in the market. The regression model is made on the consumer fixed fees, F_p, as this is the only variable that shows a statistically significant relationship with the number of consumers per card.

[17] For more information about the regression models and the null hypothesis please refer to [62], [63] and [64].

The curves corresponding to the linear regression are shown in figure 4-4. In table 4.14 we observe that the lower value of the coefficient of determination R^2, which represents the proportion of the variation in the number of consumers per card, explained by the linear regressor (the consumer fixed fee) is 0.9353. This coefficient is for the case when the consumers initially have *All* cards.

$$
\begin{aligned}
N_{\mathcal{C}_p}^{log,a} &= -0.1644358F_p + 6.478832 \\
N_{\mathcal{C}_p}^{log,r} &= -0.1556544F_p + 6.433733 \\
N_{\mathcal{C}_p}^{log,z} &= -0.1460485F_p + 6.386472
\end{aligned}
\tag{4.6}
$$

Given that the processes resulting from the simulation are stationary[18], we are able to elaborate regression models based on the statistically significant factors that determine measurements of the payment card providers' performance. These measurements are the average number of consumers, the average number of merchants, the average number of card transactions and the average profit. In the case of average number of consumers per card, the regression models used to plot the graphs are included in equation 4.6. Those estimations are calculated for each one of the different cases of *initial cards* distribution. The variables $N_{\mathcal{C}_p}^{log,a}$, $N_{\mathcal{C}_p}^{log,r}$ and $N_{\mathcal{C}_p}^{log,z}$ are the logarithm of the number of consumers per card corresponding to the cases of *initial cards* distribution: *All, Random* and *Zero* respectively.

Our study also reveals that the number of consumers per cards is sensitive to the merchants' fixed fees Γ_p, as the coefficient of determination, $R^2 = 0.4963$. In other words, there is some influence of the merchants' fixed fees over the number of consumers per card, but we could not express this relationship by an equation.

[18]Please refer to section 4.2

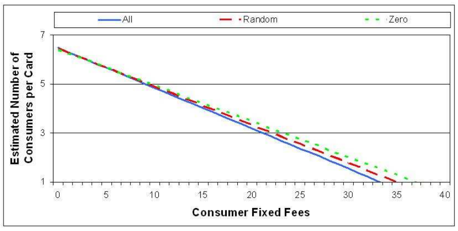


Figure 4-4: Estimated Number of Consumers per Card

Next, we estimate the average number of merchants per card based on the data series resulting from the simulation. In this case, we calculate the dependency in terms of merchants' fixed fees $\mathit{\Gamma}_p$ and consumers' fixed fees F_p, as this multivariate regression model is the one that better explains the behaviour of the number of merchants per card. The estimations are presented in equation 4.7, where the variables $N_{\mathcal{M}_p}^{log,a}$, $N_{\mathcal{M}_p}^{log,r}$ and $N_{\mathcal{M}_p}^{log,z}$ correspond to the logarithm of the number of merchants per card in each of the three cases of *initial cards* distribution. Furthermore, in figure 4-5 we show graphically this dependency in the case when the consumers and the merchants have *Zero* cards initially. We observe from the figure that the fixed fees charge to the consumers impact the number of merchant per card in the market, specially in the corner where the merchants' fixed fess are zero and the consumers' fixed fees are on their maximum level, 10.

$$\begin{aligned} N_{\mathcal{M}_p}^{log,a} &= 4.488059 - 0.1017348 F_p - 0.241483 \mathit{\Gamma}_p \\ N_{\mathcal{M}_p}^{log,r} &= 4.339315 - 0.0877077 F_p - 0.2227732 \mathit{\Gamma}_p \\ N_{\mathcal{M}_p}^{log,z} &= 4.059697 - 0.0433825 F_p - 0.2136886 \mathit{\Gamma}_p \end{aligned} \tag{4.7}$$

The third variable related to the market share is the number of transactions per card, $N_{I_p}^{log,case}$. Here, we estimated the number of transactions per card in terms of the consumers' fixed fees and the merchants' fixed fees. We present this two-dimensional

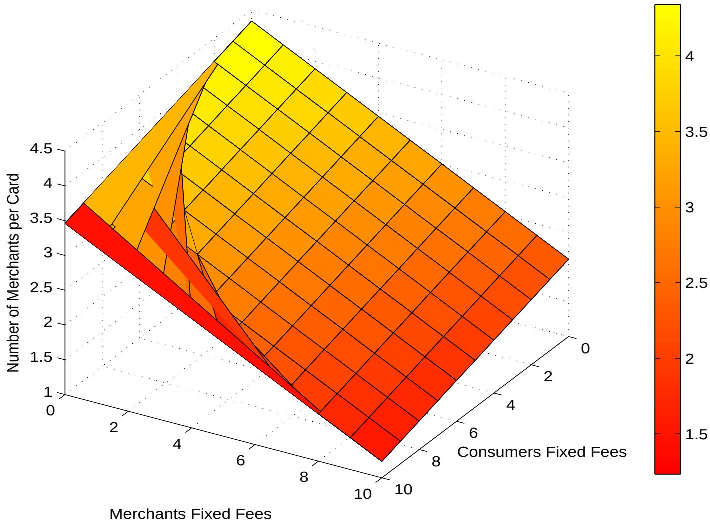


Figure 4-5: Estimated Number of Merchants per Card

dependency in equation 4.8. In addition, in figure 4-6 we show graphically this relationship in a semi-logarithmic scale for the case of *Zero* cards distributed initially. In this figure, we note that the number of transactions is strongly reduced, when the consumers' fixed fees are on their maximum level and the merchants' fixed fees are zero. The same corner effect was observe in the figure 4-5.

$$
\begin{aligned}
N_{I_p}^{log,a} &= 11.03137 - 0.2374831F_p - 0.2642109\varGamma_p \\
N_{I_p}^{log,r} &= 10.83682 - 0.2075534F_p - 0.2436478\varGamma_p \\
N_{I_p}^{log,z} &= 10.45746 - 0.1990897F_p - 0.1696916\varGamma_p
\end{aligned}
\tag{4.8}
$$

In what follows, we present our observations of the data obtained from the experiment, related to the market share of the competitors.

Observation 1: From equation 4.6 and figure 4-4, it follows that the number of consumers per card $N_{\mathcal{C}_p}$ is primarily determined by the consumers' fixed fees F_p. In addition, we notice that the merchants' fixed fees have some influence over the number of consumers per card, but this relationship could not be expressed by an

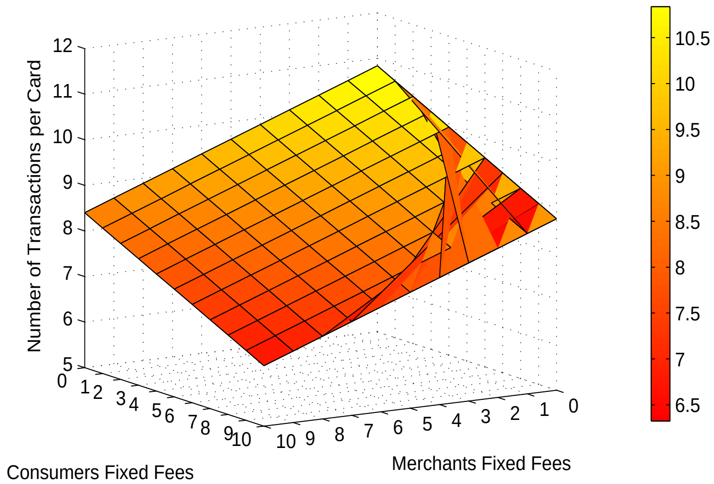


Figure 4-6: Estimated Number of Transactions per Card

equation. The rest of the price elements, namely the consumer benefits and the merchants' benefits do not represent statistically significant relationships with the number of consumers per card.

Observation 2: It follows from equation 4.7 and figure 4-5 that the merchants' fixed fees $\mathit{\Gamma}_p$ is the factor that has the most important impact on the number of merchants per card $N_{\mathcal{M}_p}$. In addition, we observe that the estimated number of merchants per card is sensitive to the consumers' fixed fees F_p. The rest of the variables do not exhibit statistically significant relationships.

Observation 3: From equation 4.8 and figure 4-6, it follows that the number of transactions, N_{I_p} is determined by the fixed price of both sides.

Observation 4: Regarding the impact of the *initial cards* distribution on the market, we observe that the market share of the card issuers is sensitive to the initial distribution of cards.

Total Profit

We start this exposition by presenting in table 4.15 the total profit Φ_p that the card issuers have obtained in the different scenarios. The notation that we use to distinguish among the profits of the different cases is the following: $\Phi_p^{log,a}$, $\Phi_p^{log,r}$ and $\Phi_p^{log,z}$. According to the definition of the total profit[19], it is calculated based on the price on both sides of the market and the logarithmic value of the number of transactions. For that reason, in this subsection we estimate the total profit Φ_p in terms of consumers' fixed fees and merchants' benefits, which are the variables that exhibit the most statistically significant relationships. In figure 4-7 we present this multivariate regression in the case where the consumers and the merchants start the simulation with *Zero* cards. In addition, in equation 4.9 we show the estimations for the three cases of study.

	All	*Random*	*Zero*
	Φ_p	Φ_p	Φ_p
1	31,038,685	28,142,230	25,231,742
2	25,108,512	27,537,335	30,665,882
3	29,651,934	29,897,790	30,105,244
4	21,661,143	22,135,408	22,618,611
5	29,977,845	30,252,772	30,502,026
6	26,689,832	24,986,493	22,170,112
7	18,290,111	19,129,390	20,313,908
8	31,064,781	30,891,251	30,561,218
9	19,163,941	18,917,058	18,956,974

Table 4.15: The total profit obtained in the *Cases All, Random* and *Zero*

$$
\begin{aligned}
\Phi_p^{log,a} &= 16.81216 + 0.0427178F_p - 0.2462289\beta_p \\
\Phi_p^{log,r} &= 16.76903 + 0.054214F_p - 0.1816333\beta_p \\
\Phi_p^{log,z} &= 16.71309 + 0.0687503F_p - 0.0914983\beta_p
\end{aligned}
\tag{4.9}
$$

[19]Please refer to subsection 4.4.2 for more details of how we calculate the total profit.

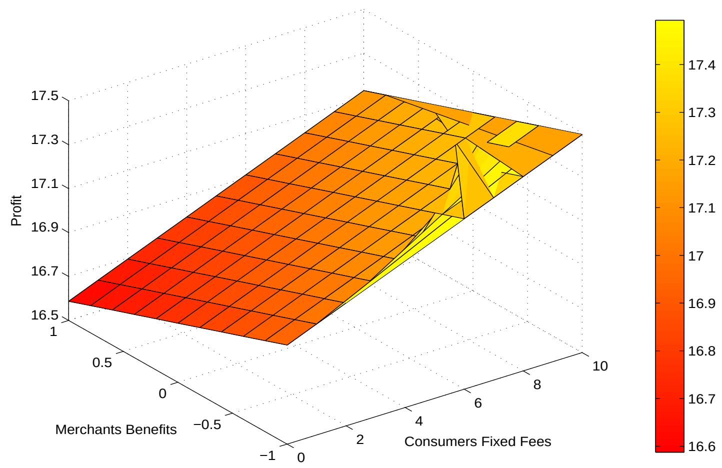


Figure 4-7: Estimated Profit per Card Issuer

Observation 5: In figure 4-7 and equation 4.9 we observe that the total profit of the competitors is related positively with the consumer fixed fees and negatively with the merchants' benefits. Furthermore, we note that areas of high profit lay over the axes of the lower merchants' benefits.

Conclusion 3: From *Observations 1* to *5* it follows that the structure and the level of the price determine the market share and the total profit of the competitors. Furthermore, we conclude that the model has reproduced explicitly the mutually constrained demands of consumers and merchants in a complex artificial environment.

4.5 General Conclusions

In this chapter we tested experimentally the model of the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket ($\mathcal{APCM}$). In this section we summarise our main findings.

First, in section 4.2 we demonstrated that the data series produced by the model are stationary. This characteristic of the data, allows us to formulate regression models and explain the dependency of the market structure, namely the market share and the profit of the payment card issuers.

We start our analysis with the experiment presented in section 4.3. It reproduces a market, in which the competitors have priced their products equally. The experiment is designed to investigate the shape of the consumers' and merchants' demands and how these demands constrain each other. We studied three different cases with 2, 5 and 9 card issuers. Using regression models, we estimate the consumers' and the merchants' demands in terms of the payment card providers' fixed price.

We observe that the demand on the merchants' side has a downward slope in terms of the merchants' fixed fees, and the demand on the consumers' side has the same slope in terms of the consumers' fixed fees. This is similar to the shape commonly used in the theoretical models.

Nevertheless, with the increase of the number of competitors the slope of the demand on the merchants' side in terms of consumers' fixed fees is getting close to zero. Given that the card issuers are independent, in order to explain our finding let us give an example. Suppose we have two competitors in the market and both of them increased their merchants' fixed fees in the same rate. Consequently, we observe a reduction in the number of merchants per card. The decrease in the merchants per card causes a reduction in the number of consumers per card as well.

Continuing with our example, let us now have more competitors in the market, which increase their merchants' fixed fees by the same rate. In this case again the number of merchants per card is reduced. Nevertheless, the number of consumers per card is reduced with lower rate. In fact, the higher the number of competitors, the lower the rate by which the number of consumers per card is reduced.

