# Hedging Volatility Dispersion Portfolios: A Comparative Analysis

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**Abstract:** Dispersion trading is a form of highly quantitative volatility trading that attempts to exploit relative mispricings between options on ETFs and options on the component assets of those ETFs. Trading opportunities are identified by relating the implied volatilities of component asset options to the implied volatilities of ETF options using Markowitz portfolio theory. After identifying viable trading opportunities, dispersion traders build portfolios of offsetting option positions and then hedge these portfolios to reduce their exposure to market risk. This research quantifies and compares the relative performances of four hedging strategies across four real-world volatility dispersion portfolios and three simulated market conditions. Portfolio outcomes are explained intuitively and justified within a quantitative financial framework. Conclusions are drawn regarding dispersion analysis, portfolio performance, order sizing, the effect of commission fees, and optimal hedging strategies. Primary results indicate that portfolio profit is strongly related to inter-asset correlations and delta-hedging is generally effective in reducing the volatility of portfolio profit.

**Keywords**: Options, ETF, Volatility Arbitrage, Dispersion Trading, Correlation, Risk Management, Hedging, Derivatives, Quantitative Finance, Markowitz Portfolio Theory

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# 1. Introduction

### 1.1 Background

Dispersion trading seeks to profit from relative volatility mispricings that exist between options on portfolios of assets, generally exchange traded funds (ETFs), and similar options on all component assets in those portfolios. Once a significant mispricing has been detected, traders build dispersion portfolios of mispriced options and manage market risk using various hedging techniques [1]. Existing academic literature does not provide an adequate comparison of relative performances for each viable hedging strategy. It is possible that certain hedging strategies outperform others under certain market conditions, or that one hedging strategy is strictly superior in all circumstances. The primary research presented in this paper quantitatively compares the performances of four hedging strategies across four real-world dispersion portfolios and three simulated market conditions. Outcomes are interpreted and recommendations are made regarding portfolio hedging. Preliminary conclusions are also drawn regarding order sizing and the effect of commission fees.

Dispersion trading emerged as a distinct trading methodology during the early 2000's as a natural extension of the statistical arbitrage pair-trading strategies prevalent throughout the 1990's. Early pioneers of dispersion trading, generally buy-side hedge funds, borrowed ideas regarding inter-asset correlation and long-short<sup>1</sup> portfolio construction from their statistical arbitrageur predecessors, and applied them to the analysis of relative option valuation [1]. They further drew on the fundamentals of portfolio theory, as formalized by Harry Markowitz, most notably the equation relating portfolio variance to the weighting of the assets within the portfolio and the inter-asset covariance relationships. Markowitz's paper "Portfolio Selection", published in the Journal of Finance in 1952, mathematically derives the formula for the variance of a portfolio shown in equation (1) as a matrix multiplication [2].

<sup>&</sup>lt;sup>1</sup> Long-Short Portfolios: Portfolios which combine purchased securities, known as "long" positions, with sold securities, known as "short" positions.

$$\sigma_{p}^{2} = \begin{pmatrix} w_{1} & w_{2} & \dots & w_{n} \end{pmatrix} \begin{pmatrix} \text{Covar}(1,1) & \text{Covar}(2,1) & \dots & \text{Covar}(n,1) \\ \text{Covar}(1,2) & \text{Covar}(2,2) & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ \text{Covar}(1,n) & \dots & \dots & \text{Covar}(n,n) \end{pmatrix} \begin{pmatrix} w_{1} \\ w_{2} \\ \dots \\ w_{n} \end{pmatrix}$$
(1)

Where:

 $\sigma$  = Portfolio volatility wi = The weight of component i Covar(i,i) = The variance of component i Covar(i,j) = The covariance between components i and j

Versatile quantitative option pricing models also play a crucial role in volatility dispersion analysis. With the advent of the Black and Scholes model in the 1970's, fair prices for European option contracts could be calculated based on their strike price, the underlying asset price, the risk free interest rate, the time to maturity, and most importantly the volatility of the underlying asset [3]. Risk management procedures mathematically derived from the Black and Scholes formulas were suggested in subsequent academic works. The most important of these procedures, known as delta-hedging, involves calculating the first derivative of the Black and Scholes option price with respect to changes in the underlying asset price, known as delta, and then buying or selling delta shares of the asset underlying the option contract. Delta is recalculated frequently throughout the life of the option and the number of shares traded on the underlying asset is adjusted accordingly [4]. The profitability of the resulting portfolio is dictated by the realized volatility of the underlying asset during the life of the option contract, as opposed to directional movements in the price of the underlying asset. While the mean expected profit for a delta-hedged option is almost the same as the mean expected profit for a naked option, assuming zero transaction costs and a reasonably accurate volatility parameter used to calculate delta, the volatility of expected profit is generally much lower when an option has been deltahedged [4]. In order to exploit this new form of risk-averse volatility trading, options traders began using the Black and Scholes equations in reverse to calculate the underlying asset volatility implied by the pricing model subject to the option's observed market price. This

"implied volatility" measures an option's price relative to the market's expectation of future realized volatility on the underlying asset [4].

Volatility traders compare an option's implied volatility to the expected realized volatility on the underlying asset, take an appropriate position in the option contract and delta-hedge the option until expiration. When the volatility implied by the price of an option is less than the expected realized volatility of the underlying asset, traders buy the option and delta-hedge the long option position accordingly [4]. Figure 1 shows the relationship between realized underlying asset volatility and profit for a long position in an option which has been delta-hedged. Conversely, when the volatility implied by the price of an option is greater than the expected realized volatility of the underlying asset, traders sell the option and delta-hedge the short option position accordingly. Figure 2 shows the relationship between realized underlying asset volatility and profit for a short position in an option which has been delta-hedged. Profitability on an option contract which has been bought and delta-hedged increases as realized volatility increases, while profitability on an option contract which has been sold and delta-hedged decreases as realized volatility increases.



Long Delta-Hedged Option

Realized Volatility

Figure 1. Effect of realized volatility on profit for a long position in an option which has been delta-hedged. Profitability is strongly positively correlated to realized volatility.



Realized Volatility

Figure 2. Effect of realized volatility on profit for a short position in an option which has been delta-hedged. Profitability is strongly negatively correlated to realized volatility.

### **1.2 Dispersion Trading Fundamentals**

### **1.2.1 Volatility Dispersion Analysis**

Dispersion traders combine the fundamentals of option volatility trading and Markowitz portfolio analysis to relate the value of ETF options, which represent options on a portfolio of assets, to the value of options on each of the component assets in that ETF. A covariance matrix interrelating the ETF portfolio components is constructed, generally using historical asset prices, in addition to a matrix containing the weights of each component within the portfolio [5]. Traders then choose an ETF option contract and pair that option with a similar option on each ETF portfolio component asset. Implied volatilities are calculated for all options chosen by the trader. Next, the implied volatilities from the options on the component assets are inserted into the covariance matrix along the diagonally bisecting axis corresponding to the variances of each component asset. A modified Markowitz portfolio variance is then calculated using the modified covariance matrix and the matrix of ETF component asset weights as shown in equation (2). Finally, the modified Markowitz portfolio variance is compared to the implied volatility of the chosen ETF option including a dispersion term beta, and portfolios of appropriate positions in the ETF option, and some number of the component asset options, are constructed [5].

$$Imp. Vol (ETF) + \beta = \begin{pmatrix} w_1 & w_2 & \dots & w_n \end{pmatrix} \begin{pmatrix} Imp. Vol (1) & Covar(2,1) & \dots & Covar(n,1) \\ Covar(1,2) & Imp. Vol (2) & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ Covar(1,n) & \dots & \dots & Imp. Vol (n) \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \\ \dots \\ w_n \end{pmatrix}$$
(2)

Where:

 $\beta$  = Dispersion term wi = The weight of component i Imp. Vol(i) = The implied volatility of component i converted into variance Covar(i,j) = The covariance between components i and j

### **1.2.2 Portfolio Construction and Expected Profit**

When beta is positive, meaning the implied volatility of the ETF option is less than the modified Markowitz portfolio volatility, dispersion traders buy the ETF option and sell similar options on some number of the component assets. When beta is negative, meaning the implied volatility of the ETF option is greater than the modified Markowitz portfolio volatility, dispersion traders sell the ETF option and buy similar options on some number of component assets. In order to achieve profitability, each leg of the trade, namely short positions and long positions, must be sized properly in absolute terms and relative to the other positions in the portfolio. Market impact and commission fees must be considered, as they may restrict the maximum or minimum viable order size per contract. Issues of order sizing, market impact and commission fees are discussed in more depth in the methodology section of this paper. Unfortunately, judgements about the relative value of individual options on component assets cannot be easily made, as individual asset volatilities would need to be accurately predicted. Dispersion traders must therefore take

the same position in all component asset options in order to fully exploit the value of theoretical mispricings. This can be difficult to accomplish when considering ETFs with large numbers of underlying assets, for example those tracking the S&P 500, as efficiently executing orders on 500 component options is nearly impossible under real trading conditions. Transaction costs may also restrict the number of options on which dispersion bets can be placed. In most cases, dispersion traders settle for taking option positions across some subset of component assets.

