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Trading Algorithms Built with Directional Changes

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***Abstract*—Algorithm trading has become more and more important to financial markets. Most existing algorithms use time series as input. Instead of relying on physical time, Directional Changes (DC) focus on the price reversion events where the reversion reaches a certain magnitude, which is referred to as the threshold. In this paper, we propose two trading algorithms based on DC – TA1 and TA2. TA1 is also based on the Average Overshoot Length scaling law (AOL). An Overshoot refers to the event of price continuing to change in the current direction before the next reversion takes place. The AOL states that on average the Overshoot length is approximately equal to the threshold of DC. We have designed two DC based trading algorithms: TA1 takes advantage of the AOL and T2 takes profit with a more conservative criteria. By testing the algorithms with five stock market indices, the results suggest that in most scenarios, the algorithms are able to generate a positive outcome. The input arguments can be changed in order to change the performance of the algorithms, so TA1 and TA2 could be tailored to trade in different markets.**

***Index Terms*—Directional Changes, AOL scaling law, trading algorithms**

I. Introduction

Directional Change (DC) is an approach to summarise the financial data as a series of upward trends and downward trends by a pre-determined percentage price change – threshold θ [8]. Under the DC framework, upward and downward trends are delimited by extremum (EXT). In an upward trend (uptrend), the Directional Change event starts at an extremum, and ends when the price is $\theta\%$ above the extremum; this is referred to as a Directional Change Confirmation (DCC) point. The trend ends at the extremum from which price drops by the same threshold $\theta\%$. The event of price changing from the DCC to the next extremum is called the Overshoot (OS) event. Similarly, in a downward trend (downtrend), the DC event starts at an extremum, and ends at a DCC point when the price is $\theta\%$ below the extremum. A trend thus comprises a DC event and an OS event. The Total Movement (TM) in a trend is the percentage price changes between the beginning and the ending extremum of the trend, divided by the threshold.

According to the Average Overshoot Length scaling law (the AOL scaling law), on average, the Overshoot length is approximately equal to the threshold – θ [8]. To explore this property of the AOL scaling law, in this paper, we are going to introduce two trading algorithms – Trading Algorithm 1 (TA1) & Trading Algorithm 2 (TA2), which are built based on the AOL scaling law.

We refer to θ, α and β as the arguments of the trading algorithms. To evaluate the effectiveness of the trading algorithms, TA1 and TA2 are tested with different input of arguments. As a standard measure of the trading algorithms, the rate of returns are calculated and compared.

As shown in later sections, both TA1 and TA2 are able to generate a positive profit. And changes in arguments could vary the outcome of both TA1 and TA2 significantly. Furthermore, TA1 – the algorithm strictly based on AOL scaling law, which is to expect the Average of Overshoot lengths to be the threshold, is generally doing better than the other one, TA2, which uses the median (as opposed to mean) of Overshoot lengths.

The remainder of the paper is organised as follows: the second section will review some of the work related to Directional Changes. The third section introduces the Trading Algorithm TA1 & TA2. The fourth section will explain the thinking behind the experiments and how the experiments will be set-up. The fifth section will list the results obtained from the experiments. While the sixth section will interpret the results, which leads to the conclusion section.

II. Literature Review

Electronic trading had transformed the financial market and provided many trading possibilities. Algorithmic trading had taken transformation one step further [1]. It not only facilitated high-frequency trading but also strengthened the powers and reach of traders. Both information acquisition and processing ability of traders were radically improved, and the reaction to change of the markets was also faster [7].

When introducing trading algorithms, some studies, such as [14], [18], examined the financial markets using physical time. However, there were researchers whom claimed that there were limitations to using physical time. For example, [11] claimed that using physical time in forecasting tools in the study of the financial markets made time series discontinuous.

To avoid discontinuous time series, the use of event-based time such as Directional Changes could be one of the solutions [2].

Avoiding data holes was not the only reason to use DC. In [11], the authors addressed the problem of forecasting trend directions in the foreign exchange market using physical time. By adopting DC, they aimed to answer the question of whether the current trend would continue for a specific percentage before the trend ended.

Combined with Genetic Programming, a trading algorithm to forecast the financial market price movements was created by [11]. The results from the experiments presented in their paper indicated that this new framework was able to generate profitable trading strategies based on Directional Changes.