Furthermore, in order to validate to what degree we have reproduced the competition among card issuers, in section 4.4 we simulated a market in which all competitors price their payment methods independently. In this experiment, the dynamics of the market is complex and consequently the estimation of the market structure is not trivial. Here we test the market with nine card issuers. We study three different initial card distributions in the market. Like in the experiment in section 4.3, in this section we are interested in estimating the number of consumers per card and the number of merchants per card. In addition, we study the behaviour of the number of transactions per card[20] and the profit achieved by the competitors.

To this end, we formulated several univariate and multivariate models in order to find which elements of the price determine the variables under study. At the end we found that the analysed variables exhibit statistically significant relationships with the following price elements.

[20]The number of consumers per card, the number of merchants per card and the number of transactions per card are the variables that represent the competitors' market share.

- The number of consumers per card is determined by the consumers' fixed fees. We found that the number of consumers per card is sensitive to the merchants' fixed fees, but this relationship could not be expressed by an equation;

- The number of merchants per card is determined by the merchants' fixed fees and it is sensitive to the consumers' fixed fees;

- The number of transactions per card is determined by the consumers' fixed fees and the merchants' fixed fees;

- The competitors' profit is determined by the consumers' fixed fees and the merchants' benefits.

Thus, the consumers' and merchants' demands are determined by the fixed fees they have to pay and exhibit certain sensitivity to the fees paid by the other side of the market. This result is consistent with the observations of the previous experiment, where the downward slope of the consumers' and the merchants' demands were determined by their own fixed fees. There, in terms of the fees paid by the other side, the slopes of the demands were getting close to zero.

In figure 4-5 we observe that the estimated number of merchants per card has different shape in the area around the corner, where the merchants' fixed fees are getting close to 0 and the consumers' fixed fees are getting close to their maximum level. Additionally, in figure 4-6 in the same corner, we notice that the shape of the estimated number of transactions per card is also different in comparison with the shape of the rest of the figure. Furthermore, in the case of the estimated profit per card issuer presented in figure 4-7, we observe another area of different shape located in the corner, where the consumers' fixed fees are getting close to their maximum level and the merchants' net benefits are getting close to their minimum.

Our econometric analysis reveals another interesting aspect of the market. According to equation 4.8 we observe that the market share of the competitors, measured in terms of transactions, could be reduced considerably if the fixed price on any side is increased. Furthermore, according to regression presented in equation 4.9 the card issuers generate profit primary from the fees payed by the consumers. From the same regression we deduce that the total profit is affected negatively by the merchants' side, as the benefits given to merchants are statistically more determinant than the fees received from them. In other words, we will expect that in the case the card issuers want to increase their profit, they will charge fixed fees on the consumers' side, which also is the side with the lower price sensitivity.

We notice as well that in the artificial environment that we have reproduced, the competition is affected by the degree of *initial cards* distribution in the market.

In general, given the two experiments presented in this chapter, we can say that the computational model allows us to study a different aspect of the market dynamics. For instance, we can estimate the impact of the price elements in an environment, where all competitors have the same price. In addition, we can study the dependency of the market structure, namely the market share and the profit, in a dynamic environment, where each card issuer determines his price independently.

In chapter 6 we are exploring further the possibilities that the computational model allows us. To that end, given the consumers' and merchants' demand, we apply a heuristic search in order to find a profit-maximising price structure and level and it will guarantee a certain level of market share measured in terms of card transactions.

Chapter 5

Designing Strategies for the Payment Card Market

In chapter 3 we introduced formally the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket, which simulates the competition among card issuers, by explicitly reproducing the demand for electronic payment instruments from consumers and merchants at the point of sale. In this chapter we propose the application of the Generalised Population Based Incremental Learning (GPBIL) algorithm in order to design a payment card providers' strategy under specific criteria.

The procedure analyses the achievements of the competitors, with the aim to identify the best-performing parameter constellation. The success of the card issuers is measured according the payment card transactions and profit achieved at the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket. This study has a normative nature, i. e. we assume that the solutions found by the GPBIL are an optimal strategy under the required criteria [60]. Here, in order to explain the application, we describe formally the Generalised Population Based Incremental Learning algorithm, whereas an empirical analysis of the suggested mechanism is presented through experimentation in the next chapter.

We have organized this chapter as follows: first in section 5.1 we describe the payment card strategies in a way that they can be used with the generalised formulation of the Population Based Incremental Learning. The presentation includes the definition of a solution space and the employed probability distribution functions. In addition, we characterise the one-step learning mechanism, which consists of positive and negative feedback. Finally in section 5.2, we present the methods used in our learning scheme regarding the initialisation of the joint probability distribution, the sampling of the individuals and the learning mechanism itself. Pseudo code is provided for each of these functions.

The chapter closes with a summary that lists the parameters of the strategy space used by the algorithm.

5.1 Designing Payment Cards Strategy

5.1.1 The Solution Space

Previously we defined the solution space of the payment card strategy as

$$\mathbb{S} = \mathbb{D}_{\mathcal{F}_p} \times \mathbb{D}_{\Gamma_p} \times \mathbb{D}_{b_p} \times \mathbb{D}_{\beta_p} \times \mathbb{D}_{l_p}$$

rewritten as

$$\begin{aligned}&\mathbb{S} = \mathbb{D}_1 \times \cdots \times \mathbb{D}_5\\&\text{with } \mathbb{D}_1 = \mathbb{D}_{F_p},\, \mathbb{D}_2 = \mathbb{D}_{\Gamma_p},\, \mathbb{D}_3 = \mathbb{D}_{b_p},\, \mathbb{D}_4 = \mathbb{D}_{\beta_p},\, \mathbb{D}_5 = \mathbb{D}_{l_p}\end{aligned} \tag{5.1}$$

Our aim in this chapter is to explore the intersections between the strategy space and the modelled demand, in order to find a distribution of values over such a space, which allows the payment methods purveyors to generate a strategy that satisfies two specific criteria. To this end, we apply a joint probability distribution function over the entire solution space and we denote it as $\mathbb{F}_{\mathbb{S}}$.

5.1.2 The Joint Probability Distribution Function $\mathbb{F}_{\mathbb{S}}$

We have said before that the strategy's domain $\mathbb{D}_i$ is over intervals of the real numbers. Assume a probability distribution function $\mathbb{F}_{\mathbb{D}_i} : \mathbb{R} \to [0,1]$ for unconditional random variables over the ranges $\mathbb{D}_i$, we define the joint probability distribution $\mathbb{F}_{\mathbb{S}}$ over $\mathbb{S}$ by

$$\mathbb{F}_{\mathbb{S}} = \mathbb{F}_{\mathbb{D}_\mathbf{1}} \cdot \ldots \cdot \mathbb{F}_{\mathbb{D}_\mathbf{5}}. \tag{5.2}$$

All electronic cards issuers have the same joint probability distribution, and we are using it firstly to sample individual strategies from the space, and secondly to modify $\mathbb{F}_{\mathbb{S}}$ through learning.

Let us now briefly explain the probability distribution over $\mathbb{D}_i$ given the real-valued interval

$$\mathbb{D}_i = [a_i, b_i]. \tag{5.3}$$

Despite the fact that this distribution can potentially be a complex function, it is possible to approximate to a certain extent such a continuous function. In the search for an optimal payment card strategy, we have followed the approach of Generalised Population Based Incremental Learning proposed by [27], which incorporates positive and negative learning, and is able to approximate versatile distributions.

In general, GPBIL limits the probability distribution function to a certain kind of step function, which is defined using an one-dimensional Self-Organising Map [1](SOM). In our case assume n_i neurons with weights

$$a_i \leq w_{i,1} < \cdots < w_{i,n_i} \leq b_i \tag{5.4}$$

These n_i neurons divide the interval $[a_i, b_i]$ into n_i sub-intervals in such a way that the k^{th} new interval contains the values closest to the k^{th} neuron weight: $\left[a_i, \frac{w_{i,1}+w_{i,2}}{2}\right)$, $\left[\frac{w_{i,1}+w_{i,2}}{2}, \frac{w_{i,2}+w_{i,3}}{2}\right), \ldots, \left[\frac{w_{i,n_{i-2}}+w_{i,n_{i-1}}}{2}, \frac{w_{i,n_{i-1}}+w_{i,n_i}}{2}\right)$,
and $\left[\frac{w_{i,n_{i-1}}+w_{i,n_i}}{2}, b_i\right]$.
The learning mechanism establishes that each of the n_i neurons is equally likely to be selected. In other words, the probability of choosing a random value from any of the n_i sub-intervals is $\frac{1}{n_i}$. Furthermore, the values of any sub-interval are equally probable, in a way that the density function within each sub-interval must be constant[2]. In order to find the prominent areas, the algorithm modifies the weights of the neurons according to specific rules, explained in the next subsection 5.1.3.

[1]This is a kind of neural network, which uses a learning mechanism to find an area with high positive feedback. At the end of the search, these areas are recognised by the concentration of neurons in them. For more information please refer to [27]

[2]For more details please refer to [27]

5.1.3 Learning Over the Joint Probability Distribution Function $\mathbb{F}_{\mathbb{S}}$

Before we present the use of the Generalised Population Based Incremental Learning algorithm for designing strategies, let us explain the learning mechanism used to differentiate between the promising and less promising areas inside the domains of the solution space, $\mathbb{D}_i \in \mathbb{S}$, where $i = 1, \ldots, 5$. This task is performed by applying rules for the modification of the probability distribution $\mathbb{F}_{\mathbb{D}_i}$ and it is an essential part of the learning process explained further in section 5.2.3. For the implementation of the procedure we have followed the standard rules of the Generalised Population Based Incremental Learning algorithm for intervals with real numbers, proposed by [27]. Here we briefly reproduce the specific rules.

We have defined so far $\mathbb{D}_i$ as an interval $[a_i, b_i]$. The probability distribution function over this domain is given by the n_i neuron weights $a_i \leq w_{i,1} < \cdots < w_{i,n_i} \leq b_i$ and it is denoted as $\mathbb{F}(w_{i,1}, \cdots, w_{i,n_i})$. The learning rule for GPBIL consists of updating these weights. It is important to emphasize that the mechanism incorporates aspects of social learning, as we use the same probability distribution for all competitors. More specifically, the card issuers implicitly share information regarding the parameter constellation and that allows them to arrive efficiently to the optimal strategy. In section 5.2 we explain specifically how this mechanism is implement, whereas in chapter 6 we present the optimal strategies obtained through experimentation.

The modification of the weights of the probability distribution function is performed in two directions: from one side the aim is to concentrate weights in areas of what is considered to be values of accurate choices, while from the other side the algorithm is moving the neuron weights away from areas of unfavorable values. The former is known as a positive feedback, while the latter is considered as negative feedback.

Positive Feedback

Assume we have a value $x \in [a_i, b_i]$ with a positive evaluation. First we need to determine which is the best matching weight $w_{i,j}$

$$j = \arg\min_{k=1}^{n_i} \left(|x - w_{i,k}| \right).$$

Thereafter, given that the positive learning rate is $\epsilon \in [0, 1]$, we modify the weights according to the following rule

$$w_{i,k}^{new} = w_{i,k}^{old} + \epsilon \cdot h(j, k) \cdot (x - w_{i,k}^{old}). \tag{5.5}$$

Here, $h(j, k)$ represents a neighbourhood kernel with cylinder size δ and it is defined as

$$h(j, k) = \begin{cases} 1 & \text{if } |j - k| \leq \delta \\ 0 & \text{else} \end{cases}.$$

Finally, we formulate the learning rule as

$$w_{i,k}^{new} = \begin{cases} w_{i,k}^{old} + \epsilon \cdot (x - w_{i,k}^{old}) & \text{if } |j - k| \leq \delta \\ w_{i,k}^{old} & \text{else} \end{cases}. \tag{5.6}$$

Negative Feedback

We have said before that the basic idea of negative feedback is the movement of neurons away from an unpromising input x, executed within a certain neighbourhood around it. This is in contrast to the weight concentration towards x in the case of positive learning, hereafter determining the closest neuron

$$j = \arg\min_{k=1}^{n_i} \left(|x - w_{i,k}| \right),$$

the weights are updated by the following equation

$$w_{i,k}^{new} = \begin{cases} w_{i,k}^{old} + \epsilon \cdot (w_{i,k-\delta} - w_{i,k}^{old}) & \text{if } |j-k| \leq \delta \text{ and } w_{i,k}^{old} \leq x \\ w_{i,k}^{old} + \epsilon \cdot (w_{i,k+\delta} - w_{i,k}^{old}) & \text{if } |j-k| \leq \delta \text{ and } w_{i,k}^{old} > x \\ w_{i,k}^{old} & \text{else} \end{cases} . \tag{5.7}$$

In the case when $w_{i,k-\delta}$ and $w_{i,k+\delta}$ are not defined, we use $w_{i,k-\delta} = a_i$ and $w_{i,k+\delta} = b_i$.

5.2 The Implementation of Generalized Population Based Incremental Learning Algorithm

Previously we have defined $\vec{s} = (\mathbf{s}_1, \ldots, \mathbf{s}_{N_{\mathcal{P}}})$ as the vector of strategies of all payment methods in one execution of the $\mathcal{APCM}$. Additionally we define $\phi_p = \left(\Phi_p, N_{T_p^*}, p\right)$ as the measurement of performance achieved in one execution of the model for one payment method. The three elements that compose it are the profit for the card issuer Φ_p, its market share measured in terms of the total number of transactions $N_{T_p^*}$ and the corresponding index of the card p. The vector $(\phi_1, \ldots, \phi_{N_{\mathcal{P}}})$ represents the performance of all payment cards in one execution of the $\mathcal{APCM}$.