The performance of a volatility dispersion portfolio behaves differently from the performance of a single delta-hedged option in some important ways. As discussed previously, the profitability of a single delta-hedged option is directly related to the realized volatility of the underlying asset. While realized volatility does have an impact on dispersion portfolio returns, the effect is lessened by the inclusion of offsetting long and short option positions. In situations where component assets become more volatile, volatility on the ETF generally increases as well. In situations where component assets become less volatile, volatility on the ETF generally decreases as well. In both cases, one leg of the dispersion trade benefits from the changes in volatility and one leg of the dispersion trade suffers, resulting in more stable relationships between realized volatility and profit. Reduced exposure to realized volatility risk is one of the most attractive characteristics of dispersion portfolios [6]. Dispersion portfolios are, however, subject to significant correlation risk as a result of the strategy's reliance on the Markowitz portfolio variance equation. Trades that looked profitable under a certain set of assumed inter-asset covariances may result in losses if realized covariances differ significantly from the assumed relationships.

In order to appreciate the interaction between the realized covariance matrix and the profitability of the dispersion portfolio, it is important to understand the ways in which inter-asset relationships affect the volatility of a portfolio. For any given set of component asset volatilities and weights, the volatility of the index increases as the sum of the covariances between the component assets increases. Conversely, index volatility approaches a minimum of zero when the sum of the covariances approaches zero. Put more intuitively, when returns on the components of the index are largely uncorrelated or hedge each other perfectly, positive returns on some components are offset by negative returns on other components, thereby decreasing the overall volatility of index returns. This is the primary benefit of proper portfolio diversification

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espoused by Markowitz in his "Portfolio Selection" paper [2]. Conversely, when the returns on the components of the index are strongly correlated, returns on a single component asset are generally associated with similar returns on many other component assets, thereby increasing the overall volatility of index returns. Therefore, in cases where ETF options have been bought and component options have been sold, profitability increases as the sum of the covariances increases, driven by increased profitability on the long ETF option positions, and profitability decreases when the sum of the covariances approaches zero, driven by decreased profitability on the long ETF option positions. In cases where ETF options have been sold and component options have been bought, profitability decreases as the sum of the covariances increases, driven by decreased profitability on the short ETF option positions, and profitability increases when the sum of the covariances approaches zero, and profitability increases when the sum of the covariances approaches zero, driven by increases increases, driven by decreased profitability on the short ETF option positions, and profitability increases when the sum of the covariances approaches zero, driven by increased profitability on the short ETF option positions [5].

Expected profit for a volatility dispersion portfolio can be calculated using any expected covariance matrix. First, Black and Scholes fair values for each option are calculated and mispricings are determined subject to bid and ask prices. Each option's mispricing is then multiplied by a scalar that accounts for order sizing, commission fees and market impacts. Long option positions have a mispricing equal to the option's Black and Scholes fair value minus the ask price for the contract. Short option positions have a mispricing equal to the bid price of the contract minus the option's Black and Scholes fair value. ETF options should be priced using the Markowitz portfolio volatility calculated using the expected individual asset volatilities. While the effects of commission fees and market impact are explored later in this work, for the sake of clarity they have been ignored in equations (3) and (4). The scalar applied to each mispricing therefore represents the number of contracts bought or sold for each option and is denoted with the letter "*C*".

Long ETF options, short component options:

$$Portfolio \ Profit = (B\&S_{(EIF)} - Ask_{(EIF)})C_{(EIF)} + \sum_{i=1}^{n} (Bid_{(Component i)} - B\&S_{(Component i)})C_{(Component i)}$$
(3)

Short ETF options, long component options:

$$Portfolio \ Profit = (Bid_{(ETF)} - B\&S_{(ETF)})C_{(ETF)} + \sum_{i=1}^{n} (B\&S_{(Component i)} - Ask_{(Component i)})C_{(Component i)}$$
(4)

Where:

Bid(i) = Current best bid price for option i Ask(i) = Current best ask price for option i B&S(i) = Black and Scholes price for option i C(i) = Number of contracts traded on option i

Previous research by Cara Marshall empirically verified the existence of profitable dispersion trading opportunities on large equity indexes subject to commission fees, market impact and spreads in quoted bid and ask prices [6]. The research presented in this paper partially verifies Marshall's findings.

### 1.2.3 Hedging and Risk Management

After building a portfolio of options, dispersion traders look for ways to control the risk associated with their positions. Typically, this involves delta-hedging each individual option contract, however more sophisticated methods involving the use of variance swaps or volatility swaps have been examined in research by Izzy Nelken, who makes a strong case for their usefulness [5]. The research presented in this paper focuses on traditional delta-hedging protocols and the performance of those protocols under various market conditions.

Delta-hedging requires the calculation of delta, the first derivative of the Black and Scholes option price with respect to the underlying asset price. Due to the importance of the volatility parameter in the Black and Scholes pricing model, the delta calculated for an option can change significantly when different volatilities are considered. Traders must therefore choose a volatility term for use in their delta calculations which performs best under the market conditions expected throughout the remaining life of the portfolio. Previous research conducted by Paul Wilmott and Ahmad Riaz into the performance of delta-hedging using various asset volatility parameters found that the standard deviation of final profit, or profit volatility, on delta-hedged options increased as the difference between the volatility parameter used to calculate delta and the realized volatility increased. The two primary volatility parameters used by Wilmott and Riaz were realized volatility and implied volatility [7].

In the case of dispersion portfolios, traders are presented with a third distinct volatility parameter to choose from when calculating delta, namely the modified Markowitz portfolio volatility calculated using equation (2). Because the Markowitz portfolio volatility incorporates inter-asset relationships between all components of the portfolio, it may be useful for hedging dispersion trades in certain market conditions. Another, perhaps more elegant way to delta-hedge the dispersion portfolio, is to buy or sell at the money straddles on the ETF and its components. Straddles are constructed by taking the same position, either long or short, in a put and a call struck at the same price. As the delta exposure of a call is equal and opposite to that of a put when at the money, positions in the asset underlying the option straddle are not necessary when the portfolio is first constructed [4]. However, once the price of the underlying asset has moved away from the strike price, the straddle will no longer be self-hedged and the trader must begin taking positions in the underlying asset. Unfortunately, delta changes more rapidly for straddle positions than for individual options, as the gamma of the long position in both the call and the put is positive. As a result, traders attempting to keep their portfolios delta-hedged will need to adjust their position in the underlying asset more frequently [4].

# 2. Research Methodology and Preliminary Results

### 2.1 Choosing Assets

### **2.1.1 ETFs and Indexes**

Traders conducting volatility dispersion analysis must make a number of important decisions when building portfolios. First and foremost, appropriate ETFs must be chosen on which profitable dispersion trading opportunities exist. ETFs that track popular equity indexes are generally used, as they are characterized by higher liquidity and boast a wide variety of quoted option contracts [6]. Weighting methods used within the ETF portfolio are important to consider as well. Most equity indexes weight the component assets according to market capitalization<sup>2</sup>, which adds additional complexity to portfolio performance stress testing calculations and dispersion analysis techniques. Price-weighted indexes provide an attractive alternative for dispersion traders, as the weights of each component asset are directly related to current market prices. Preliminary research explored real-world trading opportunities on three price-weighted indexes, the Dow Jones Utility Average (DJUA), the Dow Jones Transportation Average (DJTA) and the Dow Jones Industrial Average (DJIA). Results indicated that viable trades on the DJUA, which is composed of 15 equities, were almost non-existent due to the very limited number of option contracts quoted on both the ETF tracking the index (IDU), and the component equities of the index. The few contracts that were quoted did not fulfill the option matching criteria that were established for this research. Option matching criteria are discussed in detail later in this section. Viable trades did however, exist on both the DJTA, which is composed of twenty equities, and the DJIA, composed of thirty equities. Trades on the DJIA were, on average, significantly more profitable than trades on the DJTA, due to much smaller spreads in quoted bid and ask prices. Average quoted spreads for options on the ETF tracking the DJTA (IYT), and the components of the DJTA, were between two and three times larger than average quoted spreads for options on the ETF tracking the DJIA (DIA), and the components of the DJIA, depending on the day. These differences in observed liquidity seem reasonable given the popularity of the

<sup>&</sup>lt;sup>2</sup> Market Capitalization: The total value of outstanding shares in a company.

DJIA and the relative obscurity of the DJTA. Despite higher profitability on DJIA portfolios, the DJTA was chosen as the primary research portfolio in order to reduce the number of simulated assets in the portfolio from thirty-one to twenty-one, thereby reducing the time necessary for portfolio simulation. Four portfolios on the DJTA were constructed and subjected to stress testing. Quoted spreads on options in two of these portfolios were artificially narrowed in order to achieve profitability. In these cases, both bid and ask prices were changed by the same amount in order to maintain the mid-market price which was used to calculate implied volatility.

### **2.1.2 Component Assets**

The second important decision facing dispersion traders is how to choose which underlying assets to place bets on. As mentioned in the introduction, a trader will ideally place bets on all the assets in the ETF, however for indexes with a large number of component assets this can be nearly impossible. One method calls for the inclusion of the most heavily weighted assets in the ETF. In price-weighted indexes, these will be the assets with the highest prices. In cap-weighted indexes, these will be the assets with the largest market capitalization. In this way, traders hope to capture the majority of the inter-asset correlations used to inform their trade while simplifying their portfolio and minimizing transaction costs. Alternatively, the size of each option's quoted spread can be used to filter out positions which are likely to result in losses. If the spread on any individual contract exceeds a chosen threshold, no position in that option is taken. In the case of the DJTA, it is feasible to take an option position in each of the twenty component assets in the index, thereby avoiding the complications associated with asset subset choice. All four portfolios in this research were constructed in this way.