A Directional Change based foreign exchange forecasting algorithm (DBA) was developed by [4]. The accuracy was over 80%. Yet, under particular settings the accuracy may not outperform dummy prediction. Nevertheless, the most significant conclusion from this work was that Directional Changes were predictable.

Later, [3] proposed a trading strategy based on DC. And under particular settings, the trading strategy was able to make profits in the foreign exchange market.

Unlike the traditional ways of summarising financial data, Directional Changes sampled data by events where transaction took place. The frontier of Directional Change research was discussed by [16], which included forecasting, algorithmic trading and market tracking [15], [17].

Recently, [6] introduced a trading strategy based on Directional Change, named TSFDC. It was used in predicting the change of directions of market trends. And the result showed that the strategy outperformed the buy-and-hold strategy and some other DC based strategies.

[10] presented a contrarian trading strategy based on DC. The algorithm increased its position as OS reached each extra threshold. The threshold was adjusted to reflect the direction of the trend. The algorithm experienced big drawdown overall, but yielded high returns in the final four years of the test.

[12] studied the Chinese market with DC based volatility measures, and assessed a DC-based trading algorithm. Preliminary research suggested that positive cumulative returns could be achieved through the maximum drawdowns, and they could be substantial.

[5] presented an improved version of the Dynamic Backlash Agent (DBA), which was a trading strategy based on DC, namely Intelligent Dynamic Backlash Agent (IDBA). The result showed that the IDBA significantly performed better than its predecessor.

III. Methodology

In this section, we would like to introduce the Average Overshoot Length scaling law (the AOL scaling law) and two trading algorithms which are called TA1 and TA2. Both trading algorithms are DC based. TA1 is based on the AOL scaling law findings, while TA2 explores a variation of the AOL scaling law. And in the following sections, the results of testing both trading algorithms will be shown and compared.

A. The Average Overshoot Length Scaling Law

The AOL Scaling Law is presented as:

$$\langle|\Delta x^{OS}|\rangle = \left(\frac{\theta}{C_{x,OS}}\right)^{E_{x,OS}}$$

where $\langle\rangle$ is the average operator. $\Delta x = (x_i - x_{i-1})/x_{i-1}$ and $x_i = x(t_i)$ is the price at time t_i [9]. Δx^{OS} is the price change in an Overshoot, $\langle|\Delta x^{OS}|\rangle$ is the mean absolute price change of OSs. θ denotes the Directional Change threshold. And parameters $C_{x,OS}$, $E_{x,OS}$ are constants to be determined; the subscripts (x, OS) indicate that the parameters are related to the price x and Overshoot OS.

As shown in the paper [9], the AOL scaling law suggests, on average, a Directional Change is followed by an Overshoot with the same magnitude. That is the average length of the Overshoots is about the same size as the threshold (θ). To be more specific, that is, $\langle|\Delta x^{OS}|\rangle \approx \theta$. And according to [9], $C_{x,OS} \approx 1.06$ and $E_{x,OS} \approx 1.04$. It, on average, makes the total movement double the size of the Directional Change it is associated with. Therefore this could also be denoted as: $\langle|\Delta x^{TM}|\rangle \approx 2\theta$.

B. Trading Algorithm 1

Trading Algorithm 1, TA1, is built on the AOL scaling law, which is derived from DC. It is backed by the idea that, on average, the Overshoot lengths are approximately equal to the threshold θ that defines the Directional Changes and Overshoots [9]. TA1 serves as proof of concept. That is, to prove the AOL scaling law are able to compose trading algorithms that make profits. In this paper, therefore, for simplicity, TA1 takes long positions only.

The algorithm TA1 comprises three trading rules, one opening rule and two closing rules. The opening rule would be: opening a long position at an upward Directional Change Confirmation point (an upward DCC). A position is opened exactly once in every upward TM. Then the position is held until one of the following two conditions is satisfied. The first condition is: the price goes down by α, in which case TA1 closes the position to stop loss. The second condition is: the price goes up by another θ, in which case TA1 closes the position to make a profit.

