The Red Queen Principle and the Emergence of Efficient Financial Markets: An Agent Based Approach

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Summary. In competitive coevolution, the Red Queen principle entails constraints on performance enhancement of all individuals if each is to maintain *status quo* in relative fitness measured by an index relating to aggregate performance. This is encapsulated in Lewis Caroll's Red Queen who says "in this place it takes all the running you can do, to keep in the same place". The substantive focus of this paper is to experimentally generate stock market ecologies reflecting the Red Queen principle for an explanation of the observed highly inegalitarian power law distribution in investor income (measured here as stock holdings) and the emergence of arbitrage free conditions called market efficiency. With speculative investors modelled as using genetic programs (GPs) to evolve successful investment strategies, the analytical statement of our hypothesis on the Red Queen principle can be implemented by constraint enhanced GPs which was seminally developed in [19], [7] and [10].

1 Introduction

This paper aims to implement the methodology of computationally intelligent multi-agent models to simulate stock markets as complex adaptive systems (CAS). These markets are likened to complex ecologies involving the interaction of a large number of agents who adaptively respond to their environment and to other agents with the aid of computer programs or algorithms that mimic evolutionary principles. The environment of such agents becomes complex with statistical 'signatures' such as power laws. Associated with this soup of coevolving population of agents, each attempting to enhance its fitness relative to others, is the principle of the Red Queen based on the observation made to Alice by the Red Queen in Lewis Caroll's *Through the Looking Glass:* "in this place it takes all the running you can do, to keep in the same place".

The principle of the Red Queen was first identified by an evolutionary biologist van Valen to encapsulate features of competitive coevolution in species. In the competition that governs the fight for scarce resources or in cases of direct confrontation with zero sum payoffs such as in parasite-host or predator-prey situations what matters is relative rather than absolute performance capabilities of the individuals. Certain attributes of individuals have to be enhanced relative to the same in others to stay ahead of the race. But others are like-wise involved in performance enhancement triggering off an arms race. Since only hypothesis or conjectures but not the direct tests of the Red Queen principle can be applied to evolutionary biology, Artificial Life simulations have become the means to understand the system dynamics and distinctive if not generic features of competitive coevolution. The classic work [13] and [16] are based on competing coevolving species. In [5] parasites were deliberately introduced and it was noted that competition among coevolving species could potentially prevent stagnation in local optima. The evolution of intelligence itself is hypothesized to arise as a Red Queen type arms race giving rise to Machiavellian behaviour in social interactions, [14]. In [8] there is a fuller discussion of the relevance of the Red Queen principle for Economics.

In competitive coevolution, the Red Queen principle, therefore, entails constraints on performance enhancement of all individuals if each is to maintain *status quo* in relative fitness measured by an index relating to aggregate performance. The substantive focus of this paper is to experimentally generate stock market ecologies reflecting the Red Queen principle for a long overdue explanation of the observed highly inegalitarian power law distribution in investor income (measured here as stock holdings) and the emergence of arbitrage free conditions called market efficiency. With speculative investors modelled as using genetic programs (GPs) to evolve successful investment strategies, the analytical statement of our hypothesis of the Red Queen is as follows: each agent's strategy must 'keep up' with the average performance of all other strategies in terms of returns. This can be implemented by constraint enhanced GPs which was seminally developed in [19], [7] and [10][4].

[4] In [10] GPs were trained to detect arbitrage opportunities in a program called EDDIE-ARB which is geared toward conducting arbitrage operations in the index options and futures markets. Historical data indicated that arbitrage opportunities exist, but they are few and far between. From a large domain of search, fewer than 3% of these were found to be profitable in excess of transactions costs. While the GPs could be successfully trained to find arbitrage opportunities, many of these were being missed. The novel feature of EDDIE-ARB is a constraint satisfaction feature supplementing the fitness function that enables the GP to satisfy a degree of search intensity specified as a minimum and maximum as required by the problem. In other words, as no more than 3% of the data is likely to contain arbitrage opportunities that became the maximum. The minimum was specified to suit the needs of the user.