### 2.2 Matching Option Contracts

Next, dispersion traders must decide how to match ETF options with component asset options. Certain criteria are essential to the success of dispersion analysis and the profitability of the portfolio. European contracts must be used for all ETF and component asset options, calls must be matched with calls, puts must be matched with puts, and expiration dates must be universal across the entire portfolio. Relative moneyness must also be accounted for when matching ETF options with component options. Traders can match moneyness using simple ratios of strike price to current underlying asset price, moneyness relative to the volatility of the underlying asset, or by using the deltas for each option [5]. In practice, relative moneyness on exchange traded options, which are struck at regular price intervals, cannot be exactly matched. Therefore, an acceptable error margin when matching options must be established. Portfolios in this work were matched using simple ratios of strike price to current underlying asset price, as well as the previously mentioned criteria essential for dispersion analysis. Poorly matched portfolios were filtered out using equation (5).

$$Mean Matching Error = \underbrace{\sum_{i=1}^{n} \left( \frac{Strike Price(Component i)}{Asset Price(Component i)} - \frac{Strike Price(ETF)}{Asset Price(ETF)} \right)}_{n}$$
(5)

Where:

Strike Price(i) = Strike price on option i
Asset Price(i) = Current price of asset underlying option i

When the mean error in relative moneyness between the ETF option and the matched component options exceeded one percent, the portfolio was discarded. Additionally, when the standard deviation of moneyness across the matched component options exceeded two percent, the portfolio was discarded. Preliminary research indicated that these thresholds filtered out the majority of unprofitable portfolios while leaving a reasonable number of profitable trades to choose from. Between one-hundred-seventy and two-hundred unique option contracts were generally quoted on the DJTA index on any given day. Forty to fifty of those contracts could usually be matched with component options subject to these criteria.

### 2.3 Order Sizing

### 2.3.1 Relative Order Sizing

Finally, dispersion traders must determine optimal order sizes for each option contract in the portfolio. Orders must first be properly sized relative to the other order sizes within the portfolio. Relative order sizing between component assets should reflect relative shareholding ratios in the

ETF portfolio. In the case of a price-weighted equity index, in which a single share from each component equity is included in the ETF portfolio, dispersion traders should trade the same number of contracts on each component option. These component options must be balanced with an appropriate number of contracts on the ETF option in order to properly offset losses on either leg of the trade with profits on the other leg. An appropriate number of ETF options to be traded relative to the traded component options can be determined using a "Greek-equating" method. Traders using this method calculate the gamma<sup>3</sup> or vega<sup>4</sup> exposure of the basket of component options and then buy or sell ETF options until that exposure has been neutralized. Alternatively, the ETF leg of the trade can be balanced according to the summed weights of the component assets on which options were traded [5]. Note that the price of an ETF is almost never equal to the price of the index which it is designed to replicate. ETFs were designed to allow investors with limited capital to purchase shares in broad based index-style securities, and therefore have much lower share prices than the indexes they replicate. When considering a price-weighted index ETF, equation (6) can be used to calculate the number of ETF option contracts traded for every one contract traded on each component asset of the index.

Num. ETF Contracts = 
$$\sum_{i=1}^{n} \left( \frac{Asset Price(Component i)}{Asset Price(ETF)} \right)$$
(6)

This equation is derived from the calculation of the index price as a sum of the index's component asset prices and the ETF price as the quotient of the index price and some divisor. When the ETF price is equal to the index price, and the divisor is therefore equal to one, a single option contract should be traded on the ETF for every option contract traded on the entire basket of component options. If only a subset of component assets is chosen, a number of contracts equal to the sum of the weights for each chosen component assets. The ETF replicating the DJTA (IYT) has a divisor slightly greater than nine. Therefore, if option contracts were traded on all component assets in the index, approximately nine contracts would be traded on the ETF for every one contract traded on each component. If options were traded on a subset of component

<sup>&</sup>lt;sup>3</sup> Gamma: The second derivative of the option value with respect to changes in the underlying asset price [4].

<sup>&</sup>lt;sup>4</sup> Vega: The first derivative of the option value with respect to changes in the volatility parameter [4].

assets which comprised fifty percent of the index price, approximately four and a half options would ideally be traded on the ETF for every one option traded on each chosen component asset. Dispersion portfolios constructed for this research included trades on all twenty component assets in the DJTA as mentioned previously, and therefore included nine contracts on the ETF for every one contract on each component asset.

### 2.3.2 Absolute Order Sizing

After deciding on the relative sizing appropriate for each leg of the trade, dispersion traders must decide how many times to scale up the entire portfolio. Immediate market impact, also known as slippage, is the most important factor to consider when determining optimal absolute position sizing [6]. Slippage can be generally understood as the difference between the quoted best bid or ask price and the volume weighted average execution price (VWAP) realized for a buy or sell market order<sup>5</sup> respectively. If a trader places a market order to purchase one thousand contracts of an option, it is very likely that only some fraction of those thousand contracts can be purchased for the quoted best ask price. Once the limit order<sup>6</sup> at that best ask price has been completely filled, the remaining contracts in the trader's market order will be filled by limit orders sitting at progressively higher prices. By the time the market order has been completely executed, the average execution price per contract may be significantly higher than the quoted best ask price. The severity of slippage observed on a market order depends heavily on the market microstructure of the traded security [6]. Order books which are densely populated with large limit orders can absorb large market orders without significant slippage in execution price. In contrast, large market orders submitted to sparsely populated order books may be subjected to significant slippage in execution price. Slippage can be quantified as the change in filled price per executed contract. Suppose that the best ask price for an exchange traded security is currently one hundred dollars and a trader places a market buy order for ten units of that security. Assume that as the market order is filled, the executed purchase price increases by one dollar for each contract after the first. The total price paid for ten units of the security can now be calculated using an arithmetic sequence. The sum of an arithmetic sequence is defined by equation (7),

<sup>&</sup>lt;sup>5</sup> Market Order: An order to buy or sell an asset at the best available price. This order is filled immediately.

<sup>&</sup>lt;sup>6</sup> Limit Order: An order to buy or sell an asset at a certain price. This order may not be immediately filled, if at all.

which has been modified to reflect a market order of size "C", subject to some slippage parameter "S".

$$\frac{Total \ Transacted}{Order \ Value} = \frac{C}{2} \left( 2(Bid \ or \ Ask) + (C - 1)S \right)$$
(7)

If the total transacted order value is known, this equation can be rearranged, as shown in equation (8), and solved for "*S*" in order to quantify the realized slippage in executed price per contract.

$$S = \frac{2 (Total Transacted Order Value) - 2C(Bid or Ask)}{C^2 - C}$$
(8)

Where:

S = Slippage in executed price per contract

C = Size of market order in number of contracts

(Bid or Ask) = Current best bid or ask price for traded security

Financiers can use equations (7) and (8) in conjunction with a chosen slippage parameter to model the effects of immediate market impact on optimal order sizing. The parameter "*S*" should be defined according to empirical observations of the market microstructure for the asset in question. Assume an options trader believes an option contract is relatively cheap with respect to anticipated underlying asset volatility. That is, the trader believes the realized volatility of the underlying asset will be significantly higher than the implied volatility on the option. In order to take advantage of this trading opportunity, the trader will buy the option and delta-hedge it with the underlying asset. However, a decision regarding the number of contracts to be purchased remains to be made. If the market order is oversized, the VWAP will differ significantly from the best ask price which was used to calculate the implied volatility for the option. If the implied volatility calculated for the option using the VWAP, as opposed to the best ask price, exceeds the realized volatility on the underlying asset, the trader will lose money on the trade. Under sizing a

market order is also not optimal, as additional profit could be made on the trade by purchasing additional contracts at prices which are still significantly different from the option's fair value. It follows that the relationship between absolute profit and order sizing is certainly nonlinear.

Preliminary research found this relationship to be parabolic for profitable option trading opportunities when a constant positive slippage parameter was accounted for. In contrast, losses accrued by unprofitable opportunities accelerated exponentially with increasing order size. Similar results between profitability and order sizing were found for portfolios of options, in this case volatility dispersion portfolios. Figure 3 compares the performances of three profitable DJIA dispersion portfolios which were constructed by pairing the same ETF option with options on the largest ten, twenty and thirty component assets in the index. In all cases, portfolio profit reached a maximum at some definite order sizing scalar, however that optimal sizing scalar decreased as the number of component assets on which options were traded increased. Furthermore, the maximum profitability attained by the portfolio increased as the number of component assets on which options were traded increased. Finally, the rate at which portfolio profitability changed, with respect to changes in order sizing, increased as options were traded on larger numbers of component assets. Profitability decayed rapidly for portfolios that included options on large numbers of component assets, and more slowly for portfolios that included options on fewer component assets. These results are intuitive, as portfolios including options on a large number of component assets should capture more of the theoretical mispricing value detected in the volatility dispersion analysis, resulting in higher profit potential, while simultaneously exposing the portfolio to slippage losses on a larger number of assets. Figure 4 compares the performances of three unprofitable DJIA dispersion portfolios which were constructed by pairing the same ETF option with options on the largest ten, twenty and thirty component assets in the index. Losses accrued more gradually, with respect to order size, as the number of component assets on which options were traded decreased. Again, these results are intuitive, as these portfolios were not profitable to begin with and therefore having fewer losing positions, as well as fewer positions subject to slippage losses, proved to be advantageous.

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Figure 3. Effects of position sizing on the performance of profitable DJIA dispersion portfolios subject to a constant positive slippage in executed price per contract.