Trading Algorithm 1 could be presented as:

$$TA1 \equiv (\theta, \alpha) \tag{1}$$

where θ is the threshold used to define Directional Changes and Overshoots, α is a pre-set number to control the loss. In TA1, we make $0 < \alpha < \theta$.

As shown above, TA1 is defined by two arguments, θ and α. To better illustrate the rules, we denote the current price as P^t, for each TM there are a price at its extremum (EXT) (denoted P^{EXT}) and a price at the Directional Change Confirmation point (DCC) (denoted P^{DCC}).

Therefore, the above three rules could be listed below:

Rule 1: When $\frac{P^t - P^{EXT}}{P^{EXT}} \geq \theta$,
open a long position;

Rule 2: When $\frac{P^t - P^{DCC}}{P^{DCC}} \leq -\alpha$,
close the position (stop loss);

Rule 3: When $\frac{P^t - P^{DCC}}{P^{DCC}} \geq \theta$,
close the position (take profit).

The first rule is the entry rule of TA1, that is opening a long position when there is an upward Directional Change confirmation. In other words, when the current price P^t is $\theta\%$ higher than the price at an EXT (P^{EXT}), take a long position.

The second rule is a closing rule. If the situation expected by rule 3 does not happen before the next Directional Change[1], and the price drops over a certain degree (by α) then it is considered the timing to stop losing. In cases that Rule 2 triggers, the price would not go up by another θ before it drops by α, as a result, the algorithm would hold the position till the price to go down by $\alpha\%$. This is when the price P^t is $\alpha\%$ lower than P^{DCC}.

According to the AOL scaling law, on average the price is expected to increase after an upward DCC by another θ [9]. The third rule would be triggered if the price does go up by another θ from an upward DCC; that is when P^t is $\theta\%$ higher than P^{DCC}. Triggering Rule 3 would close the position and take profit.

These three rules make sure that a position would be opened when there is an upward Directional Change confirmed. And this position would be closed either when the price goes up by another θ, or decreases by α. For example, if θ is set to 0.05 and α is set to 0.025. The algorithm would open a long position when there is an upward 5% Directional Change confirmed. This position would be held until one of the following happens: 1) the price goes down by 2.5% or more; 2) the price goes up by another 5% or more[2]. In other words, before the next upward Directional Change takes place, when the price either goes up by another 5% or when it hits the 2.5% downward marker, either Rule 2 or 3 is going to be triggered.

C. Trading Algorithm 2

The difference between Trading Algorithm 1 (TA1) and Trading Algorithm 2 (TA2) is that TA1 always expect the mean value of the Overshoots – average Overshoot length, TA2, however, is going to use the median of Overshoot lengths instead.

The algorithm could be shown as:

$$TA2 \equiv (\theta, \alpha, \beta) \tag{2}$$

where θ is the threshold used to define Directional Changes and Overshoots, α and β are pre-set numbers to cut loss or to make profits respectively. In TA2, we make $\alpha < \theta$, β is the median of the Overshoot lengths corresponding to the chosen threshold θ. α, β and θ are bigger than 0.

As shown above, TA2 is defined by three arguments: θ, α and β. θ is the threshold used to find Directional Changes. α is the argument used to stop loss. θ and α are just similar to their counterparts in TA1. However when making profits, β is the argument used instead of θ.

[1]Two connecting Directional Changes are in two different directions by definition.

[2]Prices may not be continuous, the position would be closed at the closest price that makes the price change greater or equal to 5%

TA2 is built similar to TA1, it also comprises three trading rules. They are 1) the opening rule: opening a long position at an upward Directional Change Confirmation point (DCC), 2) the stop-losing rule: closing the position when the price goes down by α, 3) the profit-taking rule: closing the position when the price goes up by β from the DCC. When there is a long position, TA2 no longer take another long position. TA2 holds the long position until one of the closing rules is triggered.

Rule 1: When $\frac{P^t - P^{EXT}}{P^{EXT}} \geq \theta$,
open a long position;
Rule 2: When $\frac{P^t - P^{DCC}}{P^{DCC}} \leq -\alpha$,
close the position (stop loss);
Rule 3: When $\frac{P^t - P^{DCC}}{P^{DCC}} \geq \beta$,
close the position (take profit).