In the Santa Fe artificial stock market model of [1] it was found that when the forecast performance of agents as genetic programs was altered by different rates of retraining, the stock price dynamics varied correspondingly. When the genetic programs were given a slow rate of retraining, the market converged to homogenous rational expectations (HRE) while when retraining rate was speeded up, the more volatile dynamics of real stock markets was observed. However, from our perspective on the Red Queen principle, performance enhancement should not be *ad hoc* and exogenously imposed by the experimenter, but should be an endogenous constraint. The work described in [2] is exceptional in having proposed an endogenous scheme for when and by how much investors equipped with genetic programs retrain in an artificial stock market environment. The authors in [2] (pp. 377-379) discuss the process of enhancement of individual investment performance in terms of *peer pressure* and *self-realization.* They prescribe an endogenous way in which agents look for 'better' investment rules and call this procedure *Visiting the Business School.* In [2] (p. 379) the authors make an important point which can be summarized as follows: in so far as each agent's investment performance differs from one another relative to some (endogenously given) benchmark, agents now coevolve governed by a different fitness or objective function. However, the following problem may be cited with the Chen and Yeh measure based on the peer pressure criterion. It depends on a notion of rank [2] (p. 377) which presupposes knowledge by an agent of all other agents' investor performance. On the basis of decentralized information, this is not viable. Hence, we argue that relative investment performance had best be evaluated by aggregate market index returns —-the straight forward idea of 'beating the market'. Further, we move away from the Sante Fe type use of the Sharpe ratio investment decision rule used in [2] and elsewhere as it not optimal when returns are not Guassian with constant volatility. The latter is very unlikely in the case of Red Queen returns.

For above reasons, we follow the framework described in [6] which is more in keeping with the Complex Adaptive Systems framework and is concerned with power law generation of investor wealth as a manifestation of self-organized complexity. We extend the model in [6] so as to directly focus and test for the Red Queen constraint which is missing in their work. We argue that when agents retrain, the rate of retraining corresponds to the extent to which a lower bound constraint on investor wealth relative to aggregate wealth is satisfied or not: agents have to run faster or not in terms of the retraining they do. Our hypothesis is that asset market efficiency and the power law distribution in investor stock holdings requires as a necessary condition that *all* traders work harder and harder to 'beat' the market. Then, there is a point at which none of them can do so. The value of each investor's risky fund follows a stochastic multiplicative process and the distribution of stock holdings in the economy should in principle converge to the power law Pareto distribution when this lower bound Red Queen constraint on investor wealth is satisfied. By turning this Red Queen constraint on and off, we experimentally verify if the collective competitive behaviour of agents to 'beat' the market is indeed the way in which markets become efficient.

2 Power Laws in Investor Wealth and the Test for the Red Queen Principle

2.1 Steps in the Test For the Red Queen Effect

In a model with the endogenous emergence of the power law in stock returns we define the power law in terms of the distribution of investor wealth. In a large microagent system of N agents the probability distribution (or the proportion of individuals in a population with wealth of size w) being given as [5]

$$P(w) \sim w^{-1-\alpha}. \tag{1}$$

Here, w (integer valued) is a certain value of wealth w in the population. The total wealth is generated from the N micro agent systems is

$$W(t) = w_1(t) + w_2(t) + \ldots + w_N(t). \tag{2}$$

It has been discovered that dynamics characterized by generalized Lotka Voltera equations for each micro system can under certain conditions bring about the power law distribution in Equation 1. We consider here only the simplest form of this that involves $\lambda_i(t)$, the random multiplicative wealth generating factor which arises due to the performance of each agent's forecasting model and strategy to buy, sell or not to trade in relation to the market's generation of the spot price which is common to all traders.

$$w_i(t+1) = \lambda_i(t) w_i(t), i = 1, 2, \ldots, N. \tag{3}$$

The main results are outlined below. The power law in Equation 1 follows if and only if the multiplicative coefficient $\lambda_i(t)$ in equation 3 on agent's wealth becomes independent of agent i factors and all agents' payoffs from strategies are drawn from the same uniform probability distribution. That is, on average no strategy has an undue advantage over another strategy in obtaining higher than average payoff at each t. The important point here is that this is an emergent phenomena amongst traders who are each trying to find rules to 'beat' the market and assiduously try and select 'good' forecast rules by generic evolutionary fitness criteria of rewarding those rules that increase investor wealth shares, $w_i(t)/W(t)$. Thus, the emergence of market efficiency is sustained in the micro agent based model under circumstances very different from what is traditionally associated with trader rationality. Traditionally,

[5] Note that $P(w) = 0$ for $w \leq 0$ and $lim P(w) = 0$ as $w \to \infty$. That is, the distribution is zero if w is negative or tends to infinity.

the latter is at odds with agents who attempt to 'beat' the market. Note also that the power law distribution of investor wealth in an efficient market does reward a small proportion of investors with large returns while many have very little.