In the absence of a rigorous empirical study of the market microstructure observed for DJTA options, determining a reasonable slippage parameter is impossible. This work therefore chose to ignore the effects of immediate market impact and slippage. Because questions regarding optimal order sizing become meaningless under such assumptions, portfolios stress tested in this research used size scalars of one in all cases, meaning a single option contract was traded on each component asset in the index. In accordance with equation (6), nine contracts were therefore traded on the ETF in order to balance the portfolios.

### 2.4 Research Assumptions

A number of important assumptions were made when stress testing dispersion portfolios in this research. As mentioned in the previous section, slippage in execution price was ignored on the traded option contracts. Slippage in the equities underlying the traded option contracts, which are purchased and sold repeatedly during delta-hedging protocols, was also ignored. Although these assumptions do not entirely reflect reality, portfolios in which the largest option position consists of a mere nine contracts should be subject to very little slippage on average. While spreads between quoted bid and ask prices for option contracts were included in the stress testing calculations, quoted spreads for underlying equity assets were ignored. Only mid-market prices were simulated for each equity in the index. Without exception, the equities that comprise the DJTA are very liquid. Spreads between the quoted bid and ask prices for those assets tend to be no larger than two or three cents, which accounts for less than four basis points of the average asset price in the index. While paying a simulated spread would reduce average simulated profitability, the effect would be very slight. Option contracts were assumed to consist of onehundred options to buy or sell one stock. Under this assumption, the owner of one call option contract has the right to buy one-hundred shares of the underlying stock. All parameters associated with option contracts, including prices, payoffs and units of delta risk, were multiplied by one-hundred accordingly.

The final, and most noteworthy assumption made in this research, pertains to commission and trading fees. Initially, a proportional fee of fifteen basis points was included in the stress testing scenarios. Total transacted dollar value was multiplied by fifteen basis points and then either added to the final cost for the purchase of an asset, or subtracted from the final sale price of an asset. Portfolios of naked options remained mildly profitable under this proportional fee scheme, as proportional costs were paid only once and on the relatively small dollar values of option contracts. When delta-hedging was introduced however, profitability suffered dramatically, leaving no profitable opportunities. Losses were mainly due to the proportional fees paid on relatively high equity prices when first establishing and subsequently closing out delta-hedging

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positions. Further losses were accrued at every time step as option deltas were recalculated and positions in all twenty-one underlying assets were adjusted. In the interest of comparing profitable portfolio performances, commission fees were subsequently reduced to zero. Assessing the existence of real-world dispersion trading opportunities was not the primary objective of this research, therefore priority was given to the comparative analysis of relative portfolio performance under different hedging strategies and simulated market conditions. The delta-hedged portfolio outcomes, subject to a proportional commission fee, observed in this preliminary research provide empirical justification for Nelken's claim that most dispersion trades are conducted using at the money straddles [5]. Combining options on a common underlying asset in order to offset the necessary hedging position can seemingly save traders a significant amount of money.

### **2.5 Data Sourcing and Computational Methods**

Before constructing and stress testing dispersion portfolios, quoted prices on exchange-traded option chains were sourced online. Initially, option data was fetched from the Yahoo Finance website using an HTML parsing script. The validity of these quotes came into serious question however, as a significant number of large risk-free arbitrage opportunities were present in the fetched option prices. A new HTML parsing script was then written to fetch option quotes from Nasdaq, after which arbitrage opportunities completely disappeared. For the Matlab parsing function developed to fetch option chains from the Nasdaq website, see Appendix A.

In order to stress test dispersion portfolios on the DJTA, it was necessary to simulate the twenty component assets in the index over varying time periods. From these simulations, prices for the ETF asset (IYT) were extrapolated by summing the simulated prices of all twenty component equities at each simulated time-step and dividing by the ETF divisor discussed in the section on relative order sizing. Simulations were conducted using Matlab's multi-asset Monte-Carlo<sup>7</sup> simulator, "portsim". This function takes input arguments for each asset's expected return, the expected covariance matrix relating the assets, total time-frame to be simulated and the number of steps to simulate within that time-frame. Different market conditions can be simulated by

<sup>&</sup>lt;sup>7</sup> Monte-Carlo Simulation: A method for sampling discrete random outcomes of a continuous stochastic process [4].

simply altering the covariance matrix and expected return input arguments. Four different portfolios on the DJTA were stress tested under three different market conditions and subject to four different hedging protocols, yielding forty-eight unique variable combinations. For each combination of variables, the entire portfolio of twenty-one underlying assets was simulated tenthousand times over ten time-steps. These parameters were chosen in order to reduce the computational time needed to stress-test delta-hedging protocols while still providing a sufficiently large sample size on which to compute reliable portfolio performance statistics. Twelve Dell computers<sup>8</sup> running sixty-four bit Windows 7 operating systems were utilized in parallel, thereby reducing the time required to complete all experiments even further. Stress-tests took approximately one hour to compute independently when dynamic delta-hedging protocols were implemented.

As mentioned previously, component assets were simulated under three distinct market conditions using Matlab's "portsim" function. The first market condition was based on the assumptions that no true correlations exist between assets in the index and that component options are efficiently priced. An expected covariance matrix was constructed for this marketneutral method by inserting the implied variances from each component option contract down the diagonal axis of a twenty by twenty zeros matrix. Market-neutral simulation was used to test the performance of dispersion portfolios during times of low inter-asset correlation and therefore decreased volatility on the index. Figure 5 shows realized asset prices from a single portfolio simulation generated using method one. The second market condition was based on historical data, and used covariances and expected returns calculated from one year of adjusted closing prices for each component asset. Historically-based simulation served as the benchmark for portfolio performance under normal market conditions. Figure 6 shows realized asset prices from a single portfolio simulation generated using method two. The third market condition was created in two steps. First, historically-based simulations were generated using method number two as just described. An identical return shock was then applied to every asset at the same randomly chosen time period. The value of the return shock was drawn randomly for every simulation from a standard normal distribution and then multiplied by a scalar to create an

<sup>&</sup>lt;sup>8</sup> Each computer was equipped with an Intel Core i5-4590 processor clocked at 3.3 gigahertz and was installed with eight gigabytes of RAM. Matlab version 2015.a was used to run portfolio stress-testing routines on all computers.

average absolute shock value of six percent. Market-shock simulation was used to test the performance of dispersion portfolios during times of high inter-asset correlation and therefore increased volatility on the index. Figure 7 shows realized asset prices from a single portfolio simulation generated using method three. For the Matlab portfolio simulation routine, see Appendix B.



DJTA Portfolio Simulation (Market-Neutral Scenario)

Figure 5. A realized simulation of the equity portfolio replicating the DJTA and its associated ETF (IYT) under market-neutral conditions. Assets exhibit low correlations during this market condition.



Figure 6. A realized simulation of the equity portfolio replicating the DJTA and its associated ETF (IYT) under historical market conditions. Assets exhibit normal, moderate correlations during this market condition.



DJTA Portfolio Simulation (Market-Shock Scenario)

Figure 7. A realized simulation of the equity portfolio replicating the DJTA and its associated ETF (IYT) under market-shock conditions. Assets exhibit high correlations during this market condition.

Four distinct hedging protocols were used in stress testing calculations. The first method did not utilize delta-hedging, relying instead on naked options from each leg of the trade to hedge the risks associated with the other leg. Methods two, three and four delta-hedged each option contract using shares of the equities underlying those contracts. Method two used the historical volatility of each underlying asset to calculate delta at each simulated time step. Method three used the implied volatility calculated for each option contract at time zero, to calculate delta at each simulated time step. Method four used the modified Markowitz portfolio volatility calculated time step. Nethod for all option contracts at each simulated time step. Positions in equities underlying the option contracts were opened at time zero when the portfolios were first constructed, adjusted at time steps one through nine subject to new delta calculations, and closed out at the tenth and final time step. For the Matlab stress testing routine, see Appendix C.

# 3. Results

Four volatility dispersion portfolios on the DJTA were constructed from listed Nasdaq options and subsequently stress tested using Monte-Carlo simulation. Two of the constructed portfolios were built using call options and two were built using put options. Relative moneyness, as defined by the ratio of strike price to asset price, differed to some degree between the portfolios. Two portfolios consisted of options very close to the money, one portfolio consisted of options in the money and the last portfolio consisted of options out of the money. Time to maturity for each portfolio differed as well, ranging from thirteen days to forty-four days. Two portfolios were constructed using long positions in the selected ETF option and short positions in the matched component options, and two portfolios were constructed using short positions in the selected ETF option and long positions in the matched component options. Expected profit was also calculated for each portfolio using equations (3) and (4). Important portfolio information has been summarized in Table 1.

Contract Type	Strike Price / Asset Price	Days to Expiry	Markowitz Implied Vol	ETF Option Implied Vol	Position on ETF Option	Expected Portfolio Profit
ATM Call	0.9908	44	0.2105	0.2109	Short	97.36
ITM Call	0.9488	36	0.2143	0.1922	Long	216.45
ATM Put	0.9904	13	0.2096	0.2625	Short	863.94
OTM Put	0.9590	20	0.2134	0.1875	Long	47.39

#### Table 1. Option contract data for each portfolio constructed on the DJTA.

After simulations were run for each portfolio, results were compiled and descriptive statistics on portfolio performance were calculated. Tables 2, 3, 4 and 5 summarize the mean profit, volatility of profit, percentage of simulations that lost money, and expected shortfall for each portfolio subject to market condition and hedging protocol. Mean profit was calculated as the simple arithmetic average of final portfolio profit across all simulations. Volatility of profit was calculated as the standard deviation of the final portfolio profit across all simulations. The percentage of simulations resulting in a loss was calculated as the ratio of negative profit outcomes to total number of simulations. Expected shortfall was calculated as the simple arithmetic average of final portfolio profits across all simulations that ended with negative profits.