Where P^t is the current price. The extremum (EXT) price (the starting extremum of an TM) is denoted as P^{EXT}, and the price at the DCC is shown as P^{DCC}.

These rules allow the algorithm to open a long position when an upward Directional Change event is confirmed. It expects the price to increase by the median of Overshoot lengths – β, with a certain tolerance (α) of down-going of the price.

By design, Trading Algorithm 2 opens a long position at an upward Directional Change confirmation point. And if the price goes down and reaches the stop-loss point where the price P^t is $\alpha\%$ lower than P^{DCC}, Rule 2 will be triggered, the long position is closed. If the price P^t goes $\beta\%$ above the DCC price (P^{DCC}), TA2 closes the position to make profit.

IV. Experiment set-up

This section presents the data to be tested, the input of the arguments that are going to be used in testing TA1 and TA2. Also, the choice of thresholds and medians of Overshoot lengths are shown later in this section. Lastly, the way to evaluate the trading algorithms will be introduced.

A. Data

In this paper, five sets of stock indices are used to test the trading algorithms. They are the FTSE 100, Hang Seng, NASDAQ 100, Nikkei 225 and S&P 500. We use daily closing indices (treated as prices) starting from 2nd January 2009 to 1st November 2013.

B. Input of Arguments

To see how the trading algorithms perform with different input of arguments and to examine how the change of input of the arguments affect the performance of each trading algorithm; there are four sets of input for the arguments used to test TA1 and TA2 respectively. For example, for TA1, the tested input sets are $\theta = 0.05$, $\alpha = 0.02$; $\theta = 0.05$, $\alpha = 0.025$; $\theta = 0.1$, $\alpha = 0.02$ and $\theta = 0.1$, $\alpha = 0.05$ as shown in Table I. Similar Table II shows the equivalent input sets for TA2. The other arguments α is set to 0.02 or half of the threshold so it can be compared across all input sets.

TABLE I
TESTED ARGUMENTS OF TA1

θ	0.05		0.1	
α	0.02	0.025	0.02	0.05

TABLE II
TESTED ARGUMENTS OF TA2

θ	0.05		0.1	
α	0.02	0.025	0.02	0.05
β	0.0361		0.0903	

C. Choice of Thresholds

As proof of concept, in this paper, the selection of thresholds are relatively arbitrary. However the thresholds are related to the data size and the average daily return. The average daily return is roughly 0.01 and the data size is about 5 years' daily return. Together they delimit the choices of thresholds.

First, the thresholds cannot be too small. The data we adopt is daily data, and it is discontinous. If the threshold is too small and very close to the average daily price change, or even smaller than the average daily price change, the Directional Changes are going to be confirmed too often, and the actual price change during a Directional Change event might be much greater than the threshold chosen.

For example, it is known that the average daily price change rate is roughly 1%; if the threshold is set to 1%, for each price tick with a price change that is equal or greater than 1% there is a Directional Change confirmed. This is fine if the price change is exactly 1% or the price is continuous. However, the price is not continuous in this case and there is chance that the price change is greater than 1%. When the difference between two ticks (two daily prices) exceeding 1% by far, say a 1.9%, it is still confirmed as a 1% Directional Change. To put it another way, the actual price change in from a EXT to a DCC may be much bigger than the chosen threshold. If that is the case, the AOL scaling law may simply not hold, since the Overshoot should be approximately equal to the actual price change in a Directional Change event. And in this case the actual price change in each Directional Change event varies potentially significantly.

Second, the threshold cannot be too big. This is simply because if the thresholds are too big, there are not statistically enough Directional Changes to run our tests.

As a result, the thresholds used to test the algorithms and to calculate medians of Overshoot lengths are 0.05 and 0.1 which are roughly 5 to 10 times to the average daily price change. Although they are relatively arbitrary chosen, as proof of concept, we consider they satisfy the purpose reasonably.

D. Medians of Overshoot Lengths

Table III lists the medians of Overshoot lengths at different thresholds in a percentage form.