$\mathcal{MARKET} - \mathcal{GPBIL}()$
1 $I = \mathfrak{I}; \mathbb{S} = \mathfrak{S}; N_{\mathcal{P}} = \mathfrak{N}_{\mathcal{P}}; R = \mathfrak{R}$
2 $\mathbb{F}_{\mathbb{S}} = \mathit{initialisation}\ (\mathbb{S})$
3 FOR $r = 1, \ldots, R$ DO
4 FOR $p = 1, \ldots, N_{\mathcal{P}}$ DO
5 $\mathbf{s}_p = \mathit{sampling}\ (\mathbb{F}_{\mathbb{S}})$
6 $(\phi_1, \ldots, \phi_{N_{\mathcal{P}}}) = \mathcal{APCM}\left((\mathbf{s}_1, \ldots, \mathbf{s}_{N_{\mathcal{P}}}), I\right)$
7 $\vec{\phi}' = \mathit{profitDescendingSort}\ (\phi_1, \ldots, \phi_{N_{\mathcal{P}}})$
8 $\mathbb{F}_{\mathbb{S}} = \mathit{learning}\Big(\mathbb{F}_{\mathbb{S}}, \vec{\phi}', (\mathbf{s}_1, \ldots, \mathbf{s}_{N_{\mathcal{P}}})\Big)$
9 RETURN $\mathbb{F}_{\mathbb{S}}$

Figure 5-1: The modified $\mathcal{MARKET}$ procedure using GPBIL

In figure 5-1 we present the way we have used the GPBIL algorithm to find strategies under specific criteria. In our application, the strategy should fulfill two main objectives: obtain the highest possible profit Φ_p and achieve above-average market share measured in terms of the total number of transactions $N_{T_p^*}$. The first step is to initialise the joint probability function. This is performed by the function *initialisation*, which receives as a parameter the solution space $\mathbb{S}$ and returns the initialised joint probability function, $\mathbb{F}_{\mathbb{S}}$. We have explained in detail the code of this function in section 5.2.1.

The main part of the algorithm consists of a loop over R runs. At the beginning of each run every payment card selects a strategy $\mathbf{s}_p$. This process is carried out by the function *sampling* (line 4 of the GPBIL algorithm figure 5-1), which returns a strategy $\mathbf{s}_p$ for each one of the payment cards based on the probability distribution function $\mathbb{F}_{\mathbb{S}}$. Formal explanation of the latter is given in section 5.2.2.

Thereafter, in line 5, we instantiate the process $\mathcal{APCM}$ with the strategy vector $(\mathbf{s}_1, \ldots, \mathbf{s}_{N_{\mathcal{P}}})$ and number of interactions I. The pseudo code of the simulation is provided in chapter 3 in subsection 3.5.4. Here we have modified the procedure so that the process returns a vector of all payment card performance measurements $(\phi_1, \ldots, \phi_{N_{\mathcal{P}}})$.

Before the learning function is carried out, the performances of the payment card providers $(\phi_1, \ldots, \phi_{N_\mathcal{P}})$ are sorted (line 6) according to the profit Φ_p achieved in the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket ($\mathcal{APCM}$). The new vector is denoted $\vec{\phi}$. Following this step, the joint probability function $\mathbb{F}_\mathbb{S}$ is modified by a learning process (line 7). This task is accomplished considering the market share $N_{T_p^*}$ obtained in line 5. The function receives as parameters the current values of the joint probability distribution $\mathbb{F}_\mathbb{S}$, the profit based order of the performance $\vec{\phi}$ and the vector of strategies $(\mathbf{s}_1, \ldots, \mathbf{s}_{N_\mathcal{P}})$. The details of the learning mechanism are given in subsection 5.2.3.

Finally, in line 7, the GPBIL algorithm returns the resulting joint probability distribution. This function is used as a probabilistic model for generating strategies that fulfill the two main objectives: to achieve above average market share with the highest possible profit.

5.2.1 Initialisation of the Joint Probability Function

Function *initialisation*$(\mathbb{D}_1 \times \cdots \times \mathbb{D}_5)$
FOR $i = 1, \ldots, 5$ DO
    $n_i = \mathfrak{n}_i$
    FOR $k = 1, \ldots, n_i$ DO
        $w_{i,k} = a_i + \frac{2k-1}{2} \cdot \frac{b_i - a_i}{n_i}$
    $\mathbb{F}_{\mathbb{D}_1} = \mathbb{F}\left(w_{i,1}, \ldots, w_{i,n_i}\right)$
RETURN $(\mathbb{F}_{\mathbb{D}_1} \cdot \ldots \cdot \mathbb{F}_{\mathbb{D}_5})$

Figure 5-2: Pseudo code of the *initialisation*() function

The *initialisation*() process (line 1 figure 5-1) of the joint probability function, receives the strategy space $\mathbb{S}$ as a parameter and returns the initial distribution function over this space, $\mathbb{F}_\mathbb{S}$. During that function all single probability distributions over the domains receive their initial values. Given that at that point no previous knowledge is available, such functions get the same distribution. More specifically, if the i^{th} domain is an interval $[a_i, b_i]$, we apply a neuron chain with $\mathfrak{n}_i$ neurons, i.e. $n_i = \mathfrak{n}_i$ where $\mathfrak{n}_i$ is a parameter and must be predefined.

Therefore the mapping of the solution space onto the joint probability distribution function is given in the pseudo code of the *initialisation*() function (please refer to figure 5-2).

5.2.2 Sampling a Strategy

In section 3.5.3 we presented the code of the function *sampling*() corresponding to the random selection of a strategy. Here, *sampling*() (line 4 figure 5-1) returns a strategy s_p constructed according to the joint probability distribution function $\mathbb{F}_{\mathbb{S}}$. This probability distribution function is received as a parameter. The code of the new version of the function *sampling*() is shown in figure 5-3.

Function $sampling(\mathbb{F}_{\mathbb{D}_1} \cdot \ldots \cdot \mathbb{F}_{\mathbb{D}_5})$
FOR $i = 1, \ldots, 5$ DO
    $s_{p_i} = random(\mathbb{F}_{\mathbb{D}_i})$
RETURN $(s_{p_1}, \ldots, s_{p_5})$

Figure 5-3: Pseudo code of *sampling*() function

5.2.3 Learning Mechanism

In this subsection we explain our adaptation of the learning scheme of the GPBIL. The mechanism is presented in two blocks. In the first one we present the overall structure of the process, whereas in the second we explain the rules to update the joint probability function $\mathbb{F}_{\mathbb{S}}$. The pseudo code presented in figures 5-4 and 5-5 differs from the version presented in [27]. Here the modifications of the five payment methods' parameters are performed at the same time after the evaluation of the performance ϕ_p, whereas in the case of the Kern's algorithm the evaluation of the performance is done for each parameter separately.

Structure of the Learning Process

Function $learning\left(\mathbb{F}_{\mathbb{S}}, \vec{\phi}', \vec{s}\right)$

1 $e = \mathfrak{e}$
2 $\vec{\phi}^{+} = \{\phi_k | N_{T_k^*} > e\}$
3 $\vec{\phi}^{-} = \{\phi_k | N_{T_k^*} \leq e\}$
4 FOR $j = size\left(\vec{\phi}^{-}\right), \ldots, 1$ DO
5 $\quad s_p = findStrategy(\vec{s}, p)$
6 $\quad$ FOR $i = 1, \ldots, 5$ DO
7 $\quad\quad \mathbb{F}_{\mathbb{D}_i} = learning1\left(\mathbb{F}_{\mathbb{D}_i}, s_{p_i}, \frac{j}{size(\vec{\phi}^{-})}, 0\right)$
8 FOR $j = 1, \ldots, size\left(\vec{\phi}^{+}\right)$ DO
9 $\quad s_p = findStrategy(\vec{s}, p)$
10 $\quad$ FOR $i = 1, \ldots, 5$ DO
11 $\quad\quad \mathbb{F}_{\mathbb{D}_i} = learning1\left(\mathbb{F}_{\mathbb{D}_i}, s_{p_i}, \frac{size(\vec{\phi}^{+})+1-j}{size(\vec{\phi}^{+})}, 1\right)$
12 RETURN $(\mathbb{F}_{\mathbb{D}_1}, \ldots, \mathbb{F}_{\mathbb{D}_5})$

Figure 5-4: Pseudo code of *learning*() function

In figure 5-4 the overall structure of the learning process is presented. Let e be the minimum market share that the card issuers are required to achieve. In line 1 a user defined value $\mathfrak{e}$ is assigned to e. The vectors $\vec{\phi}^{+}$ and $\vec{\phi}^{-}$, corresponding to the positive and negative performance of the payment methods, are created in line 2 and 3 respectively. In order to distinguish which value is considered negative and which positive, a simple rule is applied. The cards with market share $N_{T_k^*} > e$ are included in the vector $\vec{\phi}^{+}$, whereas the rest of the cards are included in vector $\vec{\phi}^{-}$.

The modification of the probability distribution function of each domain $\mathbb{F}_{\mathbb{D}_i}$ is the aim of the loops performed from lines 4 to 7 and from lines 8 to 11, which form the main body of the function. The first loop carries out the negative feedback, considering the performance stored in the vector $\vec{\phi}^-$, whereas the second performs the positive updates of the probability distribution functions based on the evaluations represented in vector $\vec{\phi}^+$.

In lines 5 and 9 the function $findStrategy()$ receives as parameters the vector of strategies $\vec{s}$ and the index of the payment card p, in order to return the corresponding strategy s_p. In lines 6 and 10 the same loop is performed over the individual probability functions $\mathbb{F}_{\mathbb{D}_i}$, which forms the dot product of the joint probability $\mathbb{F}_{\mathbb{S}}$. The two loops are carried out, in order to execute the modifications of the $\mathbb{F}_{\mathbb{D}_i}$ throughout the $learning1()$. The pseudo code corresponding to that function is presented in figure 5-5. The function ends by returning the new value of the joint probability function $\mathbb{F}_{\mathbb{D}_1}, \ldots, \mathbb{F}_{\mathbb{D}_5}$.

Updating the Probability Distribution Function $\mathbb{F}_{\mathbb{D}_i}$

In this section we present the pseudo code of the function $learning1()$, which aims to modify the probability distribution function $\mathbb{F}_{\mathbb{D}_i}$ defined over the domain $\mathbb{D}_i$. Previously in subsection 5.1.3 we described this probability function in terms of neuron weights, $\mathbb{F}\left(w_{i,1}, \ldots, w_{i,n_i}\right)$, for a given number of neurons n_i. Here, we present how we have incorporated the rules of modification of those weights according to the positive and negative feedback.

Function $learning1()$, in addition to $\mathbb{F}_{\mathbb{D}_i}$ receives as parameters the value $x \in \mathbb{D}_i$, the impact υ of this value in the updating process and finally the flag b, which indicates if the value x belongs to the positive cases stored in vector $\vec{\phi}^+$ or to the negative ones in

Function $learning1\left(\mathbb{F}_{\mathbb{D}_i}, x, \upsilon, b\right)$

1 $j = \arg\min_{k=1}^{n_i}\left(|x - w_{i,k}|\right)$
2 IF $b = 1$ THEN
3 FOR $k = 1, \ldots, n_i$ DO
4 IF $|j - k| \leq \delta^+$ THEN
5 $w_{i,k}^{new} = w_{i,k}^{old} + \epsilon^+ \cdot \upsilon \cdot (x - w_{i,k}^{old})$
6 ELSE
7 $w_{i,k}^{new} = w_{i,k}^{old}$
8 ELSE
9 FOR $k = 1, \ldots, n_i$ DO
10 IF $(|j - k| \leq \delta^-)$ AND $\left(w_{i,k}^{old} \leq x\right)$ THEN
11 $w_{i,k}^{new} = w_{i,k}^{old} + \epsilon^- \cdot \upsilon \cdot (w_{i,k-\delta^-} - w_{i,k}^{old})$
12 ELSE
13 IF $(|j - k| \leq \delta^-)$ AND $\left(w_{i,k}^{old} > x\right)$ THEN
14 $w_{i,k}^{new} = w_{i,k}^{old} + \epsilon^- \cdot \upsilon \cdot (w_{i,k+\delta^-} - w_{i,k}^{old})$
15 ELSE
16 $w_{i,k}^{new} = w_{i,k}^{old}$
17 RETURN $\mathbb{F}\left(w_{i,1}^{new}, \ldots, w_{i,n_i}^{new}\right)$

Figure 5-5: Pseudo code of the $learning1()$ function

vector $\vec{\phi}^-$.

The process starts by finding the neuron weight $w_{i,k}$ closest to the value x. In line 2 in figure 5-5 in order to distinguish if the value x will modify the distribution function $\mathbb{F}_{\mathbb{D}_i}$ in a positive way, the process verifies if the flag b is equal to 1.

In case in which the latter is true, the code enclosed among the lines 3 to 7 is executed. Namely, the neuron weights inside the neighbourhood kernel with the cylinder size δ^+ are updated according to the equation presented in line 5. Here, ϵ^+ is the positive learning rate predefined by the user and the intention of this modification is to concentrate neuron weights closer to the positive value x.

However, if b is not equal to 1, the x value will then be considered as negative feedback.

In that case, the code enclosed among the lines from 13 to 16 is performed and the aim of this code is to modify the neuron weights inside the neighbourhood kernel with the cylinder size δ^-. The updating rule is given in lines 11 and 14, where ϵ^- is the negative learning rate. The modification is performed in order to move the neuron weights away from the value of x. The process ends by returning the new probability distribution function, defined in terms of the neuron weights $\mathbb{F}\left(w_{i,1}^{new}, \ldots, w_{i,n_i}^{new}\right)$.