	Market	-Neutral (I	Low Corre	elations)	Histo	orical (Norm	Market-Shock (High Correlations)					
OPTION TYPE	ATM CALL	ATM CALL	ATM CALL	ATM CALL	ATM CALL	ATM CALL	ATM CALL	ATM CALL	ATM CALL	ATM CALL	ATM CALL	ATM CALL
HEDGING METHOD	NAKED	$\Delta$ (HIST)	$\Delta$ (IMP)	$\Delta$ (MARK)	NAKED	$\Delta$ (HIST)	$\Delta$ (IMP)	$\Delta$ (MARK)	NAKED	$\Delta$ (HIST)	$\Delta$ (IMP)	$\Delta$ (MARK)
RESULTS:												
Mean Profit	1990.6	1982.3	1980.5	1980.2	163.5515	101.5072	96.8534	109.7823	-141.4549	-201.2228	-203.5885	-195.9771
Volatility of Profit	1034.1	586.1793	581.4009	628.5681	1226	538.6236	561.0096	619.3393	1220.2	1004.3	978.7073	963.512
% Sims Lost Money	0.0237	0.0001	0.0001	0.0002	0.5113	0.4367	0.4607	0.4572	0.6264	0.563	0.591	0.5936
Expected Shortfall	-303.3414	-80.3673	-11.7199	-77.0773	-825.2225	-345.3826	-358.6851	-411.6603	-944.9044	-768.1389	-729.4074	-734.0547

 Table 2. Simulation outcomes for portfolio of at the money calls. ETF options were sold and component options were bought in this portfolio.

	Market	-Neutral (J	Low Corre	elations)	Hist	orical (Norm	Market-Shock (High Correlations)					
OPTION TYPE	ITM CALL	ITM CALL	ITM CALL	ITM CALL	ITM CALL	ITM CALL	ITM CALL	ITM CALL	ITM CALL	ITM CALL	ITM CALL	ITM CALL
HEDGING METHOD	NAKED	$\Delta$ (HIST)	$\Delta$ (IMP)	$\Delta$ (MARK)	NAKED	$\Delta$ (HIST)	$\Delta$ (IMP)	$\Delta$ (MARK)	NAKED	$\Delta$ (HIST)	$\Delta$ (IMP)	$\Delta$ (MARK)
RESULTS:												
Mean Profit	-752.3021	-765.4908	-768.3487	-761.1634	166.9404	211.1928	212.3801	193.4817	381.2273	414.0262	415.0046	397.1201
Volatility of Profit	999.4677	515.3989	453.9857	611.822	1030.6	414.3319	446.2829	552.4448	981.8131	908.2727	1060.8	839.8839
% Sims Lost Money	0.7525	0.9652	0.9855	0.9146	0.362	0.2401	0.2919	0.3334	0.281	0.2606	0.3408	0.2753
Expected Shortfall	-1138.2	-797.0854	-780.8186	-849.016	-1002.3	-322.5619	-259.4672	-427.4257	-970.2308	-436.8738	-455.7235	-481.123

Table 3. Simulation outcomes for portfolio of in the money calls. ETF options were bought and component options were sold in this portfolio.

	Market	-Neutral (I	Low Corre	elations)	Histo	orical (Norm	Market-Shock (High Correlations)					
OPTION TYPE	ATM PUT	ATM PUT	ATM PUT	ATM PUT	ATM PUT	ATM PUT	ATM PUT	ATM PUT	ATM PUT	ATM PUT	ATM PUT	ATM PUT
HEDGING METHOD	NAKED	$\Delta$ (HIST)	$\Delta$ (IMP)	$\Delta$ (MARK)	NAKED	$\Delta$ (HIST)	$\Delta$ (IMP)	$\Delta$ (MARK)	NAKED	$\Delta$ (HIST)	$\Delta$ (IMP)	$\Delta$ (MARK)
RESULTS:												
Mean Profit	1646.7	1650.6	1650	1649.7	870.9802	860.6782	859.9166	863.2466	538.8885	531.4227	540.5068	538.1125
Volatility of Profit	534.7621	325.1032	360.5448	345.4677	670.0242	293.3327	410.5508	346.5028	611.1829	1015.6	773.7933	923.4375
% Sims Lost Money	0	0	0	0	0.0163	0.0064	0.0032	0.0053	0.0258	0.1938	0.1455	0.1537
Expected Shortfall	0	0	0	0	-22.1947	-223.5285	-107.9506	-234.8873	-20.6687	-935.3	-651.6001	-907.4321

Table 4. Simulation outcomes for portfolio of at the money puts. ETF options were sold and component options were bought in this portfolio. Note that in the case of market-neutral simulation, this portfolio was profitable across all ten-thousand simulations.

	Market	-Neutral (	Low Corre	elations)	Histo	orical (Norm	Market-Shock (High Correlations)					
OPTION TYPE	OTM PUT	OTM PUT	OTM PUT	OTM PUT	OTM PUT	OTM PUT	OTM PUT	OTM PUT	OTM PUT	OTM PUT	OTM PUT	OTM PUT
HEDGING METHOD	NAKED	$\Delta$ (HIST)	$\Delta$ (IMP)	$\Delta$ (MARK)	NAKED	$\Delta$ (HIST)	$\Delta$ (IMP)	$\Delta$ (MARK)	NAKED	$\Delta$ (HIST)	$\Delta$ (IMP)	$\Delta$ (MARK)
RESULTS:												
Mean Profit	-739.1314	-733.8587	-735.685	-735.5814	28.3188	42.8406	37.3219	33.3711	214.6358	223.8265	216.9921	213.9373
Volatility of Profit	823.0064	447.1479	333.961	520.9149	812.5559	320.7246	341.83	424.1754	754.1805	946.2602	1069.8	811.8224
% Sims Lost Money	0.8069	0.9734	0.9987	0.9351	0.4008	0.4085	0.4656	0.4275	0.2993	0.4337	0.484	0.3629
Expected Shortfall	-989.9528	-756.4186	-736.7249	-795.6959	-808.5148	-235.7848	-222.499	-362.147	-778.5954	-439.3213	-488.2348	-455.6743

 

 Table 5. Simulation outcomes for portfolio of out of the money puts. ETF options were bought and component options were sold in this portfolio.

Simulation results should be examined from two perspectives. First, the effect of simulation type on mean portfolio profit should be considered subject to the position taken on the ETF option, either long or short. Second, the effect of delta-hedging on the volatility of portfolio profit should be considered subject to simulation type. Results showed that average portfolio profit was lowest for market-neutral simulations, and highest for market-shock simulations, when a long position on the ETF option was taken. Conversely, average profit was highest for market-neutral simulations, and lowest for market-shock simulations, when a short position on the ETF option was taken. Furthermore, the volatility of portfolio profit was significantly reduced by all forms of delta-hedging for market-neutral and historical simulations, but profit volatility was only marginally reduced, and in some cases was even increased, by delta-hedging for market-shock simulations. More specifically, volatility of portfolio profit under market-neutral simulations was minimized for three out of four portfolios when implied volatilities were used to delta-hedge the portfolio. Volatility of portfolio profit under historical simulations was minimized for all portfolios when historical volatilities were used to delta-hedge the portfolio. Volatility of portfolio profit under market-shock simulations was minimized for two out of four portfolios when the modified Markowitz portfolio volatility was used to delta-hedge the portfolio.

# 4. Discussion

### 4.1 Mean Portfolio Profit and Simulation Type

The observed interactions between simulation type, mean portfolio profit and the position taken in the ETF option were expected and are rational within the context of dispersion trading. As discussed in the introduction, when inter-asset correlations are high, portfolio volatility increases. When inter-asset correlations are low, portfolio volatility decreases. Recall that market-neutral simulations were generated using inter-asset correlations of zero and market-shock simulations were generated using stronger inter-asset correlations than those observed historically. Because long option positions which have been delta-hedged benefit from increased realized volatility, portfolios that were long ETF options benefited from the higher inter-asset correlations realized during market-shock simulations, and suffered under the lower inter-asset correlations realized during market-neutral simulations. See tables 3 and 5 for the relevant data. Conversely, because short option positions which have been delta-hedged benefit from decreased realized volatility, portfolios that were short ETF options benefited from the lower inter-asset correlations realized during market-neutral simulations and suffered under the higher inter-asset correlations realized during market-shock simulations. See tables 2 and 4 for the relevant data.

### 4.2 Delta-Hedging

Delta-hedging outcomes provide a number of insights into the behavior of dispersion portfolios under various conditions. First and foremost, results generally support the research of Wilmott and Riaz discussed previously, which found that the volatility of profit for a delta-hedged option was minimized when realized volatility was used to calculate delta [7]. For market-neutral simulations, in which implied volatilities were used to simulate component asset volatility, deltahedging with implied volatilities minimized the volatility of portfolio profit in three out of four portfolios. While the portfolio consisting of at the money puts summarized in table 4 did not conform to this trend, a number of complicating factors were present which may have affected results. First, only component assets were assumed to be fairly priced under the market-neutral simulation method, meaning that the implied volatility on the ETF option which was sold may have been significantly underpriced relative to the realized volatility on the index, resulting in miscalculated delta values. Second, it is possible that an insufficient number of time steps were simulated for each asset. Because delta is recalculated at each time step, it is possible that larger numbers of time steps would result in performance improvements for each hedging protocol. Lastly, the observed volatility of portfolio profit may have been affected by the modest number of total simulations. As more simulations are generated, random statistical anomalies are flattened out by large numbers of more typical outcomes.