As shown in the table, the medians are quite different from each other even with the same threshold. For the threshold

TABLE III
MEDIANS OF OVERSHOOT LENGTHS

Data Set	Median (θ=5%)	Median (θ=10%)
FTSE 100	3.44%	6.19%
Hang Seng	4.03%	7.90%
Nasdaq 100	4.52%	5.98%
Nikkei 225	4.01%	10.31%
S & P 500	2.07%	14.76%
Average	3.61%	9.03%

0.05, all the medians are smaller than θ. Most of them are around 4%, with a minimum S & P (2.07%). For the 10% threshold, Nikkei (10.31%) and S & P (14.76%) go over 10% (bigger than the threshold), while the rest of them are roughly 5 to 8%.

E. Evaluating the Trading Algorithms

Testing a trading algorithm is all about whether it produces a profit [13]. The essence of the evaluation of the trading algorithms is to calculate the rate of returns for each trading algorithm. One of the purposes of this paper is to prove that the trading algorithm based on the AOL scaling law is able to generate positive income. And this could be judged by looking at the rate of returns created by the trading algorithms.

Since TA1 is strictly based on the AOL scaling law, and TA2 is modified from TA1 and not based on the AOL scaling law. Therefore, the other thing we can look at is the comparison of the rates of returns between the two algorithms. It may give us insight about the AOL scaling law in making trading algorithms.

We also want to examine the changes of the outcomes of both algorithms when using different input for the arguments. Therefore, the trading algorithm TA1 and TA2 are tested using the input listed in Table I and II respectively.

When testing TA2, the medians of Overshoot lengths corresponding to threshold 0.05 and 0.1 are calculated before testing TA2 (Table III).

V. EXPERIMENT RESULTS

The rate of returns for TA1 and TA2 are shown below, as the difference between the rates of returns. Beside, the AOL to threshold ratio and the overall returns of the indices are presented. The number of times that the rules are triggered is also displayed.

A. Trading Algorithm 1

TABLE IV
RATE OF RETURNS OF TA1

Data Set	Rate of Return of TA1(θ, α)			
	$\theta = 0.05$		$\theta = 0.1$	
	$\alpha = 0.02$	$\alpha = 0.025$	$\alpha = 0.02$	$\alpha = 0.05$
FTSE 100	6.91%	2.49%	-14.93%	22.26%
Hang Seng	25.60%	15.11%	21.14%	24.54%
Nasdaq 100	52.36%	62.60%	3.94%	5.95%
Nikkei 225	-2.27%	-9.39%	14.80%	6.94%
S & P 500	3.62%	16.42%	4.58%	-0.22%
Average	17.24%	17.45%	5.91%	11.89%

As TA1 is also presented as $TA1 \equiv (\theta, \alpha)$ we use TA1(0.05, 0.02) to represent that TA1 is tested with $\theta = 0.05$ and $\alpha = 0.02$. When the numbers change in the brackets, its meaning varies accordingly.

Table IV shows the rate of returns obtained by TA1, the thresholds are set to 0.05 and 0.1. The other argument of TA1 – α is set to 0.02 and 0.025 for threshold 0.05. For the other threshold 0.1, α is set to 0.02 and 0.05. The first argument of α is fixed at 0.02 for both θs, and the second α is set to half of the threshold. So they are 0.025 and 0.05. In other words, there are four combinations tested: TA1(0.05, 0.02), TA1(0.05, 0.025), TA1(0.1, 0.02) and TA1(0.1, 0.05).

From the fourth row of the table, the rate of returns of TA1 for each data set tested with an argument combination is listed. The last row lists the average returns of all data sets with each argument combination.

B. Trading Algorithm 2

TABLE V
RATE OF RETURNS OF TA2

Data Set	Rate of Return of TA2(θ, α, β)			
	$\theta = 0.05$		$\theta = 0.1$	
	$\alpha = 0.02$	$\alpha = 0.025$	$\alpha = 0.02$	$\alpha = 0.05$
	$\beta = 0.0361$		$\beta = 0.0903$	
FTSE 100	1.66%	-4.30%	-14.93%	18.88%
Hang Seng	23.58%	14.51%	20.08%	23.45%
Nasdaq 100	44.94%	54.13%	3.47%	4.79%
Nikkei 225	0.90%	-5.68%	13.45%	22.47%
S & P 500	1.33%	11.07%	3.33%	-1.41%
Average	14.48%	13.95%	5.08%	13.64%

Similar to TA1, we use TA2(0.05, 0.02, 0.0361) to represent TA2 using arguments $\theta = 0.05$, $\alpha = 0.02$ and $\beta = 0.0361$. The numbers in the brackets change accordingly when the arguments vary.