The steps in the proof of the result requires that the α parameter in Equation 1 to be positive. For this there has to be a lower bound, $w_{min}(t)$, which dictates that the central limit theorem no longer applies at large t and log normal distributions do not follow for $w_i(t)$. The lower bound $w_{min}(t)$ is specified as

$$w_{min}(t) = q\overline{w}(t). \tag{4}$$

This yields

$$q = w_{min}(t)/\overline{w}(t), \tag{5}$$

where $\overline{w}(t) = W(t)/N$, ie. $\overline{w}(t)$ is the mean wealth at time t.

On placing the lower bound constraint on minimum wealth given above, the wealth dynamics for each agent is defined as

$$w_i(t+1) = \lambda_i(t)w_i(t), \tag{6}$$

with the lower cut-off [6]

$$w_i(t+1) \geq q\overline{w}(t). \tag{7}$$

When the power law in Equation 1 holds for the system dynamics in Equations 6, 7, for given N and q in range $1 > q > 1/\ln N$, the exponent α in Equation 1 is given by [7]

$$\alpha = 1/(1-q). \tag{8}$$

2.2 Constraint Satisfaction And Power Law

To underscore the importance of the role of constraint satisfaction and how the Red Queen Effect works, we rewrite the lower bound condition in Equation 7 by substituting for $w_i(t+1)$ from Equation 6 and using the equality,

$$\lambda_i^{con}(t) = q\frac{\overline{w_t}}{w_i(t)}, 1 > q \geq 1/\ln N. \tag{9}$$

[6] In [17] the author discusses a number of ways in which agents whose wealth violated the lower bound constraint could be fixed. Unlike social security payments, in investment markets it is unlikely that public subsidies are given to agents whose performance fails to satisfy some lower bound constraint.

[7] The derivation for this is given in [17]. For this note that $\int_{w_{min}}^{\infty} P(w)dw = \int_{w_{min}}^{\infty} w^{-1-\alpha}dw = 1$ and average wealth is given by $\int_{w_{min}}^{\infty} wP(w)dw = \int_{w_{min}}^{\infty} w^{-\alpha}dw = \overline{w}$

As the first term in Equation 9 is common to all agents, the implication of Equation 9 is that $\lambda_i(t)$ has to be proportionately greater (smaller) when the mean wealth of the population of traders $\overline{w}(t) = W(t)/N$ exceeds (is less than) that of the ith agent. In other words, as we will see, agents are constrained to improve their forecast/investment performance when it is below average. This follows the point made above in [2] (p.379), GPs now effectively coevolve with different fitness functions. While in numerous studies, criteria governing search intensity for better forecast/investment rules have been included, often in an *ad hoc* way, what has not been precisely stated as above nor tested is how the emergence of i-independence of λ_{it} in Equation 3 and the emergence of the power law in investor wealth distribution follows as a population of N agents apply the constraint in Equations 6, 7 and 9 in their selection of investment strategies which collectively determine investor wealth dynamics. The failure to finely discriminate the reason for the emergence of power laws in multi-agent stock market models has led to vague conclusions such as "almost every realistic microscopic market model we have studied in the past shares this characteristic of w-independent $\Pi(\lambda)$ distribution", [18].

The problem is to follow in detail the link between agent strategies which are GPs selected by the evolutionary principle of proportional fitness and the market impact of individual trades which together determine $\lambda_i(t)$ in Equation 9. The details regarding the latter are kept to a minimum as our immediate purpose here is to illustrate how the use of the constraint enhanced GPs ([7] and [10]) is ideally suited to model and test for the role of constraints in Equation 9.