For historical simulations, in which historical volatilities were used to simulate component asset volatility, delta-hedging with historical volatilities minimized the volatility of portfolio profit for all four portfolios. These results perfectly corroborate work by Wilmott and Riaz [7].

Lastly, for market-shock simulations, in which historical volatilities were used in conjunction with an average return shock of approximately six percent applied to all component assets, deltahedging with the modified Markowitz portfolio volatility as calculated using equation (2) minimized the volatility of portfolio profit for the portfolios summarized in tables 2 and 3. However, profit volatility was minimized for the portfolios summarized in tables 4 and 5 when options were not delta-hedged at all. Again, the small number of simulated time steps is likely responsible for these results, especially given the large shock returns that were applied to each asset in the portfolio for every simulation. Within the Black and Scholes framework, delta should be recalculated and the hedge adjusted at every instant throughout the life of the option in order to account for continuous changes in the stochastic geometric Brownian motion underlying the model [3]. When simulated movements in the underlying asset price are exceptionally large between time steps, the hedge cannot be adjusted throughout the price change, and risk management outcomes suffer as a result [4]. Because each portfolio was shocked with the same severity, and each portfolio was simulated using the same number of time steps, the portfolio of at the money puts experienced the most severe unhedged price changes relative to their remaining time to maturity.

Apart from validating previous industry research, these simulation results provide new insights into the performance of delta-hedged dispersion portfolios relative to naked dispersion portfolios. While differences in observed mean profits between each delta-hedging protocol were small across all portfolios and simulation methods, differences between mean naked profits and mean delta-hedged profits were significant in some cases. Standard deviations of the three deltahedged mean portfolio profits, calculated across all portfolios and simulation methods, ranged from \$0.46, on at the money puts simulated using the market-neutral method, to \$10.91, on in the money calls simulated using the historical method. The average of these standard deviations was only \$4.49. Closely clustered mean profits for the three delta-hedging methods indicate that despite significant effects on the volatility of portfolio profit, the underlying asset volatility used to calculate delta had only a minimal effect on mean portfolio profit. In contrast, the largest difference between mean naked profit and mean delta-hedged profit across portfolios and simulation methods was \$60.84, on at the money calls simulated under historical conditions. A large discrepancy of \$38.74 between mean naked profit and mean delta-hedged profit was also observed for in the money calls simulated under historical conditions. When mean delta-hedged profits and mean naked profits observed under historical simulations across all portfolios are compared to expected profits for each portfolio, as calculated using equations (3) and (4) and listed in Table 1, the effect of delta-hedging on mean dispersion portfolio profit becomes clear. In all cases, mean naked profit is corrected, either upward or downward, towards the expected

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portfolio profit by delta-hedging the entire portfolio. Further research is needed to explain the causes for deviation of mean naked profit from expected portfolio profit, however recommendations for effective dispersion trading can still be made in its absence.

### 4.3 Recommendations for Dispersion Trading

Dispersion traders should understand the effects of inter-asset correlation on their dispersion portfolios. As evidenced by simulation results, dispersion trades that are long ETF options and short component options benefit when correlations increase, and suffer when correlations decrease. Dispersion trades that are short ETF options and long component options benefit when correlations decrease, and suffer when correlations increase. Traders can therefore use current correlations as a signal for dispersion trading opportunities. When inter-asset correlations are particularly high, it may be a good time to sell ETF options and buy component options, in anticipation of correlations normalizing downwards in the future. Conversely, when inter-asset correlations are particularly low, it may be a good time to buy ETF options and sell component options, in anticipation of correlations normalizing upwards in the future.

Dispersion traders looking to reliably realize expected profits on their portfolio should certainly delta-hedge each option traded. While particularly risk hungry traders may wish to forgo delta-hedging in the hopes of capturing directional payoffs on individual equity options, naked portfolios will almost certainly experience higher profit volatility, and may also be subject to mean potential profits that deviate from expected portfolio profit under various circumstances. If commission fees are high, traders may wish to take positions in at the money straddles in order to minimize the delta exposure on each underlying asset. Gamma exposure across the portfolio will be relatively high in this case, meaning that traders must adjust hedging positions taken in the underlying assets on a more frequent basis. Simulation results indicate that if commission fees are low enough, it is viable to delta-hedge options without the use of straddles, in which case gamma exposure across the portfolio will be fairly low. This will translate to a more stable portfolio which requires fewer adjustments to hedging positions in the underlying assets. Choosing the proper volatility with which to calculate delta is also important when attempting to minimize portfolio profit volatility. Ideally, traders will have constructed an expected covariance matrix before placing their trade. Using the asset variances in this matrix to hedge each

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component option, and the expected Markowitz portfolio volatility to hedge ETF options, is likely the best approach. Research results indicate that in some cases, when a trader believes a large market-shock may be imminent, using the modified Markowitz portfolio volatility to deltahedge all options in the portfolio may be beneficial, assuming the trader can frequently adjust his hedging positions.

### **4.4 Opportunities for Further Research**

Further research into dispersion trading is necessary. By building on existing work, academics can more explicitly address questions regarding component asset subset selection, optimal portfolio sizing, option matching criteria and alternative hedging techniques. While relationships between mean portfolio profit, volatility of portfolio profit and simulation method were considered for portfolios constructed using options on all components in the index, selection of a subset of component equities may change the relationships examined in this work. Therefore, further research into the effect of component asset subset selection is recommended. Different methods for component selection, including selection by largest weight and smallest spread in quoted option prices, should be compared.

Options in this work were matched using a relative moneyness term defined as the ratio of the strike price to current underlying asset price. Alternative measures of moneyness were also discussed in this work, including moneyness relative to the volatility of the underlying asset and the delta of the option. Further research is needed to determine the effect of alternative option matching criteria on risk management outcomes and portfolio profitability.

This research examined basic relationships between absolute portfolio profit and portfolio sizing subject to immediate market impact, however questions regarding the effect of hedging protocol on that relationship remain. While it is likely that a similar, generally parabolic relationship exists, the characteristics of the function defining that relationship may change subject to alternative hedging strategies.

Finally, an extensive and quantitative comparison between the performances of all potential dispersion portfolio hedging strategies needs to be conducted. Such a study should examine the hedging performance of variance swaps and option straddles, in addition to the standard delta-

hedging protocols examined in this work. Large samples of dispersion portfolios should be stress tested under each hedging method and under different market conditions in order to inform dispersion traders regarding optimal portfolio outcomes in various scenarios. A decision making framework should be constructed according to observed results.

# **5.** Conclusion

Dispersion trading is a form of relative value volatility trading conducted using options on portfolios and options on the component assets of those portfolios. Traders building dispersion portfolios attempt to control market risk exposure with various hedging techniques. Unfortunately, no existing academic literature quantitatively compares the performances of different hedging methods with respect to dispersion portfolio performance. The research presented in this paper partially addresses this research gap.

Real-world volatility dispersion portfolios were constructed on the Dow Jones Transportation Average, hedged using four distinct protocols and stress tested subject to three simulated market conditions. Delta-hedged portfolio outcomes were compared to naked portfolio outcomes across each portfolio and simulation method. Preliminary research results indicated that absolute portfolio profit is parabolic with respect to portfolio sizing subject to a positive slippage parameter, and that commission fees are notably detrimental to portfolios hedged without the use of straddles. Primary research results confirmed that mean portfolio profit is strongly related to inter-asset correlations and showed that delta-hedging is generally effective in reducing the volatility of portfolio profit.

This work only compared the performances of naked portfolios to the performances of portfolios delta-hedged using three different volatility parameters. Future research should primarily seek to provide quantitative comparisons between the performances of variance swap hedging and delta-hedging strategies with respect to dispersion portfolios, however there are opportunities for further research into other dispersion trading topics as well. It should be the goal of academics in

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this field to formalize an entire decision making framework which highlights the strengths and weaknesses of each dispersion portfolio hedging strategy across a wide variety of potential risk scenarios. Such a framework would ensure the continued growth and profitability of dispersion trading into the future.