TA2 is also tested with two thresholds – $\theta = 0.05$ and $\theta = 0.1$. Argument α is the same as it is in testing TA1. β is the average median of Overshoot lengths across five data sets at $\theta = 0.05$ and $\theta = 0.1$. Since β is the replacement for θ closing the position when making profit. Therefore, there are two βs listed in Table V. That is, β is used instead of θ when Rule 3 is triggered, so there are only two β needed.

TABLE VI
DIFFERENCE OF PERFORMANCE BETWEEN TA1 AND TA2

Data Set	Rate of Return of (TA1-TA2)			
FTSE 100	5.25%	6.79%	0.00%	3.38%
Hang Seng	2.02%	0.60%	1.06%	1.09%
Nasdaq 100	7.36%	8.47%	0.47%	1.16%
Nikkei 225	-3.17%	-3.71%	1.35%	-15.53%
S & P 500	2.29%	5.35%	1.25%	1.19%

C. Other Results

Table VII lists the average Overshoot lengths over threshold ratio. As can be seen in the table, the ratios are very close to 1. That means the average Overshoot lengths are indeed approximately equal to the thresholds.

TABLE VII
AVERAGE ESTIMATED AOL OVER θ

Data Set	AOL/θ
FTSE 100	0.100
Hang Seng	0.996
Nasdaq 100	1.112
Nikkei 225	1.096
S & P 500	0.903

Table VIII shows the overall returns for each data set tested. They are the growth rate over the whole tested period – from 2nd January 2009 to 1st November 2013.

TABLE VIII
OVERALL RETURNS IN EACH TESTED DATA SET

Data Set	Overall Return
FTSE 100	47.63%
Hang Seng	54.56%
Nasdaq 100	167.45%
Nikkei 225	57.04%
S & P 500	89.06%
Average	83.15%

Table IX and X are tables showing how many times Rule 2 and Rule 3 are triggered in TA1 with threshold 0.05 and threshold 0.1 respectively. In both tables α is set to either 0.02 or half of the threshold. Rule3/2 denotes the ratio of numbers rule 3 triggered over numbers rule 2 triggered.

TABLE IX
RULES TRIGGERED AND RULE 3 TO RULE 2 RATIOS FOR TA1

Data Set	Rate of Return of TA1(θ, α)					
	$\theta = 0.05$					
	$\alpha = 0.02$			$\alpha = 0.025$		
	Rule2	Rule3	Rule3/2	Rule2	Rule3	Rule3/2
FTSE 100	13	8	0.62	12	9	0.75
Hang Seng	14	12	0.86	14	12	0.86
Nasdaq 100	8	12	1.50	7	13	1.86
Nikkei 225	17	9	0.53	17	9	0.53
S & P 500	13	8	0.62	10	10	1.00
Average	13	9.8	0.82	12	10.6	1.00

TABLE X
RULES TRIGGERED AND RULE 3 TO RULE 2 RATIOS FOR TA1

Data Set	Rate of Return of TA1(θ, α)					
	$\theta = 0.1$					
	$\alpha = 0.02$			$\alpha = 0.05$		
	Rule2	Rule3	Rule3/2	Rule2	Rule3	Rule3/2
FTSE 100	6	0	0.00	2	3	1.50
Hang Seng	4	3	0.75	3	4	1.33
Nasdaq 100	5	2	0.40	4	3	0.75
Nikkei 225	5	3	0.60	4	3	0.75
S & P 500	2	1	0.50	2	1	0.50
Average	4.4	1.8	0.45	3	2.8	0.97

Similar to the previous two tables, Table XI and XII are the TA2 equivalents to Table IX and X. And the tested threshold are 0.05 and 0.1 as well. However, what different is that TA2 expects the median instead of the mean of Overshoot lengths.