5.3 Summary

In this chapter we propose the application of the Generalised Population Based Incremental Learning algorithm in finding a strategy under specific criteria. To this end, the GPBIL explores the consumers' and merchants' demands reproduced by the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket in order to fulfil the established criteria.

Symbol	Payment Method's Variables Domains
$D_{\mathcal{F}_p}$	Consumer Fixed Fee Domain
D_{Γ_p}	Merchant Fixed Fee Domain
D_{b_p}	Domain of the Consumers' Benefits
D_{β_p}	Domain of the Merchants' Benefits
D_{l_p}	Publicity Cost

Table 5.1: Strategy's Domains

For this reason, we have divided the chapter in two main sections. First in section 5.1 we described the payment card strategies in a suitable way as it is presented in table 5.1 and we explained the one-step learning mechanism. Further in section 5.2 we provided the set of procedures used to implement the learning scheme of the algorithm.

Chapter 6

Profit-maximising strategy

6.1 Introduction

In this chapter, we present the empirical study of the algorithm used to design a strategy under specific criteria, the mechanism that was formally explained in the previous chapter. The exposition is divided in two parts. In the first part we validate the effectiveness of the learning process, i.e. to what degree the resulting strategy accomplishes the purpose it has been designed for. For this reason, we have performed an experiment aimed to find a strategy that satisfies two specific criteria. The results of this study are presented in section 6.2. Furthermore, in section 6.3 we evaluate how efficient the obtained strategy is by comparing its performance with a randomly generated strategy. At the end of the chapter we conclude with a summary of the results presented.

6.2 Is learning possible?

In this section we explore through experimentation how accurate the Generalised Population Based Incremental Learning (GPBIL) algorithm is in finding a profit-maximising strategy, which in addition has to achieve an average market share measured in terms of card transactions. In other words, we want to test how efficient our heuristic is and to what degree the evolved strategy[1] satisfies the searching criteria. In order to test the model, we have performed five different cases of study.

We have organised the section as follows: first in 6.2.1 we explain the structure of the experiment and its settings. Next, in section 6.2.2 we list the measurements used to assess the accuracy of the algorithm. Following on, in order to illustrate how the algorithm has resolved the criteria of the average market share, we have included two of the five cases studied. We present the observations and conclusions of the first case in section 6.2.3, whereas in section 6.2.4 we include the observations and conclusions of the second case. Next, in order to answer how the algorithm has performed the criteria of the highest possible profit, in section 6.2.5 we present the results of the five cases regarding the total profit of the strategy, found by the GPBIL. Finally in 6.2.6 we give general conclusions with respect to the performance of the algorithm.

[1]The evolved strategy is the strategy obtained at the end of the search.

6.2.1 Testing the Profit-Maximising Strategy of the Competitor

The test of the profit-maximising strategy has been conducted in order to evaluate how accurate is the GPBIL in finding for the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket a strategy with the highest possible profit and the average market share in terms of card transactions. In this section our experiment consists of exploring the areas of intersections between competitors' price and the consumers' and merchants' demand in order to find out if there are any areas in the strategy domain, where the payment card providers could maximise their profit and at the same time could obtain an average market share. In order to relate the results obtained by the current experiment and the results observed in sections 4.4 and 4.3, here we used the same parameters for the consumers' and merchants' inertia for adding and dropping cards as used in chapter 4.

The experiment consists of executing the process $\mathcal{MARKET} - \mathcal{GPBIL}$ for a considerable number of runs. At the end of the execution we validate if the GPBIL has found one or more than one prominent area in the domain of each of the strategy's elements. In that case, it means that the joint probability distribution of the algorithm has settled down and we will compare the average performance of the initial strategies with the average performance of the final strategies. On the other hand, if the joint probability has not settled down, i.e. the algorithm has not found a prominent area in the strategy's domain, we would not have elements to evaluate the effectiveness of the algorithm.

In order to test the accuracy of the algorithm, we have specified five different cases of study.

Symbol	Description	Cardinality	Value
$\mathcal{M}$	Set of Merchants	$N_{\mathcal{M}}$	125
$\mathcal{C}$	Set of Consumers	$N_{\mathcal{C}}$	1100
$\mathcal{M}_c$	Consumers' Set of Merchants	$N_{\mathcal{M}_c}$	5

Table 6.1: The sets of the agents

Case $9L$*:*

The number of cards $N_{\mathcal{P}} = 9$ and the network connection among consumers and merchants $nc = l$

Case $9SW$*:*

The number of cards $N_{\mathcal{P}} = 9$ and the network connection among consumers and merchants $nc = sw$

Case $9R$*:*

The number of cards $N_{\mathcal{P}} = 9$ and the network connection among consumers and merchants $nc = r$

Case $5L$*:*

The number of cards $N_{\mathcal{P}} = 5$ and the network connection among consumers and merchants $nc = l$

Special Case[2] $2L$*:*

The number of cards $N_{\mathcal{P}} = 2$ and the network connection among consumers and merchants $nc = l$

We have executed ten examples for each one of the five cases, using the same random seed in each example. This guarantees that the only differences, among the five cases for each example, are the number of competitors and the network connections. In this way, we can make a clear comparison among the results obtained for each case. Additionally, the fact that the competitors are trying to achieve an average market share, effectively means that the aim of the payment card providers in the cases of 2, 5 and 9 cards is different. For that reason we have chosen for this section data obtained in two of the five cases, which have different objectives: *Case* 9L and *Case* 5L.

Symbol	Description of the Constants	Value
ε	common constant for the inertia to changes	1
x_c^-	the consumers' inertia to drop cards	0.05
x_c^+	the consumers' inertia to add new cards	2
x_m^-	the merchants' inertia to drop cards	0.05
x_m^+	the merchants' inertia to add new cards	9
α	the impact of the publicity cost	0.1
φ	the impact of the publicity cost	5

Table 6.2: Constants used in the agents' decisions

The setting we have used to perform the experiment is the following: the number of runs $R = 6000$, the number of interactions $I = 3000$, the poisson distribution used to determine the decision period of consumers and merchants has a mean $\lambda = 20$; the rest of the user defined parameters are divided in three groups. The first group (table 6.1) presents the setting of the different sets of agents; in the second group we have listed the values of the constants, which impact the decisions of the end-users (table 6.2) and finally the domain of each element of the strategy space is presented in table 6.3.

Symbol	Domains	Value
$\mathbb{D}_{\mathcal{F}_\mathbf{p}}$	Consumer Fixed Fee Domain	$[0, 10]$
$\mathbb{D}_{\Gamma_p}$	Merchant Fixed Fee Domain	$[0, 10]$
$\mathbb{D}_{b_p}$	Domain of the Consumers' Benefits	$[-1, 1]$
$\mathbb{D}_{\beta_p}$	Domain of the Merchants' Benefits	$[-1, 1]$
$\mathbb{D}_{l_p}$	Publicity Cost's Domain	$[0, \infty]$

Table 6.3: Strategy's Domains

6.2.2 Measures for the test of the Profit-Maximising Strategy

In this section we list the measurements implemented in order to perform the test of the profit-maximising strategy.

- Φ_f^{100} is the average profit from the ten executions of all cards in the first 100 runs;
- Φ_l^{100} is the average profit from the ten executions of all cards in the last 100 runs;
- ms_r^p is the difference between the average market share and the market share obtained by a specific card in a particular run, for each example;
- δ is a threshold that delimited the interval in which ms_r^p is acceptable;
- μ is the average value selected by the payment card providers from the domain $\mathbb{D}_i \in \mathbb{S}$ in the last 100 runs;
- σ is the standard deviation of the average value selected by the payment card providers from the domain $\mathbb{D}_i \in \mathbb{S}$ in the last 100 runs;

6.2.3 The *Case* $9L$

In this subsection we present the *Case* $9L$, in which nine payment card providers compete in a market, where the consumers and merchants are connected locally among them. In the first part of the subsection we present the results and the observations related to them, whereas in the second part we list our conclusions.

Observations

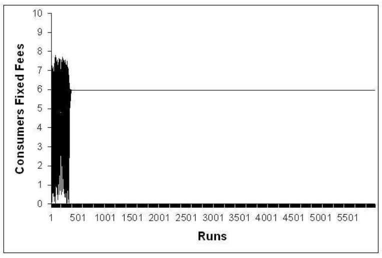
In this part we present in three tables the results corresponding to the competition with 9 payment card providers in a market, with local connections among consumers and merchants. Together with these results we list our observations.

In the first table, 6.4 the results for each one of the ten executions are presented in two rows. In the first row is shown the average value μ that the payment card providers have been selecting from the domain $\mathbb{D}_i$ in the last 100 runs. The second row represents the standard deviation σ from this value. The columns correspond to each element of the strategy.

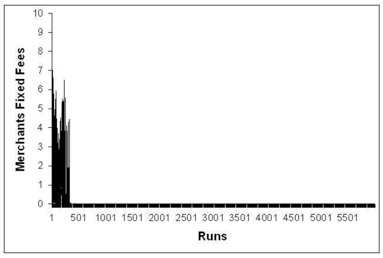
	F_p	$\mathit{\Gamma}_p$	b_p	β_p	l_p	Φ_p
μ	**7.57**	**0.00**	**-1.00**	**-1.00**	**11.11**	**6,048,995.23**
σ	0.00	0.00	0.00	0.00	2.45	
μ	**5.33**	**0.00**	**-1.00**	**-1.00**	**7.66**	**5,275,214.86**
σ	0.00	0.00	0.00	0.00	0.00	
μ	**3.51**	**0.00**	**1.00**	**-1.00**	**11.81**	**3,204,527.52**
σ	0.00	0.00	0.00	0.00	0.00	
μ	**6.03**	**0.00**	**0.48**	**-1.00**	**11.82**	**4,356,514.63**
σ	0.00	0.00	0.00	0.00	0.00	
μ	**5.46**	**0.00**	**-1.00**	**-1.00**	**10.49**	**5,333,885.81**
σ	0.00	0.00	0.00	0.00	0.00	
μ	**6.03**	**0.00**	**-1.00**	**-1.00**	**13.85**	**5,562,761.79**
σ	0.00	0.00	0.00	0.00	0.00	
μ	**5.98**	**0.00**	**-1.00**	**-1.00**	**8.39**	**5,551,276.47**
σ	0.00	0.00	0.00	0.00	0.00	
μ	**6.48**	**0.00**	**-1.00**	**-1.00**	**9.97**	**5,738,453.78**
σ	0.00	0.00	0.00	0.00	0.00	
μ	**5.38**	**0.00**	**-1.00**	**-1.00**	**10.24**	**5,299,438.88**
σ	0.00	0.00	0.00	0.00	0.00	
μ	**5.66**	**0.00**	**-1.00**	**-1.00**	**10.82**	**5,423,793.36**
σ	0.00	0.00	0.00	0.00	0.00	

Table 6.4: Resulting strategies from the ten executions in the *Case* $9L$ considering the last 100 runs

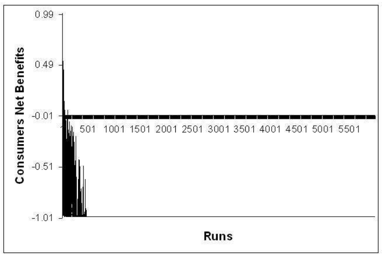
In addition in figure 6-1 we present how each one of the strategy elements has evolved during one execution of the model. The x axis represents the number of runs, whereas the y axis represents the values of the variables' domains. The curves inside the figures represent the limits of the sub-intervals, in which each domain is divided.



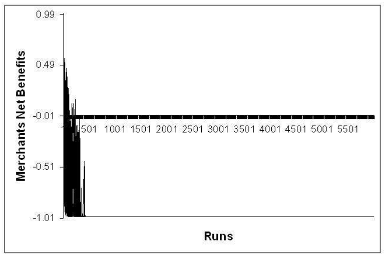
(a) Consumers Fixed Fees $\mathcal{F}_p$



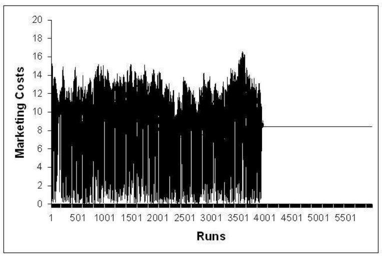
(b) Merchant Fixed Fees Γ_p



(c) Consumers Net Benefits b_p



(d) Merchants Net Benefits β_p



(e) Marketing Cost l_p

Figure 6-1: Evolving Strategy

Observation 1: Considering the standard deviation σ reported in table 6.4, in which from 50 σ reported 49 are equal to zero, we can say that the joint probability distribution $\mathbb{F}_{\mathbb{S}}$ has settled down in the case of nine cards competing in the market;

Observation 2: The competitor strategies from the ten executions exhibit certain similarities, which could be listed as follows:

- In nine out of the ten examples the consumers pay high fixed fees($F_p > 5$ with $max(F_p) = 10$);
- The merchants do not pay fixed fees;
- In eight out of the ten examples, the consumers pay the highest possible variable fee;
- The merchants pay the highest possible variable fee in all examples.

ms_r^p	ms_1^1	ms_1^2	ms_1^3	ms_1^4	ms_1^5	ms_1^6	ms_1^7	ms_1^8	ms_1^9
	-0.10	-0.10	-0.06	0.05	0.01	0.06	0.02	0.04	0.07
	0.06	0.02	-0.07	0.06	-0.01	0.04	0.05	-0.11	-0.06
	0.03	0.05	0.05	0.02	-0.06	-0.20	0.01	0.08	0.03
	-0.04	-0.05	0.06	-0.23	0.08	0.07	0.04	0.02	0.06
	0.04	0.04	-0.09	0.05	-0.01	0.00	-0.06	-0.04	0.08
	-0.07	0.09	0.08	0.04	0.08	-0.05	-0.05	0.07	-0.18
	-0.06	0.01	-0.14	0.01	0.07	-0.04	0.02	0.08	0.03
	0.04	0.06	-0.02	-0.06	0.02	-0.05	0.03	-0.02	-0.02
	0.06	-0.03	-0.03	0.00	0.04	0.03	-0.12	0.03	0.03
	0.05	0.05	-0.02	0.04	-0.07	0.04	0.05	0.01	-0.15

Table 6.5: The difference between the average market share and the individual market share obtained by each card in the first run for each execution

In the second table, 6.5 we present ms_r^p, the difference between the average market share and the individual market share obtained by the issuers for each card $p \in \mathcal{P}$. These data are reported for each of the ten executions for the first run $r = 1$. The rows represent the results for each example and the columns - the difference of each card.