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# **Appendix A**

```
function [ optiondata ] = GetNasdaqOptionChain( ticker )
html = urlread(['http://www.nasdaq.com/symbol/' ticker '/option-chain?dateindex=-1&page=1']);
numpages = regexpi(html, 'dateindex=-1&page=(\d+)" id="quotes_content_left_lb_LastPage"', 'tokens');
if isempty(numpages) == 0 % Found page number data on first HTML page
            numpages = str2num(numpages{1}{1}); %Converts numpages from cell string to number
else %Didn't find page number data on first HTML page
            numpages = 1;
end
datearray = [];
contractarray = [];
strikearray = [];
bidarray = [];
askarray = [];
volumearray = [];
trys = 5;
parfor page = 1:numpages %Every page of option data
            for trynum = 1:trys %Number of times it has tried to fetch the data
                         try
                                   html = urlread(['http://www.nasdaq.com/symbol/' ticker '/option-chain?dateindex=-1&page=' num2str(page]]);
                                   datedata = regexpi(html, ['(d{6}).d{8}-' ticker '-.{3,4}''>.{12}</a>], 'tokens'); %Fetches expiration dates
                                   contractdata = regexpi(html, ['\d{6}(.)\d{8}-' ticker '-.{3,4}''>.{12}</a>', 'tokens'); %Fetches contracts, calls / puts
                                    strikedata = regexpi(html, ['\d{6}.(\d{8})-' ticker '-.\{3,4\}''>.\{12\}<\sqrt{a>'}, 'tokens'); % Fetches strike prices
                                   biddata = regexpi(html, ['\d{6}.\d{8}-'ticker'-.{3,4}''>.{12}<\ad{b} < td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^>]*>[^<]*<\td[^]*>[^<]*<\td[^]*>[^<]*<\td[^
                                               s^{<td}^{>}=([^{<}])^{+}([^{<}])^{+}(td>'], tokens'); % Fetches bid prices
                                   askdata = regexpi(html, ['\d{6}.\d{8}-'ticker'-.{3,4}''>.{12}<\askdata = regexpi(html, ['\d{6}-'ticker'-.{3,4}''>.{12}<\askdata = regexpi(html, [
                                               s^{<td}^{>}=(<)^{<td}, \ b^{<td}^{>}=((<)^{+})^{<td}, \ b^{<td}^{+}=((<)^{+})^{<td}, \ b^{<
                                    td>s*<td[^>]*>[^<]*</td>s*<td[^>]*>[^<]*</td>s*<td[^>]*>[^<]*</td>s*<td[^>]*>([^<]*</td>s*<td[^>]*>([^<]*</td>s*<td[^>]*<([^<]*</td>s*<td[^>]*<([^<]*</td>s*<td[^>]*<([^<]*</td>s*<td[^>]*<([^<]*</td>s*<td[^>]*<([^<]*</td>s*<td[^>]*<([^<]*</td>s*<td[^>]*<([^<]*</td>s*<td[^>]*<([^<]*</td>s*<td[^>]*<([^<]*</s*<td[^>]*<([^<]*<([^<]*<[^]*<([^<]*<[^]*<([^<]*<([^<]*<[^]*<([^<]*<([^<]*<[^]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<]*<([^<)*:([^<]*<([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:([^<)*:
```

```
datedata = [datedata{:}]; %Reduces all 2-dimensional fetched cell arrays to 1-dimensional cell arrays
contractdata = [contractdata{:}];
strikedata = [strikedata{:}];
biddata = [biddata{:}];
askdata = [askdata{:}];
volumedata = [volumedata{:}];
```

datearray = [datearray; datenum(datedata, 'yymmdd') - today]; %Converts into number of days from today contractdata = strrep(contractdata, 'C', '1'); %Converts C's and P's into 1's and 0's contractdata = strrep(contractdata, 'P', '0'); contractarray = [contractarray; transpose(cellfun(@str2num, contractdata))]; %Converts data from strings to numbers strikearray = [strikearray; transpose(cellfun(@str2num, strikedata) / 1000)];

biddata(cellfun(@isempty, biddata)) = {'0'}; %Replaces empty strings with 0's
bidarray = [bidarray; transpose(cellfun(@str2num, biddata))];
askdata(cellfun(@isempty, askdata)) = {'0'};
askarray = [askarray; transpose(cellfun(@str2num, askdata))];
volumedata(cellfun(@isempty, volumedata)) = {'0'};
volumearray = [volumearray; transpose(cellfun(@str2num, volumedata))];

[strikedata; biddata; askdata; volumedata; contractdata; datedata];

#### break

```
catch %Error was thrown
if trynum == trys
disp(['Error Fetching Option Data for ' ticker ' Page ' page])
end
continue
end
end
```

optiondata = [strikearray, bidarray, askarray, volumearray, contractarray, datearray]; % Compile option data into output variable

end

end

# **Appendix B**

optionrow = input('Row Number of Portfolio to Simulate: ');

numpers = input('Number of Periods to Simulate Per Asset: ');

numsims = input('Number of Portfolio Simulations to Generate: ');

simmeth = input('Portfolio Asset Simulation Method (Correlated Historical Return Based Simulation = 1, Risk-Neutral Simulation = 2, Correlated Shock Simulation = 3): ');

simulatedprices = [];

if simmeth == 1 %Simulates asset returns using variances, covariances and mean returns from historical daily return data varcovars = cov(allreturns(:,2:end)) \* (252 / 365); %Converts trading day covars with calendar day covars simulatedrets = portsim(((mean(allreturns(:,2:end)) + 1).^(252/365) - 1), varcovars, numpers, alloptiondata(optionrow, 6, 1) /

numpers, numsims, 'Expected');

elseif simmeth == 2 %Simulates neutral returns with user defined variances, zero covariances and mean returns = risk free rate inputvols = input('Vector of Independent Asset Annual Volatilities: ');

varcovars = zeros(size(allprices, 2) - 1); % Creates matrix of zeros for varcovar matrix

 $varcovars(eye(size(varcovars)) \sim = 0) = inputvols.^2 / 365; % Replaces diagonal of varcovar matrix with daily variances$ 

simulatedrets = portsim(ones(1, size(allprices, 2) - 1) \* (exp(rfr / 365) - 1), varcovars, numpers, alloptiondata(optionrow, 6, 1) / numpers, numsims, 'Expected');

elseif simmeth == 3 %Simulates correlated extreme market events

magnitude = input('Expected Magnitude Decimal Percentage of Market Shock: ');

varcovars = cov(allreturns(:,2:end)) \* (252 / 365); % Converts trading day covars with calendar day covars

simulatedrets = portsim(((mean(allreturns(:,2:end)) + 1).^(252/365) - 1), varcovars, numpers, alloptiondata(optionrow, 6, 1) / numpers, numsims, 'Expected');

for sim = 1:numsims

shockreturn = randn(1)\*1.2663\*magnitude; %Shock return from the standard normal distribution scaled to magnitude shockindex = randi([1 numpers],1,1);

simulatedrets(shockindex, :, sim) = simulatedrets(shockindex, :, sim) + shockreturn;

end

end for sim = 1:numsims %Converts simulated returns to simulated prices

simulatedprices = cat(3, simulatedprices, ret2tick(simulatedrets(:, :, sim), todayprices(2:end)));

#### end

simulatedprices = [sum(simulatedprices, 2) / divisor, simulatedprices]; %Calculates index prices with component prices

simulatedrets = [zeros(size(simulatedrets, 1), 1, size(simulatedrets, 3)), simulatedrets]; %Adds column of placeholder zeros for sim = 1:numsims

simulatedrets(:, 1, sim) = tick2ret(simulatedprices(:, 1, sim)); %Calculates simulated returns from index prices

end

squaredrets = simulatedrets.^2; %Squares all simulated returns

simulatedvols = sqrt(mean(squaredrets, 1)) / sqrt((alloptiondata(optionrow, 6, 1) / 365) / size(simulatedrets, 1));

# **Appendix C**

numplayedassets = input('Number of Largest Components to Pair With Index in Portfolio: ');

hedgemeth = input('Option Hedging Method (Naked = 1, Delta With Historical Vols = 2, Delta With Implied Vols = 3, Delta
With Markowitz Implied Vol = 4): ');

slippage = input('Average Slippage Per Option Contract After First: ');
optimize = input('Optimize Order Sizing? (Yes = 1, No = 0): ');

if optimize == 0

scaleup = input('Scalar to Upsize Dispersion Portfolio (1 For Least Slippage): '); %How many times to scale up portfolio
elseif optimize == 1

scaleup = (1:20); %Range of order sizing scalars

end

if slippage == 0 % Avoids bug in calculating slippage slippage = 0.0000000000001;

end

[~, sortpageindex] = sort(todayprices(2:end), 'descend'); %Creates list of indexes for largest components
sortpageindex = [1, sortpageindex + 1]; %Adds ETF to list of traded asset indexes
nameplayedassets = indexfile(sortpageindex(1:numplayedassets + 1)); %Tickers of assets on which options are traded
numcontracts = zeros(1, length(indexfile)); %Number of contracts traded for each contract
allassetprofits = zeros(1, length(indexfile), numsims); %Profits across all assets for each simulation
allaverageprofits = []; %Average simulated portfolio profit for each portfolio sizing

hedgevols = zeros(1, length(indexfile)); % Vols used to calculate delta for each option contract

if hedgemeth == 2 % Historical vols

hedgevols = sqrt(var(allreturns) \* 252);

elseif hedgemeth == 3 % Implied vols

hedgevols = transpose(squeeze(alloptiondata(optionrow, 9, :)));

```
elseif hedgemeth == 4 % Markowitz implied vol
```

hedgevols = ones(1, length(indexfile)) \* alloptiondata(optionrow, 8, 1);

```
end
```

for scale = scaleup

numcontracts(1) = round((sum(todayprices(sortpageindex(2:numplayedassets + 1))) / sum(todayprices(2:end))) \* divisor \*
scale); %Number of contracts to trade for index option

numcontracts(sortpageindex(2:numplayedassets + 1)) = scale; %Number of contracts to trade for each component in index if hedgemeth == 1 %Naked portfolio

for simindex = 1:size(simulatedprices, 3) %Each simulation

for assetindex = sortpageindex(1:numplayedassets + 1) %Each played asset

if alloptiondata(optionrow, 5, assetindex) == 1 % Call option

if sign(alloptiondata(optionrow, 10, assetindex)) == 1 %Long position

allassetprofits(1, assetindex, simindex) = max(simulatedprices(end, assetindex, simindex) -

```
alloptiondata(optionrow, 1, assetindex), 0) * numcontracts(assetindex) - sum(alloptiondata(optionrow, 3,
```

```
assetindex) * [1:slippage:1 + slippage * (numcontracts(assetindex) - 1)]) * (1 + commission);
```