The price is determined solely as a function of excess demand with respect to aggregate bids, B, and offers, O, at each t. Denoting the total number of agents/traders in the market as N, the aggregate bids and offers at t is

$$B_t = \sum_i b_{it} \; and \; O_t = \sum_j o_{jt}, i \neq j \tag{10}$$

We follow the price adjustment scheme discussed in [2] which is based on excess demand $(B_t - O_t)$

$$P_{t+1} = P_t(1 + \beta(B_t - O_t)) \tag{11}$$

Here β can be interpreted as the speed of adjustment parameter. The form that this takes is identical to that in [2],[8]

$$\beta(B_t - O_t) = \begin{cases} \tanh(\beta_1(B_t - O_t)) \; if \; B_t \geq O_t, \\ \tanh(\beta_2(B_t - O_t)) \; if \; B_t < O_t \end{cases} \tag{12}$$

The stock return, R_t, is defined as

[8]Here tanh is the hyperbolic tangent function : $tanh(x) \equiv \frac{\exp_x - \exp_{-x}}{\exp_x + \exp_{-x}}$.

$$R_t = (\ln P_{t+1} - \ln P_t) \tag{13}$$

In Equation 10, as will be explained later, whether bids or offers are made by traders depend on the recommendations made by their respective GPs. The Palmer rationing scheme is then used for the allocation of shares to each agent. Denoting the ith agent's holdings of shares at time t by h_{it},

$$h_{it} = h_{it-1} + \frac{V_t}{B_t} b_t \; or \; \left(-\frac{V_t}{O_t} o_{it}\right) \tag{14}$$

Here $V_t \equiv min(B_t, O_t)$ is the volume of trade in the stock and the expression in the bracket is the case that applies when the agent is going short.

Agents have to commit to buy and sell orders based on their expectations of capital gains or losses. Expectations are a function of an information set which includes only current and past prices, total market value or W_t , number of traders in the market, their own wealth and strategy set. Associated with each agent's choice of trading strategy is an indicator function $\Im_i^{B,O}(+1, -1)$ which determines whether the strategy is profitable $(+1)$ or loss making (-1) with respect to the returns R_{t+1}. Thus, the distribution over time $f(\lambda_{it})$ on agent's investment returns from (6) is given by

$$f(\lambda_{it}) = f(\Im_i^{B,O}(+1, -1), R_{w(i)t+1}) \tag{15}$$

$R_{w(i)t+1} \equiv d\ln(w_{it+1}/w_{it})$ is referred to as the agent's return factor.

To include the constraint on agent's investment performance in Equation 9 which we claim is the direct characterization of the Red Queen, we define the range in which each agent's Red Queen constraint applies. The minimum search effort for better forecasting rules is defined as a function of $\frac{\overline{w}}{w_i(t)}$ and with $q = 1/\ln N$, it is inversely proportional to the log of the number of agents N. Thus, [9]

$$\mu_{min} = \frac{1}{\ln N} \; \frac{\overline{w_t}}{w_i(t)} > 0 \tag{16}$$

Indeed, if $q = 0$, agents do not make any effort, then $\alpha = 1$ from Equation 8 and deterioration of agents' wealth share with below average investment performance can also lead to α to be less than one.

The maximum search effort is more problematic to specify. If q is set to 1 (ie. $w_{min} = \overline{w}$), effectively α becomes infinite in Equation 8 and the ownership distribution becomes highly egalitarian or degenerate with the entire population having the same wealth share. The value of q has to be one that implies a reasonable power law coefficient α, $1 < \alpha < 2$. For this, $0.5 > q > 1/\ln N$ with $q \cong 0.4$ is recommended. Thus

$$\mu_{max} = 0.4 \frac{\overline{w_t}}{w_i(t)} > 0 \tag{17}$$

[9] It is convenient to express Equation 16 in percentage terms. Thus $\mu_{min} = -\ln(\ln N) + \ln \overline{w}(t) - \ln w_i(t)$.

2.3 EDDIE and Constraint Satisfaction in GPs: Design For Test of Red Queen

We retain the architecture for EDDIE explained in [19], [7] for the selection of decision rules which recommend whether to buy or sell. As is standard with GPs, each agent is assigned an initial population of decision rules. These include well known fundamentals based forecasting rules or trend following moving average type technical rules. Candidate solutions are selected randomly, biased by their fitness, for involvement in generating members of the next generation. General mechanisms (referred to as *genetic operators*, e.g. reproduction, crossover, mutation) are used to combine or change the selected candidate solutions to generate offspring, which will form the population in the next generation.