Observation 3: In table 6.5 we observe considerable differences between the individual market share and the average market share. For instance, suppose $\delta = 0.3$, the minimum negative and the maximum positive ms_r^p registered are -0.23 and 0.09 respectively; we observe that in twenty eight out of the ninety possible examples, the values of ms_r^p are in the interval $[-0.3, 0.3]$ and in two occasions the value of ms_r^p is equal to 0.

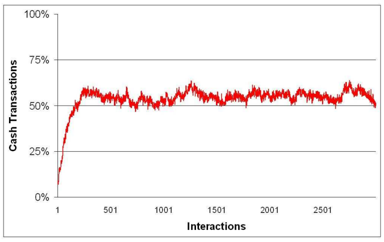
ms_r^p	ms_{6000}^1	ms_{6000}^2	ms_{6000}^3	ms_{6000}^4	ms_{6000}^5	ms_{6000}^6	ms_{6000}^7	ms_{6000}^8	ms_{6000}^9
	-0.02	-0.02	-0.01	-0.01	0.01	0.02	0.00	0.01	0.01
	0.00	-0.02	-0.01	0.00	0.00	-0.01	0.01	0.01	0.02
	0.00	-0.01	-0.02	0.00	0.00	0.01	0.01	0.00	0.01
	0.00	-0.02	-0.01	-0.01	0.00	0.00	0.01	0.02	0.01
	-0.03	-0.01	0.01	0.00	0.00	0.01	0.01	-0.01	0.01
	-0.02	-0.01	-0.01	0.01	-0.01	0.00	-0.01	0.02	0.02
	-0.02	-0.02	-0.01	0.00	0.00	0.01	0.01	0.02	0.00
	-0.01	-0.01	-0.01	0.00	0.00	-0.02	0.00	0.03	0.01
	-0.02	-0.02	-0.01	0.00	0.00	0.00	0.01	0.01	0.01
	-0.01	-0.03	-0.01	-0.01	0.00	0.00	0.02	0.02	0.03

Table 6.6: The difference between the average market share and the individual market share obtained by each card in the final run for each execution

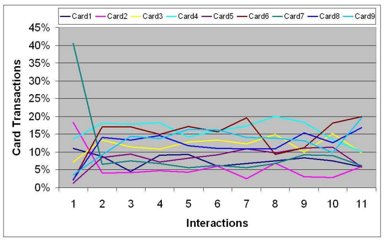
In table 6.6 we present ms_r^p, the difference between the average market share and the individual market share obtained by the nine cards providers for each $p \in \mathcal{P}$. The data is reported for each one of the ten executions, where the final run is $r = 6000$. The rows represent the executions and the columns the difference of each card.

Observation 4: The differences between the individual market share and the average market share have been reduced significantly. Given that the absolute value of the threshold $\delta = 0.3$, in all ninety examples of table 6.6 the values of ms_r^p are in the interval $[-0.3, 0.3]$ and in twenty eight of the ninety cases the value of ms_r^p is equal to 0.

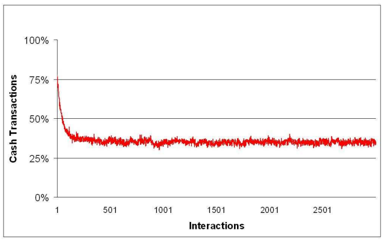
Furthermore, in order to illustrate the dynamics reproduced by the model, in figure 6-2 we present a comparison between the cash and card transactions in the first and in the last run of one execution. In the x axis the number of interactions among consumers and merchants are shown, whereas the y axis represents the percentage of transactions. In figures 6-2(b) and 6-2(d) the transactions achieved by all competitors are illustrated.



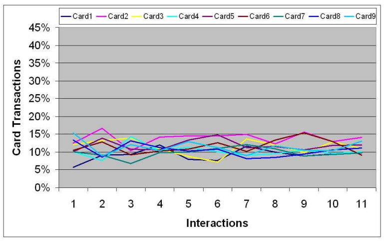
(a) Cash Transactions in the first run



(b) Card Transactions in the first run



(c) Cash Transactions in the last run



(d) Card Transactions in the last run

Figure 6-2: Market Dynamics

Analysis

In this section we present our conclusions related to the test of the profit-maximising strategy corresponding to the case with 9 competitors with local connections in the market.

Conclusion 1: We conclude from *Observations 1* and *2* that the algorithm has found a unique solution for each one of the ten executions and those solutions exhibit clear similarities.

Conclusion 2: From *Observations 3* and *4* it follows that the algorithm has reduced the differences among market shares of the payment card providers as one of the criteria applied in the search of the profit-maximising strategy.

In order to explain our result from an economic point of view, let us recall our observations from chapter 4. We have used the same parameter setting for the current experiment and the experiments presented in that chapter. In section 4.3.5 we said that there are several conditions in the market that could help us to advance further the understanding of the model. These conditions are listed as follows:

- The sensitivity of the consumers' demand in terms of consumers' fixed fees is lower in the case of nine competitors;

- With comparison to the merchants' demand, we notice that the sensitivity of the consumers in terms of consumers' fixed fees is also lower than the sensitivity of the merchants in terms of merchants' fixed fees;
- The slope of the consumers' demand in terms of merchants' fixed fees is getting close to zero, i.e. the consumers' demand is adjusted more slowly to changes in the number of merchants per card;
- The slope of the merchants' demand in terms of consumers' fixed fees is also getting close to zero.

Further, in subsection 4.4.3 we observe that the total profit of the competitors is primary generated by the consumers' and merchants' fixed fees, being the consumers' fees the most important source. At the same time the profit is negatively correlated to the merchants' benefits. These specifications characterise the market we have reproduced. Further, with the use of the GPBIL, we are able to observe the emerging price structure and level.

Given that the consumers exhibit lower demand sensitivity, in order to maximise their profit, the card issuers charge them high fixed fees, and such fees are the most important source of profit for the issuers. In this context, a relevant question is: why consumers stay in the market, given the high fixed fees the issuers charge them? A possible explanation of the interest of the consumers in the usage of cards, could be the presence of a considerable number of merchants accepting cards. The willingness of the merchants to accept cards is because they do not pay fixed fees. Nevertheless, while the merchants do not pay fixed fees, they do pay variable fees (negative net benefits), which in addition are the other important source of profit for the issuers[3]. Furthermore, in figure 4-7 we observed that one of the estimated areas of maximum profit is located where the merchants' net benefits are equal to -1 and the consumer fixed fees are either between 2 and 4, or between 7 and 8. The area located where the merchants' net benefits are equal to -1 and the consumers' fixed fees are between 4 and 7, is also an area of high profit. For that reason, the solutions found are consistent with our finding in chapter 4 related to the case of nine competitors.

6.2.4 The *Case* $5L$

Similarly to the previous part, here we present the observations and the analysis corresponding to the case of five competitors in a market with local connections among consumers and merchants.

[3]The sources of the profit were analysed in subsection 4.4.3.

Observations

In this section the results from the *Case* $5L$ are presented in three tables with their corresponding observations.

In table 6.7 we present the strategies obtained in the ten executions of the model. The results of each execution are presented in two rows. In the first row, we list the average value μ that the payment card providers have selected from the domain $\mathbb{D}_i$ in the last 100 runs. In the second row, we present the standard deviation σ from this value. The columns show the elements of the strategy.

Observation 5: According to the data reported in table 6.7, only in three executions the joint probability distribution has settled down completely, considering the standard deviation σ. In the rest of the cases, one or two of the elements of the strategy present a standard deviation σ bigger than zero. In total, from fifty possibilities in eight occasions it is the case that $\sigma > 0$.

Observation 6: The resulting strategies of the ten executions present similar characteristics. These could be listed as follows:

- In seven out of the ten examples the consumers pay fixed fee;
- The merchants do not pay fixed fees;
- The consumers received positive benefits;
- The merchants pay the higher possible variable fees.

	$\mathcal{F}_p$	$\mathit{\Gamma}_p$	b_p	β_p	l_p	Φ_p
μ	**0.07**	**0.00**	**1.00**	**-1.00**	**7.81**	**83,193.46**
σ	0.16	0.00	0.00	0.00	0.00	
μ	**3.33**	**0.00**	**0.43**	**-1.00**	**9.52**	**4,030,092.77**
σ	0.00	0.00	0.00	0.00	1.77	
μ	**4.21**	**0.00**	**0.53**	**-1.00**	**10.56**	**4,527,125.71**
σ	0.00	0.00	0.00	0.00	0.00	
μ	**0.00**	**0.00**	**1.00**	**-1.00**	**2.23**	**-5,576.79**
σ	0.00	0.00	0.00	0.00	1.28	
μ	**1.40**	**0.00**	**0.37**	**-1.00**	**8.74**	**2,202,551.73**
σ	0.68	0.00	0.00	0.00	0.00	
μ	**3.82**	**0.00**	**0.78**	**-1.00**	**10.83**	**4,213,727.65**
σ	0.00	0.00	0.00	0.00	0.00	
μ	**0.00**	**0.93**	**0.10**	**-1.00**	**8.75**	**561,356.43**
σ	0.00	0.58	0.00	0.00	1.89	
μ	**3.71**	**0.00**	**0.61**	**-1.00**	**10.64**	**4,210,577.77**
σ	0.00	0.00	0.00	0.00	0.00	
μ	**0.37**	**0.00**	**0.71**	**-1.00**	**8.64**	**706,220.40**
σ	0.29	0.00	0.00	0.00	0.00	
μ	**0.00**	**0.00**	**0.57**	**-1.00**	**7.17**	**203,547.22**
σ	0.00	0.00	0.00	0.00	2.48	

Table 6.7: Resulting Strategy with 5 cards in Local Network

In table 6.8 we present ms_r^p, the difference between the average market share and the individual market share obtained by the five card providers for each card $p \in \mathcal{P}$. These data are reported for each of the ten executions for the first run $r = 1$. The rows represent the data from each execution, whereas in the columns the differences of each card are listed.

Observation 7: The differences between the average market share and the individual market share reported in table 6.8 are considerable. For instance, suppose absolute value of the threshold $\delta = 0.3$, the minimum negative and the maximum positive ms_r^p registered are -0.31 and 0.16 respectively. Apart from that, in eleven out of the fifty possible examples, the values of ms_r^p are in the interval $[-0.3, 0.3]$; in one

ms_r^p	ms_1^1	ms_1^2	ms_1^3	ms_1^4	ms_1^5
	-0.06	-0.08	-0.05	0.13	0.06
	0.10	0.03	-0.22	0.10	-0.01
	0.06	0.08	0.07	0.02	-0.23
	0.01	0.00	0.13	-0.30	0.16
	0.05	0.09	-0.20	0.10	-0.03
	-0.31	0.13	0.11	-0.01	0.09
	-0.03	0.04	-0.21	0.07	0.14
	0.08	0.11	-0.03	-0.18	0.02
	0.09	-0.06	-0.08	-0.01	0.06
	0.08	0.07	-0.05	0.08	-0.18

Table 6.8: The difference between the average market share and the individual market share obtained by each card in the initial run for each execution

occasion the value of ms_r^p is equal to 0.

ms_r^p	ms_{6000}^1	ms_{6000}^2	ms_{6000}^3	ms_{6000}^4	ms_{6000}^5
	-0.03	0.01	-0.02	0.02	0.01
	-0.02	-0.01	-0.01	0.01	0.02
	-0.03	-0.01	-0.01	0.02	0.03
	-0.07	0.02	0.01	0.02	0.02
	-0.05	0.03	0.02	-0.02	0.02
	-0.03	0.01	-0.01	-0.01	0.04
	-0.02	-0.02	-0.02	0.02	0.04
	-0.01	-0.01	0.02	-0.02	0.02
	-0.05	-0.02	0.01	0.03	0.04
	-0.02	-0.01	0.01	0.01	0.01

Table 6.9: The difference between the average market share and the individual market share obtained by each card in the final run for each execution

In table 6.9 we present ms_r^p, the difference between the average market share and the individual market share obtained by each one of the five card issuers for each $p \in \mathcal{P}$. The data is reported for the final run $r = 6000$ for each of the ten executions. The rows represent the data from each example, whereas the columns show the differences of each card.

Observation 8: According to the results presented in table 6.9, the algorithm has managed to reduce the differences between the average market share and the individual market share of each competitor. We can see from the data that the minimum negative and maximum positive ms_r^p registered are -0.07 and 0.04 respectively. In addition, in forty four out of the fifty possible examples, the value of ms_r^p is in the interval $[-0.3, 0.3]$.

Analysis

In this section, we present our conclusions related to the test of the profit-maximising strategies corresponding to the case of 5 competitors in a market with local connections among consumers and merchants.