TABLE XI
RULES TRIGGERED AND RULE 3 TO RULE 2 RATIOS FOR TA2

Data Set	Rate of Return of TA2(θ, α, β)					
	$\theta = 0.05$					
	$\alpha = 0.02$			$\alpha = 0.025$		
	$\beta = 0.0361$					
	Rule2	**Rule3**	**Rule3/2**	**Rule2**	**Rule3**	**Rule3/2**
FTSE 100	12	9	0.75	11	10	0.91
Hang Seng	13	14	1.08	13	14	1.08
Nasdaq 100	6	14	2.33	5	15	3.00
Nikkei 225	16	11	0.69	16	11	0.69
S & P 500	12	9	0.75	9	11	1.22
Average	11.8	11.4	1.12	10.8	12.2	1.38

TABLE XII
RULES TRIGGERED AND RULE 3 TO RULE 2 RATIOS FOR TA2

Data Set	Rate of Return of TA2(θ, α, β)					
	$\theta = 0.1$					
	$\alpha = 0.02$			$\alpha = 0.05$		
	$\beta = 0.0903$					
	Rule2	**Rule3**	**Rule3/2**	**Rule2**	**Rule3**	**Rule3/2**
FTSE 100	6	0	0.00	2	3	1.50
Hang Seng	4	3	0.75	3	4	1.33
Nasdaq 100	5	2	0.40	4	3	0.75
Nikkei 225	5	3	0.60	3	4	1.33
S & P 500	2	1	0.50	2	1	0.5
Average	4.4	1.8	0.45	2.8	3	1.08

VI. INTERPRETATION OF RESULTS

To exploit the AOL scaling law, we proposed the trading algorithm TA1(θ, α). The original thought on α is to set it to half of θ. That is, the algorithm waits for the price to go up by another θ from an upward DCC as long as the prices do not drop half of a threshold.

TA1(0.05, 0.025) is used as the bench mark. As can be seen in Table IV (3rd column), TA1(0.05, 0.025) can make a profit for all data sets except Nikkei 225. And it reaches a 17.45% average rate of return.

When the threshold is doubled (TA1(0.1, 0.05), shown in 5th column Table IV), the algorithm can still make a profit for most of the data sets except S & P. However, the average rate of return is 11.89% which is smaller than TA1(0.05, 0.025)'s 17.45%. What is more, performance of the two algorithms could vary in different data sets. For example, the rate of return for FTSE 100 is 2.49% at TA1(0.05, 0.025), and it is 22.26% at TA1(0.1, 0.05). And TA1(0.05, 0.025) makes a 62.60% profit while TA1(0.1, 0.05) makes 5.95% with Nasdaq 100. The other difference is that TA1(0.05, 0.025) loses -9.39% while TA1(0.1, 0.05) gains 6.94% with Nikkei 225, and TA1(0.1, 0.05) loses -0.22%, TA1(0.05, 0.025) gains 16.42% with S & P 500.

It does seem like that although TA1(0.1, 0.05) is outperformed by TA1(0.05, 0.025) on average, it does not apply to each individual data set. One possible explanation is that different arguments are more suitable for different data sets.

As TA1(0.05, 0.025) is only made to compare with TA1(0.05, 0.02), and its α is only 0.005 smaller than TA1(0.05, 0.025), the outcome does not exhibit a big difference on average. They are 17.25% and 17.45% correspondingly. However, it is a surprise to see how they are different with individual data sets. The smallest difference is with FTSE 100, the rates of returns are 6.91% and 2.49% for TA1(0.05, 0.02) and TA1(0.05, 0.025). The biggest difference is roughly 13% with S & P 500.

The difference between TA1(0.05, 0.02) and TA1(0.05, 0.025) could imply that only a small twist in the arguments could lead to a comparatively big change in the rate of return.

As for TA1(0.1, 0.02) and TA1(0.1, 0.05), the difference becomes bigger on average. Individually, TA1(0.1, 0.05) does better than TA1(0.1, 0.02) with FTSE 100, Hang Seng and Nasdaq 100, and worse with Nikkei 225 and S & P 500.