In EDDIE, a candidate solution is represented by a genetic decision tree (GDT). The basic elements of GDTs are *rules* and *forecast values.* A single rule consists of one useful indicator for prediction, one relational operator such as "greater than", or "less than", etc, and a threshold (real value). Such a single rule interacts with other rules in one GDT through logic operators such as "Or", "And", "Not", and "If-Then-Else". Forecast values in this model are directions of price movements, either a positive trend (i.e. positive x% return within specified time interval can be achievable) or negative trend (i.e. negative x% return within a specified time interval can be achievable).

Table 1. A contingency table for two-class classification/prediction problem

	Predicted negative trend $(\widehat{Q}_-)$ **SELL**	**Predicted positive trend** $(\widehat{Q}_+)$ **BUY**
Actual negative trends (Q_-)	# of True Negative (TN) $\Im_i^O = +1$	# of False Positive (FP) $\Im_i^B = -1$
Actual positive trends (Q_+)	# of False Negative (FN) $\Im_i^O = -1$	# of True Positive (TP) $\Im_i^B = +1$

RC: Rate of correctness; RF: Rate of failure

$$RC_i = \frac{TP + TN}{Q_+ + Q_-} \leq \frac{TP + TN}{\widehat{Q}_+ + \widehat{Q}_-}; RF_i = \frac{FP}{\widehat{Q}_+} + \frac{FN}{\widehat{Q}_-}, \quad (18)$$

where $Q_+ = FN + TP; Q_- = TN + FP; \widehat{Q}_- = TN + FN; \widehat{Q}_+ = FP + TP$

Recommendation to BUY at t follows from the prediction of a price rise (positive trend) over a given period and recommendation to SELL follows from the prediction of a price fall (x% negative trend). Note different returns thresholds and horizons exist for different classes of traders. Since GDTs are used to predict directions of price changes and make recommendations for trade, the success or failure of recommendations can be categorised as a two-class classification problem. Each prediction point for every GDT can be classified into either a positive position or a negative position. For each GDT, we define RC (Rate of Correctness), and RF (Rate of Failure) as its prediction performance criteria. Formula for each criterion is given through a contingency table in Table 1.

Each agent selects the GDT which constitute trading strategy to buy or sell that maximizes the fitness function

$$\Gamma_{(1)} = \varphi(rc)RC - \varphi(rf)RF \tag{19}$$

The fitness function involves two performance values, i.e. RC and RF, each of which is assigned a different weight $\varphi(rc)$ or $\varphi(rf)$ respectively. While the fitness function can guard against loss making positions, the population of GDTs from which agents conduct their search may lead to investment income under-performance as there is no constraint as to how intense the search for suitable GDTs should be. Hence, the role of the constraint satisfaction enhanced GPs was devised in [7]. A new parameter set

$$\Re = [\mu_{min}, \mu_{max}] \tag{20}$$

is specified to supplement the fitness function (19) in EDDIE. Using the conditions defining μ_{min} and μ_{max} in Equations 16 and 17 agents intensify their search for GDTs to enhance their investment performance to satisfy the analytical conditions for the emergence of power law in investor wealth distribution in the stock market. Such a constraint enhanced coevolving population of trading strategies is allowed to run for a large number of time periods. This completes the description of the design of the framework that aims to test for the Red Queen Effect in the emergence of power laws and efficiency of asset markets.

2.4 Implementation of Wealth Dynamics

All EDDIE agents have the same initial wealth w_0 and are given a fixed and equal number S_0 of stocks and C_0 cash at the beginning of time. Then, wealth at $t+1$

$$w_{it+1} = (P_t h_{it} + C_{it}). \tag{21}$$

Here, h_{it} is holdings of stocks by the i_{th} agent.