Conclusion 3: From *Observation 5* and *6* it follows that the algorithm has found a solution in each one of the ten executions, even though some elements of the strategy do not represent a unique value. In addition, the evolved strategies exhibit a similar price structure.

Conclusion 4: From *Observations 7* and *8* it follows that the algorithm has reduced the differences among market shares of the payment card providers, as one of the criteria applied in the search for the strategy.

In general, in the cases of nine and five competitors in the market, we observe that the maximisation of the profit is obtained with strategies that present a similar price structure. The common elements of this structure are:

- The merchants do not pay fixed fees;

- The merchants pay the highest possible variable fee;
- The consumers pay fixed fees.

In section 6.2.3 we attempt to explain how the price level of the listed elements is related to our findings of chapter 4. Here we would like to analyse why there is a difference between the levels of consumer fixed fees and the consumer net benefits in the cases of five and nine competitors. In the case of five competitors the consumers receive net benefits and pay lower fixed fees, whereas in the case of nine competitors the consumers instead of receiving benefits, they pay variable fees and higher fixed fees. In order to explain these differences, let us recall our findings of chapter 4, where we have used the same parameter setting with five and nine competitors.

In section 4.3 we observed that the consumers reduce their demand sensitivity in terms of consumer fixed fees, when the number of competitors increase. In other words, with more competitors in the market, the consumers react more slowly if their fixed fees are increased. In addition, the sensitivity of the consumers' side in terms of merchants' fixed fees is higher in the case of five competitors in the market than in the case of nine. This means that in case the merchants' fixed fees are increased and consequently the number of merchants accepting card is decreased, the number of consumers having cards will be adjusted more slowly in the case of nine competitors than in the case of five.

Furthermore, the target of the heuristic used to obtained the profit-maximising strategies, is established as the average market share. In other words, having a different number of competitors, the GPBIL has to achieve a different target, e.g it is approximately 11% in the case of nine competitors and 20% in the case of five. We said earlier in section 4.5 that the number of transactions is reduced with the increase of the consumers' and merchants' fixed fees. Thus, the different market target explains why the consumers are charged less fixed fees in the case of five competitors than in the case of nine.

These factors explain the corner solution we have obtained in the case of five card issuers in the market, where merchants do not pay fixed fees, the consumers received high net benefits and the merchants are charged with high variable fees.

6.2.5 The Profit Achieved

In the present subsection, we present the profit achieved in each one of the four cases of the study and the observations and analysis related to these results.

$N_{\mathcal{P}}$	nc	Φ_f^{100}	Φ_l^{100}
9	l	4,092,329.50	5,179,486.23
9	sw	4,110,964.27	5,088,416.74
9	r	4,084,039.59	5,271,941.41
5	l	3,888,038.14	2,073,281.64

Table 6.10: Comparison between the average profit obtained in the first and last 100 runs

Observation 9: According to the results presented in table 6.10, we observe that in the cases where we have nine competitors in the market, the average profit obtained in the last 100 runs is higher than the average profit obtained in the first 100 runs.

Observation 10: In the same table, in the case of 5 competitors in the market the final average profit obtained is lower than the average profit obtained in the first 100 runs.

Conclusion 5: Given that the payment card providers in the case of 9 competitors want to achieve approximately 11% of the market share, from *Observation 9* we conclude that the GPBIL has found a profit-maximising strategy applying higher fees to the consumers than to the merchants.

Conclusion 6: In the case of 5 competitors, the aimed market share is approximately 20%. Under this condition we conclude from *Conclusion 4* and *Observation 10* that the GPBIL has found a strategy that approximates the target of 20% of the market share. Given this high target, the average profit of the competitors has been reduced, but they still achieve a relevant profit, $\Phi_l^{100} = 2{,}073{,}281.64$.

6.2.6 Analysis of the Learning

In this section we present general conclusions regarding the comparison among the two cases of study presented in sections 6.2.3 and 6.2.4 together with the GPBIL performance in finding the profit-maximising strategy.

In sections 6.2.3 and 6.2.4 we analyse the particular conditions of the 9 and 5 competitors, taking into account the observations made in chapter 4. We noticed that the number of competitors affects the consumers' and merchants' demand sensitivity in terms of the fixed prices on both sides. We also observed that the higher the number of competitors, the lower the impact of the merchants' fixed fees on the number of consumers per cards and vise versa. In addition, as one of the search criteria in the current experiment was established in terms of the average market share, the GPBIL had to achieve different targets, given the different number of payment card providers. These conditions give us the reason why the solutions of the case of 9 competitors have a slightly different structure than the solutions of the case of 5 competitors.

Conclusion 7: From *Conclusions 5* and *6* it follows that learning is possible.

6.3 Are the evolved strategies efficient?

In the previous section 6.2 we concluded that the algorithm has found a profit-maximising strategy in the case of nine competitors in the market. In this section we present an experiment designed to verify the efficiency of the strategies, obtained by the GPBIL for this case.

The section is organized as follows. First in section 6.3.1 we explain the design of the experiment; next in section 6.3.2 we list the measures used to evaluate the efficiency of the strategy. Further, in section 6.3.3 the results of the experiment together with the observations are presented and we finalise the section with the related analysis in section 6.3.4.

6.3.1 The efficiency test

The efficiency test consists of comparing the performance of the profit-maximising strategy with the performance of randomly generated strategy. In order to do so, we have created the same environment as described in section 6.2.1 for the case of nine competitors in a market with local interactions among consumers and merchants. The experiment consists of executing the process $\mathcal{MARKET} - \mathcal{GPBIL}$ for one run $R = 1$. In the simulation, eight of the nine competitors use the same profit-maximising strategy, whereas one competitor applies a randomly generated strategy. We have tested five of the ten strategies presented in section 6.2.3 by the procedure described. Each strategy is tested against ten different random strategies in ten independent executions of the model. In table 6.11 we list the randomly generated strategies[4] used in the experiment. In the first column of the table we list the number of the execution in which this strategy was used, whereas in the rest of the columns we present the elements of the strategy.

Execution	F_p	Γ_p	b_p	β_p	l_p
1	5.50	0.49	-0.10	-0.79	14.84
2	5.82	0.42	-0.74	0.36	14.24
3	6.62	3.96	0.12	-0.25	0.70
4	0.92	4.49	-0.66	-0.09	5.68
5	6.07	6.17	-0.87	0.04	15.42
6	3.16	3.95	-0.66	-0.88	0.69
7	6.16	1.87	0.48	-0.72	1.92
8	7.80	5.87	-0.15	-0.64	0.51
9	6.31	3.57	-0.07	0.53	8.29
10	6.57	3.93	0.56	-0.47	12.80

Table 6.11: Randomly generated strategies

At the end of the execution we compare the profit and market share of the randomly generated strategy with the average profit and average market share obtained by the competitors, using the profit-maximising strategy. The payment card provider applying randomly generated strategy needs to obtain better market share and better profit, in order for this to be considered a more efficient strategy, than the strategies found by the GPBIL in the previous section. In section 6.3.3 we present the results of two of the five strategies tested.

6.3.2 Measures

In this section we list the measurements used to evaluate the performance of the different kinds of strategies.

[4]For each strategy obtained by the GPBIL we used exactly the same set of randomly generated strategies.

- Φ_p^{gp} is the average profit of the eight competitors using the strategy obtained by the GPBIL;
- Φ_p^{rm} is the profit of the competitor with the randomly generated strategy;
- $N_{I_p}^{gp}$ is the average market share of the eight payment card providers, using the strategy obtained by the GPBIL, measured in terms of number of transactions;
- $N_{I_p}^{rm}$ is the market share of the competitor, using the randomly generated strategy, measured in terms of number of transactions.

6.3.3 Observations

In this section we present the results obtained from two of the five strategies tested. The section is divided in two parts, each one corresponding to the performance of one of the profit-maximising strategies.

Test of Efficiency Case 1 (TEC1)

This part is dedicated to the results obtained in the Efficiency Test of the performance of the profit-maximising strategy presented in row 3 of table 6.4 in section 6.2.3. In table 6.12 we list the elements of that strategy.

F_p	$\mathit{\Gamma}_p$	b_p	β_p	l_p
3.51	0.00	1.00	-1.00	11.81

Table 6.12: Profit-Maximising Strategy, Case 1

In figure 6-3 we present graphically the comparison between the performance of the randomly generated strategies[5] and the profit-maximising strategy from the ten executions. In table 6.13 the same data are shown organized in the following way: in the first column the number of the execution is presented, in the second and third columns the market share and the profit of the random strategy are listed, and finally in the last two columns the average market share and the average profit of the profit-maximising strategy are presented.

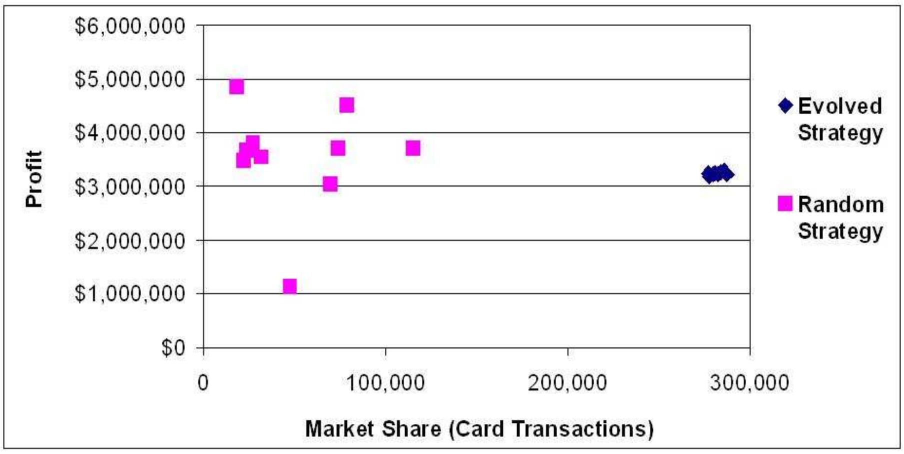


Figure 6-3: Test of Efficiency, Case 1

Ex.	$N_{I_p}^{rm}$	Φ_p^{rm}	$N_{I_p}^{gp}$	Φ_p^{gp}
1	115,092	3,711,873	276,873	3,235,839
2	78,536	4,502,427	281,802	3,226,917
3	31,653	3,539,388	285,434	3,273,227
4	47,468	1,130,966	277,161	3,172,209
5	17,945	4,842,936	286,763	3,231,322
6	69,554	3,037,368	279,898	3,224,163
7	73,756	3,711,376	280,569	3,242,955
8	23,724	3,669,594	285,970	3,291,332
9	21,821	3,476,859	286,019	3,264,873
10	27,007	3,807,948	283,580	3,256,304

Table 6.13: Test of efficiency, case 1

[5]Please refer to table 6.11

Observation TEC1: The profit-maximising strategies have performed statistically better than the randomly generated strategies. Regarding the market share achieved, the performance of the evolved strategy is much better than the other strategies. Nevertheless in terms of profit, the strategy of GPBIL is less profitable than the majority of the randomly generated strategies, despite the fact that the random strategies have not been able to penetrate considerably in the market.

Test of Efficiency Case 2 (TEC2)

In this part we test the performance of the profit-maximising strategy presented in row 10 of table 6.4 in section 6.2.3. We present the elements of the strategy in table 6.14.

F_p	Γ_p	b_p	β_p	l_p
5.66	0.00	-1.00	-1.00	10.82

Table 6.14: Profit-maximising strategy, case 2

In figure 6-4 we present graphically the comparison of the performance of the randomly generated strategy[6] and the profit-maximising strategy in terms of market share and profit. The same data are presented in table 6.15. The table is organized as follows: in the first column we present the number of the execution, next in the second and in the third columns the market share and profit obtained by the competitor with randomly generated strategy are listed and, finally, in the fourth and in the fifth columns we show the average market share and average profit of the competitors using the profit maximising strategy.

Observation TEC2: The profit-maximising strategy has performed better than the

[6]Please refer to table 6.11.

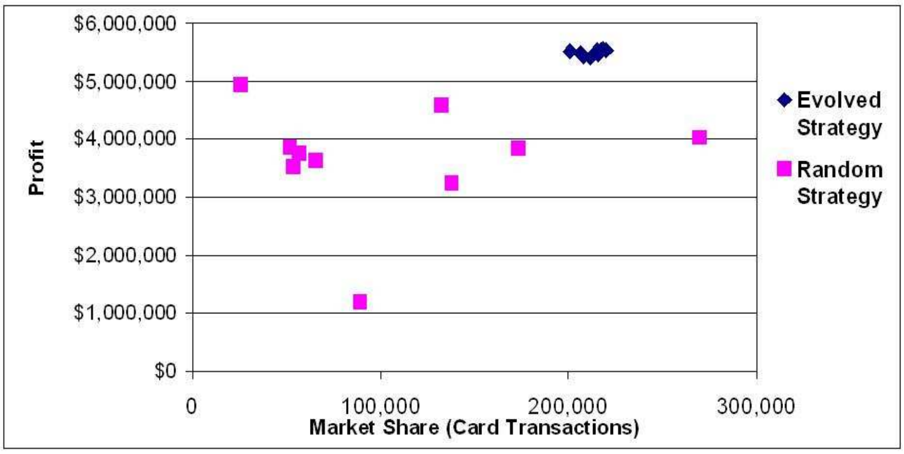


Figure 6-4: Test of Efficiency, Case 2

Ex.	$N_{I_p}^{rm}$	Φ_p^{rm}	$N_{I_p}^{gp}$	Φ_p^{gp}
1	270,086	4,028,220	200,859	5,508,231
2	132,753	4,580,05	206,786	5,467,251
3	65,463	3,638,983	215,090	5,531,546
4	89,048	1,195,634	211,330	5,404,561
5	25,704	4,932,694	220,308	5,537,538
6	138,006	3,235,442	208,256	5,429,815
7	173,742	3,837,502	206,364	5,482,818
8	56,985	3,745,939	217,497	5,532,962
9	53,913	3,525,837	215,560	5,458,349
10	51,820	3,860,624	218,343	5,554,859

Table 6.15: Test of Efficiency, Case 2

randomly generated strategy. Only in execution 1 the random strategy has managed to achieve better market share, but worst profit. In the rest of the cases the strategy found by GPBIL has performed better in terms of profit and market share.