The other comparison we would like to make with Table IV is between TA1(0.05, 0.02) and TA1(0.1, 0.02), their rates of returns are shown in 2nd and 4th column respectively. On average, the rate of return of TA1(0.05, 0.02) is 17.24% and it is 5.91% for TA1(0.1, 0.02). This comparison emphasises that effect that the algorithms take when the ratio of $\frac{\alpha}{\theta}$ changes. As can be seen in the table, the rates of returns generated by TA1(0.1, 0.02) are almost always worse than what TA1(0.05, 0.02) generates except for Nikkei 225 and S & P 500. Although, on average TA1(0.1, 0.02) is much worse than TA1(0.05, 0.02), this could be caused by the positively skewed rate of return of Nasdaq 100.

On average, TA1(0.1, 0.02) is the worst argument set. One possible explanation could be that TA1(0.1, 0.02) misses more profitable chances that the price increases another θ% from an upward DCC as the tolerance is 1/5 to the threshold. That is the argument set is comparatively much more sensitive to the price drop (risk averse).

To summarise, there does not seem to be a universal best argument set with in TA1, for each individual data set, there is a certain combination of arguments that can generate a comparatively better outcome.

Similar to TA1, we would also like to make few comparisons for the results generated by the arguments of TA2 as shown in Table V. Apart from TA2(0.1, 0.05, 0.0903) with S & P, the tables presents highly correlated numbers. Similar conclusion might be drawn from the comparison within TA2 using different arguments. However, an overview of the two tables would display another picture, that is TA2 is outperformed by almost every argument set, compared to TA1. Note that basically β in TA2 replaces θ in TA1.

As mentioned before, the major difference between the two algorithms is that TA2 is not closely following the AOL scaling law. As shown in Table VII, the average Overshoot lengths is roughly equal to the threshold θ. As the average is also the expected number statistically, using the medians may close the position prematurely or too late.

As can be seen in Table VI, apart from Nikkei 225 with some arguments, TA1 always performs better than TA2. These results suggest that following the AOL scaling law (TA1) tightly tend to give a better performance than not (TA2).

This is also presented in Table IX to XII. In Table IX and XI, the numbers of rules triggers does not exhibit significant difference. The Rule 3 to Rule 2 ratios for TA2 is always higher than TA1's. This could be explained by the fact that the median Overshoot is smaller than the average Overshoot lengths, and the tolerance argument α is kept the same. Therefore, Rule 3 has a higher chance of being triggered in TA2 than in TA1. Despite Rule 3 is triggered more often, it is not enough compensate the lost rate of returns due to closing the position before the expected values.

The situation listed in Table X and XII is another display. The rules triggered for both TA1 and TA2 is essentially the same except TA1(0.1, 0.05) and TA2(0.1, 0,05, 0.0903) with Nikkei 225. The reason for this might be that the comparatively big arguments could not react to small changes of the price. Since the price is not continuous, they close positions at similar points, that is when the position is closed by Rule 3, the price is already bigger than both the average and the median of the Overshoot length. And at 0.1 threshold with Nikkei the medians are bigger than the threshold for this argument combination as shown in Table III, so that, in this particular case, one more Rule 3 is triggered.

As proof of concept, and it is shown that the trading algorithms built on the AOL scaling law are proven to be able to make money, and by changing the arguments of the trading algorithms the outcome could be adjusted.

VII. Conclusion

This paper has introduced the two trading algorithms – TA1 & TA2, which are built based on Directional Changes. TA1 is also built with the AOL scaling law. TA1 comprises three rules. It opens a long position at an upward DCC, and hold the position till the price either goes down by $\alpha\%$ (to stop loss) or goes up by another $\theta\%$ (to take profit). Like TA1, TA2 opens a position at an upward DCC, and hold the position till either the price goes down by $\alpha\%$ or goes up by $\beta\%$, where β is the median of Overshoot lengths. TA1 and TA2 are tested with five data sets, and different argument (θ, α and β) combinations.

From the results, we can conclude that, first, in most cases TA1 and TA2, the algorithms built with DC are able to generate positive outcomes (making money). Second, there does not seem to be a set of arguments to suit to all data sets. Third, their performance could be adjusted by varying the arguments, even a very small change in the arguments could lead to a relatively big change in the rate of returns. Fourth, in this experiment, the algorithm based on the AOL scaling law outperforms the one without the AOL scaling law in general. Better understanding of the markets would help setting arguments to suit individual markets. Research in profiling [17] and market regimes [15] could help in this aspect.

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