Wealth dynamics follows a simple rule based on a agent's GP recommendation: agents go 100% in the direction recommended by their GP forecasts for price trends. That is, if the required x% (-x%) return is achievable in a given time horizon, the agent will place bids (offers) to buy (sell) stocks to the full extent that his budget will permit.

$$b_{it} = h_{it-1} + \frac{C_{it-1}}{P_t} \text{ ,if } C_{it} > 0 \tag{22a}$$

$$o_{it} = h_{it-1} \text{ ,if } h_{it-1} > 0 \tag{22b}$$

The actual holdings, h_{it}, that this will result in is determined by the rationing scheme in (14). Every BUY recommendation at t automatically sets up a limit order to sell the first time the required return is achieved or at the specified horizon whether or not the forecast is fulfilled. Likewise, SELL recommendation at t is followed by a buy limit order the first time a predicted price fall occurs, but no buy follows if the forecast is not fulfilled at the investment horizon. An investment scheme of this kind will result in the maximization of the growth of wealth in a manner that is entirely predicated by the agent's forecasting capability (see, [15]). At the end of his investment horizon, an agent whose prediction is wrong in the sense that he bought predicting a price rise will have lost a substantial part of his wealth in cash. The agent who predicted a price fall wrongly will end up failing to recoup his holdings of stocks. As only holdings of stocks can ultimately enhance wealth, individual wealth shares $w_i / \sum_{i=1}^{N} w_i$ can grow only as an agent increases his holdings of stocks relative to total stocks. For this reason in the next section we will report the distribution of stock holdings.

3 Results from Experimental Data

In this section, we report the results on price data and the distribution of stock holdings in the final run of simulations in the two cases : with the Red Queen constraints on GP performance and when GP retraining is undertaken in an *ad hoc* fashion specified by the experimenter. For the latter we simply set the different classes of agents to retrain at fixed intervals of time rather than prompted by any endogenous constraint on their investment performance. Further, it must be noted that in this preliminary analysis, the Red Queen constraint only involves that agents retrain when their wealth falls below average wealth. The experiments with and without Red Queen constraints were done for 200 and 1000 periods. Appendix A summarizes genetic programs, agent and market related parameters. Traders are organized into 4 groups of upto 20 in each. Each group is given a subset of the some of the well known technical trading rules. Each trader/agent uses a single population GP with 70 generations to generate the returns forecasts at each t.

Well known stylized facts on stock market prices and returns are that they fail to be normally distributed and also stock returns are unpredictable. By the Bera-Jarque test, we find significant departures from normality for returns data for all the runs. Further, the price series generated for both lengths of runs with the Red Queen constraint satisfies the null hypothesis that their respective unit root coefficients are not statistically different from zero.

What is important for our analysis is the empirical cumulative density function for stock holdings on a log-log scale, given in Figures 1-4 in Appendix B. The distribution of stock holdings is more egalitarian with *ad hoc* retraining case than for the Red Queen case. The results of the regression on the log of the number of agents with stock holdings greater than h on the log of h gives the estimate of the relevant power law exponent. What is interesting, is that in the case of a 1000 runs, the Red Queen data shows values for the power law exponent of 1.466 which is close to the classic 3/2 value found by Pareto for income distribution.

Table 2. Estimates for Power Law Coeficients

	ξ **GEV Shape Parameter #**	$1/\xi$ **Tail Index for Fat Tails**	**Ln-Ln OLS Slope for** α
Red Queen			
200 runs	1.03	0.97	0.448
	(.544, ± 0.89)		
1000 runs	0.57	1.75	1.466
	(.378, ± 0.62)		
Ad Hoc Retraining			
200 runs	1.205	0.83	0.1985
	(0.5017, ± 0.82)		
1000 runs	1.92	0.52	3.68
	(.0009, ± .0014)		

In the brackets are given the standard errors for the estimates and the critical values at 95% confidence level for the null hypothesis that $\xi = 0$.

As seen from Table 4 the power law exponent $\alpha = 3.68$ in the case of *ad hoc* retraining indicates a more equal distribution of stock holdings than the simulations with the Red Queen constraints. However, in the latter case, it should be noted that the GEV Generalized Extreme Value [10] shape parameter ξ which results in the tail index of 1.75 is not statistically significantly different from $\xi = 0$ at 95% confidence level.