#### else %Short position

```
allassetprofits(1, assetindex, simindex) = sum(alloptiondata(optionrow, 2, assetindex) * [1 - slippage *
  (numcontracts(assetindex) - 1):slippage:1]) * (1 - commission) - max(simulatedprices(end, assetindex,
  simindex) - alloptiondata(optionrow, 1, assetindex), 0) * numcontracts(assetindex);
```

#### end

#### else %Put option

```
if sign(alloptiondata(optionrow, 10, assetindex)) == 1 %Long position
```

allassetprofits(1, assetindex, simindex) = max(alloptiondata(optionrow, 1, assetindex) - simulatedprices(end, assetindex, simindex), 0) \* numcontracts(assetindex) - sum(alloptiondata(optionrow, 3, assetindex) \*

```
[1:slippage:1 + slippage * (numcontracts(assetindex) - 1)]) * (1 + commission);
```

#### else %Short position

```
allassetprofits(1, assetindex, simindex) = sum(alloptiondata(optionrow, 2, assetindex) * [1 - slippage *
  (numcontracts(assetindex) - 1):slippage:1]) * (1 - commission) - max(alloptiondata(optionrow, 1, assetindex) -
  simulatedprices(end, assetindex, simindex), 0) * numcontracts(assetindex);
```

```
end
```

```
end
```

end

#### end

```
else %Delta-hedged portfolio
```

```
for simindex = 1:size(simulatedprices, 3) % Each simulation
```

for assetindex = sortpageindex(1:numplayedassets + 1) % Each played asset

```
currenthedge = 0;
```

```
for timeindex = 1:size(simulatedprices, 1) %Each simulated time step
```

#### if alloptiondata(optionrow, 5, assetindex) == 1 %Call option

if sign(alloptiondata(optionrow, 10, assetindex)) == 1 %Long call

```
if timeindex == size(simulatedprices, 1) % Final time period
```

newhedge = 0; % Close hedge position

allass etprofits (1, asset index, simindex) = allass etprofits (1, asset index, simindex) + (currenthedge - allass etprofits) + (current

newhedge) \* simulatedprices(timeindex, assetindex, simindex) \* (1 + commission) +

max(simulatedprices(end, assetindex, simindex) - alloptiondata(optionrow, 1, assetindex), 0) \*

numcontracts(assetindex) - sum(alloptiondata(optionrow, 3, assetindex) \* [1:slippage:1 + slippage \*

```
(numcontracts(assetindex) - 1)]) * (1 + commission);
```

#### else %Not final time period

newhedge = - EuropeanOptionDelta(simulatedprices(timeindex, assetindex, simindex),

alloptiondata(optionrow, 1, assetindex), rfr, (alloptiondata(optionrow, 6, assetindex) - (timeindex - 1) \* (alloptiondata(optionrow, 6, assetindex) / numpers)) / 365, hedgevols(assetindex), alloptiondata(optionrow,

5, assetindex), 0) \* numcontracts(assetindex);

if newhedge > currenthedge % Buying shares

allassetprofits(1, assetindex, simindex) = allassetprofits(1, assetindex, simindex) + (currenthedge -

newhedge) \* simulatedprices(timeindex, assetindex, simindex) \* (1 + commission);

#### else %Selling shares

allassetprofits(1, assetindex, simindex) = allassetprofits(1, assetindex, simindex) + (currenthedge -

newhedge) \* simulatedprices(timeindex, assetindex, simindex) \* (1 - commission);

#### end

currenthedge = newhedge;

#### end

#### else %Short call

if timeindex == size(simulatedprices, 1) % Final time period

newhedge = 0; % Close hedge position

allassetprofits(1, assetindex, simindex) = allassetprofits(1, assetindex, simindex) + (currenthedge -

newhedge) \* simulatedprices(timeindex, assetindex, simindex) \* (1 - commission)

+ sum(alloptiondata(optionrow, 2, assetindex) \* [1 - slippage \* (numcontracts(assetindex) - 1):slippage:1])

\* (1 - commission) - max(simulatedprices(end, assetindex, simindex) - alloptiondata(optionrow, 1,

assetindex), 0) \* numcontracts(assetindex);

#### else %Not final time period

newhedge = EuropeanOptionDelta(simulatedprices(timeindex, assetindex, simindex),

alloptiondata(optionrow, 1, assetindex), rfr, (alloptiondata(optionrow, 6, assetindex) - (timeindex - 1) \*

(alloptiondata(optionrow, 6, assetindex) / numpers)) / 365, hedgevols(assetindex), alloptiondata(optionrow,

5, assetindex), 0) \* numcontracts(assetindex);

if newhedge > currenthedge % Buying shares

allassetprofits(1, assetindex, simindex) = allassetprofits(1, assetindex, simindex) + (currenthedge -

newhedge) \* simulatedprices(timeindex, assetindex, simindex) \* (1 + commission);

#### else %Selling shares

allassetprofits(1, assetindex, simindex) = allassetprofits(1, assetindex, simindex) + (currenthedge – newhedge) \* simulatedprices(timeindex, assetindex, simindex) \* (1 - commission);

#### end

currenthedge = newhedge;

#### end

end

#### else %Put option

if sign(alloptiondata(optionrow, 10, assetindex)) == 1 %Long put

if timeindex == size(simulatedprices, 1) % Final time period

newhedge = 0; % Close hedge position

allassetprofits(1, assetindex, simindex) = allassetprofits(1, assetindex, simindex) + (currenthedge – newhedge) \* simulatedprices(timeindex, assetindex, simindex) \* (1 - commission)

+ max(alloptiondata(optionrow, 1, assetindex) - simulatedprices(end, assetindex, simindex), 0) \* numcontracts(assetindex) - sum(alloptiondata(optionrow, 3, assetindex) \* [1:slippage:1 + slippage \*

(numcontracts(assetindex) - 1)]) \* (1 + commission);

#### else %Not final time period

newhedge = EuropeanOptionDelta(simulatedprices(timeindex, assetindex, simindex),

alloptiondata(optionrow, 1, assetindex), rfr, (alloptiondata(optionrow, 6, assetindex) - (timeindex - 1) \* (alloptiondata(optionrow, 6, assetindex) / numpers)) / 365, hedgevols(assetindex), alloptiondata(optionrow,

5, asset index), 0) \* numcontracts(asset index);

if newhedge > currenthedge % Buying shares

allassetprofits(1, assetindex, simindex) = allassetprofits(1, assetindex, simindex) + (currenthedge -

newhedge) \* simulatedprices(timeindex, assetindex, simindex) \* (1 + commission);

#### else %Selling shares

allass etprofits (1, asset index, simindex) = allass etprofits (1, asset index, simindex) + (current hedge - allass etprofits) + (current hedge - allass etpr

newhedge) \* simulatedprices(timeindex, assetindex, simindex) \* (1 - commission);

#### end

currenthedge = newhedge;

#### end

#### else %Short put

if timeindex == size(simulatedprices, 1) % Final time period

newhedge = 0; % Close hedge position

allassetprofits(1, assetindex, simindex) = allassetprofits(1, assetindex, simindex) + (currenthedge -

```
newhedge) * simulatedprices(timeindex, assetindex, simindex) * (1 + commission)
```

+ sum(alloptiondata(optionrow, 2, assetindex) \* [1 - slippage \* (numcontracts(assetindex) - 1):slippage:1])

\* (1 - commission) - max(alloptiondata(optionrow, 1, assetindex) - simulatedprices(end, assetindex,

simindex), 0) \* numcontracts(assetindex);

#### else %Not final time period

newhedge = - EuropeanOptionDelta(simulatedprices(timeindex, assetindex, simindex),

alloptiondata(optionrow, 1, assetindex), rfr, (alloptiondata(optionrow, 6, assetindex) - (timeindex - 1) \* (alloptiondata(optionrow, 6, assetindex) / numpers)) / 365, hedgevols(assetindex), alloptiondata(optionrow,

5, assetindex), 0) \* numcontracts(assetindex);

if newhedge > currenthedge % Buying shares

allassetprofits(1, assetindex, simindex) = allassetprofits(1, assetindex, simindex) + (currenthedge -

newhedge) \* simulatedprices(timeindex, assetindex, simindex) \* (1 + commission);

#### else %Selling shares

allassetprofits(1, assetindex, simindex) = allassetprofits(1, assetindex, simindex) + (currenthedge – newhedge) \* simulatedprices(timeindex, assetindex, simindex) \* (1 - commission);

#### end

```
currenthedge = newhedge;
```

```
end
```

```
end
```

end

end

end

```
end
```

end

allassetprofits = allassetprofits \* 100; % Multiply profits by 100 to reflect contract sizing

if optimize == 0

results = {'Average Portfolio Profit: ', mean(sum(allassetprofits, 2));

'Volatility of Portfolio Profit: ', std(sum(allassetprofits, 2));

'% of Simulations Resulting in Loss:', length(find(sign(sum(allassetprofits, 2)) == -1)) / numsims;

'Expected Shortfall:', mean(sum(allassetprofits(1, :, find(sign(sum(allassetprofits, 2)) == -1))))}

elseif optimize == 1

allaverageprofits = [allaverageprofits, mean(sum(allassetprofits, 2))];

end

end