6.3.4 Analysis

Conclusion 8: From *Observations TEC1* and *TEC2* it follows that the strategies found by the Generalised Population Based Incremental Learning algorithm are efficient strategies in achieving market share and profit. In other words, they fulfill the purpose they have been designed for.

6.4 Summary

In this chapter we validated through experimentation the effectiveness of the Generalised Population Based Incremental Learning algorithm in finding profit-maximising strategy. To this end, in section 6.2 we explained the experiment, in which we used the GPBIL to explore the search areas formed by the interactions among consumers, merchants and card issuers. The search was performed in order to find a profit-maximising strategies. We have included the data obtained for the cases of nine and five competitors in the market. We conclude that the algorithm has successfully found the required strategies.

Further in section 6.3, using the strategies obtained in the case of nine competitors, we compared the performance of those strategies against randomly generated strategies. We concluded that the evolved strategies obtain better results and are more efficient in penetrating the market than the other strategies.

Chapter 7

Conclusions

This thesis introduced the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket model, an agent-based approach for studying the payment card market. The final chapter is a summary of this study. It looks at the work presented, lists the main contributions, discusses limitations and takes a look at further research.

7.1 Summary of the Work Presented

The current thesis is organised in seven chapters. We start our exposition by presenting a general explanation of the two-sided nature of the payment card market in chapter 1. In the same chapter, given the importance of studying the competition among payment card providers, we give the reasons why we believe that a different approach is needed to gain a better understanding of the market dynamics.

Next, chapter 2 is divided in two main parts. In section 2.1 we briefly provide an overview of the existing analytical models aimed at studying the payment card market dynamics. Following on in section 2.2 we present the foundations behind the existence of Agent-based Computational Economics and the reasons why we use this approach to study the complex interactions among card providers, consumers and merchants in the market.

Further, in chapter 3 we formally introduce the specification of the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket model. We start with the presentation of the key elements of the model in section 3.2; next in 3.3 we explain the decisions behind the interactions among consumers and merchants. The exposition continues in 3.4 with the presentation of the basic rule used to select the competitors' strategies. Finally in section 3.5 we give the complete set of procedures, applied to reproduce the commercial transactions at the point of sale.

In chapter 4 we explore different scenarios in order to further our understanding of the payment card market. To this end, in section 4.3 we demonstrate how using the agent-based methodology we can estimate the price sensitivity of consumers' and merchants' demands. The study was performed in a market, where all competitors priced equally their products. In section 4.4 we analysed the sensitivity of the end-users' demand in a dynamic environment, where each card issuer priced his card independently.

Next, in chapter 5 we propose the implementation of the Generalised Population Based Incremental Learning (GPBIL) algorithm [27] in the design of strategies with specific criteria. For that reason in section 5.1 we present the main features of the algorithm, and in section 5.2 we explain the set of procedures used to perform the heuristic search for the required strategy.

The aim of chapter 6 is to evaluate the performance of the GPBIL algorithm, when it is applied to find a profit-maximising strategy. The heuristic is performed over the search space formed by the interactions among consumers, merchants and card issuers. In section 6.2 we explain the experiment used and the results obtained, whereas in section 6.3 we test how efficient are the evolved strategies presented in the previous section.

The final chapter is a summary of the thesis and presents the main contributions of our work.

7.2 Contributions

7.2.1 Modelling of Agent-based Payment Card Market

The electronic payment cards are a well known example of two-sided platforms [2]. It is two-sided, because the payment card issuer, in order to place his product in the market, needs to attract two different types of clients: on one side there are merchants, who want to sell their goods, and on the other side there are consumers, who want to buy goods. The goal of the payment card provider is to persuades the merchants and the consumers to use the card that he is issuing, in the commercial transactions between them. Additionally, the usage of the payment method among consumers and merchants give rise to indirect externalities, i. e. if the number of consumers increases, the number of merchants will increase as well, and vice versa.

The existing analytical studies recognize the importance of the consumers' and merchants' decisions at the point of sale, as the analysis is based on representative players:

consumers, merchants and payment card providers. Nevertheless, these models are unable to incorporate explicitly the complex relationships, resulting from the interactions among consumers and merchants, which we consider crucial for the understanding of the market dynamics. For this reason we believe that modeling the heterogeneous interactions among the market participants will advance our knowledge of the card market. This is necessary, given the worldwide acceptance of payment cards and the economic importance of the market.

To this end, in this thesis we have created the first, to our knowledge, Agent-based computational model, which reproduces explicitly the dynamics of interactions among the three groups of players involved in the industry: the payment card providers, the consumers and the merchants. More specifically, our aim was to simulate at the micro level the social phenomenon which occurs at the point of sale, and thus observe and study the emerging patterns at the macro level.

Nevertheless, the conceptualization of the studied phenomena is not a trivial task. All the more so, if in the process of modeling two contrary trends are involved. On one hand, we firmly believe that the more factors are included in the representation of the market, the closer to reality the model will be and as a consequence a better understanding of the studied phenomenon we will have. On the other hand, if too many factors are incorporated in the artificial simulation of the market, it could lead to chaotic results, without the possibility of understanding them. To this end, finding an adequate number of factors to incorporate in the process of modeling is crucial for the success of the project. In this context, in the model we have looked for a degree of complexity, which allows us to study the intrinsic relationships in the market.

7.2.2 The Two-sided Market Structure Dependency

The payment cards allow consumers and merchants to interact with each other through commercial transactions. The card issuers used a complex price structure in order to address each side independently. This structure usually consists of two components: fixed and variable fees [18]. In the literature it is widely accepted that the setting of this complex price structure depends on the elasticity of demand on both sides and the indirect effects arising from the interactions with the other side of the market.

From the interactions among consumers, merchants and card issuers, a market structure emerges, which is not easy to analyse. For instance, consumers face a complex choice which cards to hold. This decision is not only affected by the fees charged and benefits provided by each card, but has also to take into account the acceptance of a card by merchants. Thus, the decisions by merchants to accept such cards have to be considered too. Similarly, the merchants choose which card to accept based not only on its costs and benefits, but also influenced by the consumers' decisions to hold a certain card. Consequently, in setting the fees and benefits of the payment card, the card issuers have to take these aspects of consumer and merchant choice into account, leading to a wide range of complex interactions between these three parties.

For this reason, modeling the mutually constrained consumers' and merchants' demands for payment cards is crucial task in the understanding of the competition among card issuers. In order to advance this understanding, in this thesis we have created the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket $\mathcal{APCM}$, which with simple and reasonable premises explicitly reproduces the commercial transactions among consumers and merchants at the point of sale and allows to analyse the emerging phenomena. Furthermore, given the stationary time series (see section 4.2) produced by the model, we can study the market structure dependency on the consumers' and merchants' decisions, under different scenarios and conditions.

7.2.3 The Competition among Several Card Issuers

Given the growing importance of the payment card market, the competition among payment card providers has attracted the attention of academics [29, 10, 9, 6, 65, 66] and regulators [19, 31, 20, 21, 23]. Nevertheless, the major challenge faced by these experts lies in the complexity of the two-side markets. This complexity arises from the considerable number of participants in each one of the two groups split between the sides of the platform, the inherent externalities inside and outside each group and the price structure established by the competitors. These factors are the reason why the competition among several card issuers is very difficult to study analytically.

The first model of competition in two-sided markets was proposed in [2], following by the models in [15] and [16]. These models analysed the market dynamics with two competitors in the market. In [16] the authors conclude that the competition improves the consumers' and merchants' welfare, while the profit of the competitors is reduced.

In our model, in the case of two competitors studied in section 4.3, we observed that the consumers and merchants react accordingly to movements of the price, which is consistent with the findings in [16]. Nevertheless, by using the agent-based methodology, we had the possibility to go further and to reproduce the complex dynamics of competition among larger number of card issuers. Thus, in the experiment presented in section 6.2, where we study the cases of profit maximisation in markets with five and nine card issuers separately, we noticed that in the case of nine competitors fees charged to consumers are considerably higher. In a way that our findings imply that competition through a larger number of competitors in non-interconnected networks is not necessarily in the interest of consumers [67].

7.2.4 Methodology for studying the payment card market

In this thesis we have proposed a novel methodology for studying the payment card market. We have structured the interactions among consumers, merchants and card issuers by assuming that the different kind of participants' decisions are taken with different frequencies through time. In addition we have incorporated in the model a geographic location of consumers and merchants.

The structure of the simulation is based on time dependent decisions. To this end, we explicitly reproduce the commercial transactions at the point of sale among consumers and merchants, in order to capture the properties of the market dynamics. Consumers and merchants are located in a lattice, as presented in chapter 3 and their interactions are built on three decisions. The first two are the consumers' decisions "where to shop?" and "which card to use", which are taken in each time period, whereas the third decision is "to which card to subscribe" and it is taken by consumers and merchants after certain number of commercial transactions have occurred[1].

In addition, the competitors' strategic decision "how to price their card" is incorporated in the model through machine learning mechanism presented in chapter 5. This decision is taken with the lower frequency and it is modeled under a normative framework.

In this way, the methodology proposed, incorporated two concepts in the study of

[1]Each consumer and merchant take this decision with different frequency.

the payment card market: the time dependent decisions and the physical location of consumers and merchants. It also combines a descriptive simulation, in the case of consumers and merchants interactions, and a normative one, in the case of the card issuers' strategies. Using this framework we can easily incorporate into the model (or if necessary substitute) other consumers', merchants' and card issuers' decisions, which will allow us to investigate different aspect of the market.

7.2.5 Payment card provider's strategies

Different instantiations of the $\mathcal{APCM}$ open the opportunity to study a variety of aspects of the payment card market. In this context, in this thesis we proposed the application of the Generalised Population Based Incremental Learning (GPBIL) algorithm in order to explore the search area formed by the interactions among a large number of heterogeneous individuals: consumers, merchants and card issuers. The aim of the heuristic was to design a card issuer's strategy, under specific criteria. For illustrative purposes we developed an experiment, in which the evolved strategy was required to achieve the highest possible profit and to obtain better than the average market share measured in terms of card transactions.

In chapter 6 in section 6.2, we present the result from the search. We concluded that the algorithm has successfully found strategies that compete well under the given criteria. In addition, in section 6.3 we verify that the evolved strategies are better than randomly generated strategies.

7.3 Limitations

Laws and rules in the society form the framework for individuals in order to relate to each other. Nevertheless, interactions among people create complex phenomena far beyond the personal decisions and the original framework of rules. Nowadays, due to the technological development of the computational tools, some of these phenomena could be studied with certain accuracy, but the abstract idealization of them remains a difficult task.

In this context, the $\mathcal{APCM}$ model has achieved to certain extent the representation of the consumers' and merchants' decisions in a commercial transaction. For instance, in the equations presented in section 3.3 we have selected certain factors and we have established relations among them. Nevertheless, there exist variety of ways, in which these factors could relate to each other. Nevertheless, given the limitation of time and possibilities we have explored only one possibility.

In addition the factors involved in the competitors', the consumers' and merchants' decisions are modeled with multiple parameters. These parameters have an extended domain and the possible trajectories of the simulation are countless. In this thesis, we have limited our experimentation to certain number of trajectories.

Furthermore, in the real market, the preferences of the consumers and merchants could change over time. In the artificial market, we observed in chapter 4 in section 4.2 that the modeled behaviour of the consumers and the merchants produces a stationary process. Furthermore, under different sets of parameters the simulated the market dynamics will produce different stationary processes.

Another important limitation is the fact that the domains of the strategies' elements have not been adjusted to numbers closer to reality. For instance, the consumers' fixed fees and the merchants' fixed fees could have completely different domains in the real world, nevertheless in our experiment we have used the same domains for both of them. This limitation could explain why in our model the consumers' fixed fees are the main source of the competitors profit, when in the real world the relevant source for the issuers are usually the merchants fixed fees.

In general we can conclude that the use of agent-based models requires extensive experimentation, which needs computational and other resources and considerable amounts of time. The scope of any project is bound by these constrains. Nevertheless, the use of agent-based models allows us to capture the emerging properties of the market and further our understanding of the payment card industry.

7.4 Future Work

Despite the fact that the competition in the payment card market is crucial issue, many aspects of it still remain unclear. In this context, the combination of the agent-based modelling and the machine learning techniques have opened vast opportunities in studying the market competition from the perspectives of economics, business, law and policy making. For instance, the results presented in chapter 6 imply that having more competitors in the market increase the price for the consumers, as it is getting more difficult for card holders to switch from one issuer to another. We also observed that the lower the competitors' target market share, the higher the prices of the cards. In this sense, will be interesting to investigate which will be the implications on the market competition, if the card issuers learn how to maximise profit having constrains on market share, which is different from the one explored in this thesis.