4 Concluding Remarks

The endogenous explanation for the emergence of a power law in phenomena as varied as earth quakes, size of cities and income distribution is currently of interest with the framework of complex systems theory (see, [11] for a survey). It is alleged in [11] that the power law generated in the [6] model "depends on an arbitrarily imposed lower bound "constraint on individual agent's wealth defined as a function of the mean wealth of the population (see, Equation

[10]This has been estimated using the EVIM program described in [4].

(7)). In this paper, we argue that this is far from an arbitrary constraint. The satisfaction of this lower bound constraint is modelled here as a behavioural one that dictates performance enhancement in adaptive learning. In artificial stock market models in which agents have to individually learn and adapt in a multi population GP environment, retraining of GPs is mostly done in an *ad hoc* way. Reference [2] is an exception here. The bulk of the modelling effort in this paper has gone to explicitly set up the multi-population form of the constraint enhanced GPs of [7] in an artificial stock market environment.

Though interesting results supportive of the Red Queen hypothesis have been obtained, the limited experimental runs done so far, should make the experimental results reported here rather tentative. Further, the Red Queen constraint was implemented only in a limited way. More detailed results will be reported in the near future.

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A Appendix 1

This appendix summarizes the GP, market and agent related parameters

A.1 Genetic Programming Parameters

Table 3. Genetic Programming Parameters

Initial tree depth	3
Maximum tree depth	7
Percent of population to copy	30
Probability of mutation	0.01
Probability of maximization	0.01
Number of generations	70
Population size	60
Maximization step	10

Table 4. Functions

Relational Functions	$=, <, >$
Logical Functions	AND, OR, NOT

A.2 Market Parameters

Table 5. Market Parameters

Number of Groups of Traders	4
Number of trading Periods	200 or 1000
β_1	0.0001
β_2	0.00008

A.3 Indicators

See [19] for explanation of these well known technical rules.

A.4 Traders' Conditions

Table 6. Indicators

Group One	Group Two	Group Three	Group Four
Moving Average 12	Volatility 12	Trend Rule 5	Moving Average 12
Moving Average 50	Volatility 50	Trend Rule 50	Moving Average 50
		Filter 5	Volatility 12
		Filter 63	Volatility 50
			Trend Rule 5
			Trend Rule 50
			Filter 5
			Filter 63

Table 7. Traders' groups

Group	One	Two	Three	Four
Number of Traders	20	20	10	10
Horizon(Days)	5	10	21	30
Rate of Return	1	1.5	2	3.5
Initial Cash	4000	4000	4000	4000
Initial Stocks	100	100	100	100

B Appendix 2

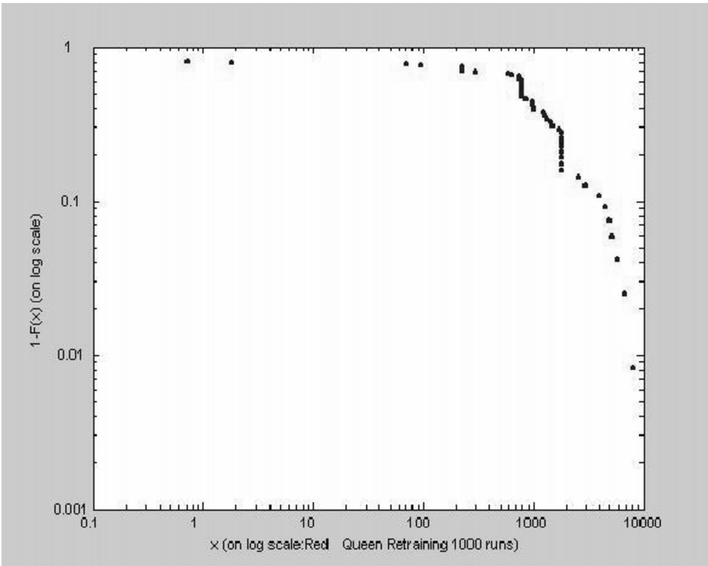

Fig. 1. Cummulative Density Function of Stock Holdings: Red Queen Retraining 1000 periods.