For instance, suppose we have the same setting for consumers and merchants. If we repeat the experiment with 2, 5 and 9 competitors presented in chapter 6, we can study what will be the resulting price structure if the competitors consider only the profit maximisation in their objective function. Another possible variation could be to repeat the experiment with 2, 5 and 10 competitors and update the distribution probability function used by the GPBIL with exactly the same number of solutions for each case. In other words, in the case of 2 competitors the probability distribution function will be updated every 5 runs, in the case of 5 competitors, the function will be updated every 2 runs and finally in the case of 10 competitors the probability distribution function will be updated in each run. In this way, the probability distribution function will be updated having in consideration 10 different solutions, regardless the number of competitors.

On the other hand, the model could be improved by increasing the degree of heterogeneity in each group of market participants. For instance, the consumers could have different level of income. Furthermore, we can divide consumers in groups according to their income and we can establish different behaviour rules for each group. Similarly, the merchants could have different degree of market power and their behaviour rules could be established accordingly. We can increase as well the heterogeneity of the card issuers by assigning initially a specific market share for each competitor. Nevertheless, establishing behaviour rules requires a specific market data, which either is very difficult to obtain or it is expensive.

Another possible line of research could be focussed on the interchange fees. In the payment card market, the optimum level of interchange fee is a matter of serious discussions among researchers. Economists and business managers believe that different levels of interchange fees will have an impact on the economic efficiency of the market outcome, [1]. For that reason, a relevant extension of the model will be to split the card provider entity in two different kind of agents: the issuer that negotiates with consumers and the acquirer that is deling with the merchants. Consequently we will need to incorporate into the price structure the interchange fee and study how different levels of this fee impact the market dynamics. In this context, a special attention must be given into the relationship between the interchange fee and the final prices established by the competitors [30].

Furthermore, given the setting of four party scheme (consumers, merchants, issuers and acquirers), in the model could be incorporate a policy maker agent. This agent could be used to explore different scenarios under possible regulation policies in order to prevent undesirable consequences of them.

In general, given that the $\mathcal{A}$rtificial $\mathcal{P}$ayment $\mathcal{C}$ard $\mathcal{M}$arket incorporates substantial numbers of participants and represents realistically their relationships, it allows the researcher to explore different aspects of the market dynamics. For instance, in the model we can incorporate budget constrains on the consumers' and the merchants' side; we can also study the affect of the elasticity of demand, when the merchants side exhibits the lower sensibility; in addition we can consider imperfect competition on the merchant side and compare our findings with the results of [28]; furthermore calibrating the model with empirical data will give us more insights in the market nature.

As a final remark, we can say that scientists have come long way through in the process of modeling complex social phenomena. Due to the use of computational power and tools we have the opportunity to incorporate more realistic features in our models in order to reproduce explicitly the interactions among individuals. Nevertheless representing the reality as such, still remains a remote possibility.

Appendix A

Data obtained from the Experiments

The appendix consists of the main results obtained in the four mayor experiments presented in the thesis. To that end we have attached a compact disc with the related data. The disc is organised in four folders with the following structure.

Folder Name Experiment_section_4.2

This folder contains the following files:

APCM-20000Interactions-Example1.xls

APCM-20000Interactions-Example2.xls

APCM-20000Interactions-Example3.xls

APCM-20000Interactions-Example4.xls

APCM-20000Interactions-Example5.xls

APCM-20000Interactions-Example6.xls

APCM-20000Interactions-Example7.xls

APCM-20000Interactions-Example8.xls

APCM-20000Interactions-Example9.xls

APCM-20000Interactions-Example10.xls

Folder Name Experiment_section_4.3

This folder contains the following files:

APCM-analysisDemand-2cards.xls

APCM-analysisDemand-5cards.xls

APCM-analysisDemand-9cards.xls

APCM-PaymentPerformance-2cards.xls

APCM-PaymentPerformance-5cards.xls

APCM-PaymentPerformance-9cards.xls

Folder Name Experiment_section_6.2

This folder contains the following files:

Graph_Strategy_2Cards_LocalConnections.xls

Graph_Strategy_5Cards_LocalConnections.xls

Graph_Strategy_9Cards_LocalConnections.xls

Graph_Strategy_9Cards_RandomConnections.xls

Graph_Strategy_9Cards_SmallWorldConnections.xls

Table_Strategy_2Cards_LocalConnections.xls

Table_Strategy_5Cards_LocalConnections.xls

Table_Strategy_9Cards_LocalConnections.xls

Table_Strategy_9Cards_RandomConnections.xls

Table_Strategy_9Cards_SmallWorldConnections.xls

Transactions_2Cards_LocalConnections.xls

Transactions_5Cards_LocalConnections.xls

Transactions_9Cards_LocalConnections.xls

Transactions_9Cards_RandomConnections.xls

Transactions_9Cards_SmallWorldConnections.xls

Folder Name Experiment_section_6.3

This folder contains the following file:

EfficientStrategy.xls

The files are organized into categories, which are distinguished by name. In each category of excel files, the data is presented in the following sheets.

Folder Name Experiment_section_4.2

File Name APCM-20000Interactions-Example*Number*.xls

Sheet Name Totals

This sheet contains the data related to the transactions of cash and the market share of all electronic payments.

Sheet Name Transactions

The sheet contains the graphical representations of the cash transactions and several figures of the market share after specific number of interactions.

Sheet Name OnlyCards

This contains the data related to the share of each electronic payment method from the set of transactions performed by card.

Sheet Name Performance

This sheet contains the figure of the transactions performed by each card.

Sheet Name Card*Number*

The sheet contains three figures: the number of transactions the number of consumers and the number of merchants related to the card in question.

Folder Name Experiment_section_4.3

File Name APCM-analysisDemand-*Number*cards.xls

Sheet Name ConsumerFixedFee

Sheet Name MerchantFixedFee

Sheet Name ConsumerBenefits

Sheet Name MerchantBenefits

Sheet Name PublicityCost

These sheets contain the variations of the market share in terms of number of transactions, consumers and merchants, after specific change of the corresponding strategy element. Each sheet has ten examples.

File Name APCM-PaymentPerformance-*Number*cards.xls

Sheet Name Analysis

This sheet represents all variations of the market share in terms of number of transactions, consumers and merchants, after specific change of the each strategy element. The sheet contains ten examples.

Folder Name Experiment_section_6.2

File Name Graph_Strategy_*Number*Cards_*Kind*Connections.xls

Sheet Name Sheet 1

The sheet contains 50 figures organized in 5 columns and 10 rows. In each row is presented one of the 10 examples obtained from the experiment. In each column is presented one of the five elements of the strategy.

File Name Table_Strategy_*Number*Cards_*Kind*Connections.xls

Sheet Name Initial-FinalStrategy[2]

This sheet contains a comparison among the average chosen strategies in the first 100 runs and the average chosen strategies in the last 100 runs.

Sheet Name FinalStrategy*Number*Cards*Kind*

In this sheet is presented the selected strategies of the last 100 runs. The data is obtained from the 10 examples of the experiment.

Sheet Name *Number*Cards*Kind*Network

In this sheet are presented some statistics related to the strategies obtained from the first and last 100 runs for each payment.

File Name Transactions_*Number*Cards_*Kind*Connections.xls

Sheet Name Example*Number*

In this kind of sheet for each example four figures are presented. The figures correspond to the cash and card transactions in the first and final runs.

Folder Name Experiment_section_6.3

File Name EfficientStrategy.xls

Sheet Name Sheet1

In this sheet are included the data and the figures obtained in experiment that tests the efficiency of the strategy presented in chapter 6 section 6.3.

These folders and files are the contained in the compact disc attached at the end of the thesis.

Appendix B

The effect of the fixed price on the Merchants' side

In this appendix we present a complementary experiment of the test explained in section 4.3. In the mentioned section we presented the results obtained from the changing in consumer fixed fees, whereas here we present the data obtained from the changes in merchant fixed fees. We have organized the section as follows. First we explain briefly the experiment and we give the parameter setting, next we present the measurements used to assess the outcome of the study, and finally we present the results of the test.

The Parameter setting

We start by recalling the list of symbols used to represent the elements of the strategy. Next, we explain how we have conducted the test and finally we present the setting of the parameters.

Symbol	Description
F_p	Consumer Fixed Fee
Γ_p	Merchant Fixed Fee
b_p	Benefits of the Consumers
β_p	Benefits of the Merchants
l_p	Publicity Cost

The experiment consists of three scenarios with different number of competitors $N_{\mathcal{P}} \in [2, 5, 9]$. For each scenario we have executed five cases. In each case the payment card issuers are using the same price. We distinguish from one case to another, because the merchants fixed fees have a different price level. The *starting case* uses the strategy, presented in the table, that presents the unique payment card strategy. In each one of the remaining four cases the merchant fixed fees take one of the following values $[0, 1, 3, 4]$. In other words, the merchant fixed fees are decreased with 50% and 100% and consequently increased with 50% and 100%. The rest of the strategy elements remain without changes.

F_p	Γ_p	b_p	β_p	l_p
4	2	0.3	-0.3	7

Additionally, each case is executed ten times with a different random seed. The random seed is used to ensure that the values of the simulation parameters are kept the same and the only change in the model is the change in the level of the merchant fixed fees and the number of competitors. For our analysis we report the average number of consumers per card and the average number of merchants per card. Given the procedure we have followed, for each scenario these two figures[3] represent the marginal change on the consumers' and on the merchants' sides arising from the changes in the merchant fixed fees.

The number of interaction is set to $\mathcal{I} = 3000$. The decision period of consumers and merchants is determined by a poisson distribution with $\lambda = 20$. The rest of the parameters are presented in two tables: the first lists the parameters setting for the agents, whereas the second shows the constants[4].

Symbol	Description	Cardinality	Value
$\mathcal{M}$	Set of Merchants	$N_{\mathcal{M}}$	125
$\mathcal{C}$	Set of Consumers	$N_{\mathcal{C}}$	1100
$\mathcal{M}_c$	Consumers' Set of Merchants	$N_{\mathcal{M}_c}$	5

Measuring the outcome of the experiment

In this subsection we present the notation of the variables used to evaluate the outcome of the experiment.

[3]The average number of consumers per card and the average number of merchants per card

[4]We use exactly the same consumers' and merchants' inertia as in section 4.3

Symbol	Description of the Constants	Value
ε	common constant for the inertia to change	1
x_c^-	accounting for the consumers' inertia to drop cards	0.05
x_c^+	account for the consumers' inertia to add new cards	2
x_m^-	account for the merchant' inertia to drop cards	0.05
x_m^+	account for the merchant' inertia to add new cards	9
α	account for the impact of the publicity cost	0.1
φ	account for the impact of the publicity cost	5

- $\Delta\mathit{\Gamma}_p$ - The percentage of changes in the fixed price;
- $N_{\mathcal{M}_p^*}$ - The average number of merchants per card, calculated over ten executions;
- $N_{\mathcal{C}_p^*}$ - The average number of consumers per card, calculated over ten executions;
- $N_{\mathcal{M}_p^*}^{log}$ - The average number of merchants per card, on logarithmic scale;
- $N_{\mathcal{C}_p^*}^{log}$ - The average number of consumers per card, on logarithmic scale;
- $N_{\mathcal{M}_p^*}^{N_{\mathcal{P}}}$ - The estimated over $N_{\mathcal{M}_p^*}^{log}$ average number of merchants per card, where $N_{\mathcal{P}}$ is the number of competitors in the market;
- $N_{\mathcal{C}_p^*}^{N_{\mathcal{P}}}$ - The estimated over $N_{\mathcal{C}_p^*}^{log}$ average number of consumers per card;
- $E_{\mathit{\Gamma}_p}$ - The demand sensitivity on the merchants' side;
- $\mathcal{E}_{\mathit{\Gamma}_p}$ - The demand sensitivity on the consumers' side arising from changes in the consumers fixed fees;

The effect of changes in the merchants fixed fees

In this section we present the results obtained from the five cases of different levels of merchant fixed fees. In table following we have summarised the results of the experiment. In addition the two figures show respectively the demand of the merchants and the consumers in logarithmic scale given the changes in the merchant fixed fees.

$N_{\mathcal{P}}$	Γ_p	$\Delta\Gamma_p$	$N_{\mathcal{C}_p^*}$	$N_{\mathcal{C}_p^*}^{log}$	$N_{\mathcal{C}_p^*}^{N_{\mathcal{P}}}$	$N_{\mathcal{M}_p^*}$	$N_{\mathcal{M}_p^*}^{log}$	$N_{\mathcal{M}_p^*}^{N_{\mathcal{P}}}$
2	**0**	-100%	494	6.202536	6.187096	67	4.204693	4.201329
	1	-50%	469	6.150603	6.161618	53	3.970292	3.985337
	2	—	**456**	**6.122493**	**6.136141**	**44**	**3.784190**	**3.769345**
	3	50%	450	6.109248	6.110663	35	3.555348	3.553353
	4	100%	444	6.095825	6.085185	28	3.332205	3.337361
5	**0**	-100%	397	5.983936	5.975831	47	3.850148	3.840062
	1	-50%	387	5.958425	5.965251	38	3.637586	3.650292
	2	—	**383**	**5.948035**	**5.954670**	**32**	**3.465736**	**3.460522**
	3	50%	382	5.945421	5.944090	26	3.258097	3.270752
	4	100%	379	5.937536	5.933509	22	3.091043	3.080982
9	**0**	-100%	337	5.820083	5.816523	35	3.555348	3.556419
	1	-50%	333	5.808143	5.809596	30	3.401197	3.403222
	2	—	**330**	5.799093	**5.802669**	**26**	**3.258097**	**3.250024**
	3	50%	328	5.793014	5.795743	22	3.091043	3.096827
	4	100%	328	5.793014	5.788816	19	2.944439	2.943630

Appendix C

Augmented Dickey-Fuller Test

Appendix D

Statistics of the Linear Regression Models

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