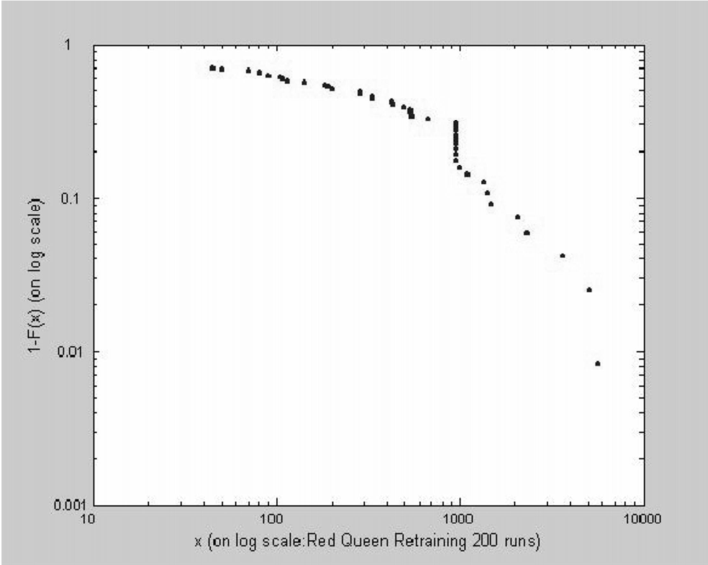

Fig. 2. Cummulative Density Function of Stock Holdings: Red Queen Retraining 200 periods.

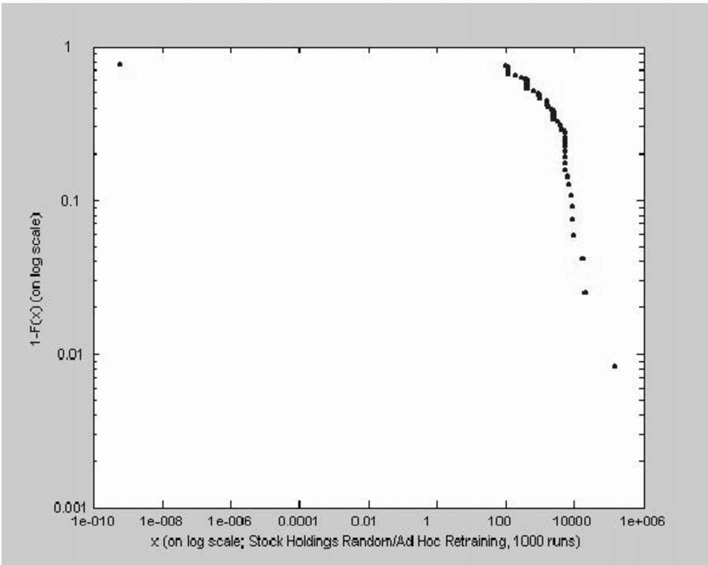

Fig. 3. Cummulative Density Function of Stock Holdings: *Ad Hoc* Retraining 1000 periods.

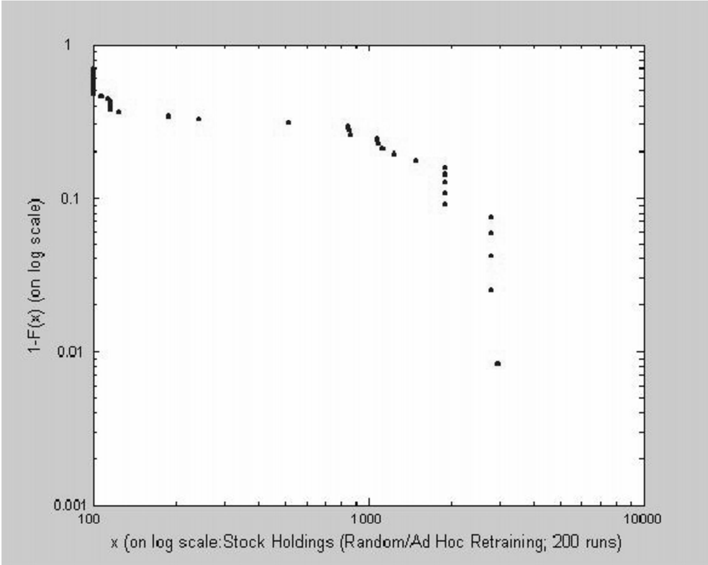

Fig. 4. Cummulative Density Function of Stock Holdings: *Ad Hoc* Retraining 200 periods.