Stress-testing SPEI:

Policy recommendations about the Mexican Payment System simulating distressed liquidity scenarios

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Abstract

The purpose of this project is to achieve a better understanding of the Mexican Payment System SPEI in distressed liquidity conditions. To this extent, we simulate the operational failure of one of the participant to the system and measure its consequent impact onto the remaining institutions in terms of the additional funds (Extraordinary Liquidity) needed in order to fulfill the outstanding obligations as originally scheduled. The main finding of this study is that the severity of the consequences triggered by a failure depends strongly on the time of the day in which it takes place. Moreover, given the tiered structure of the network, and the different access to liquidity provided by the Central Bank, it is observed that disruptions to the system could force participants to completely change their borrowing behaviour in order to be able to settle all their daily payments. A further result is the construction of an index based on the topological properties of the network capable of providing a proxy for the Systemic Impact of single institutions.

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The propriety of the data used for this work belongs to Banco de México. Therefore, because of its confidential nature, it has not been possible to include the source data from which the results showed in this report have been drawn.

For any enquiry, please contact Dr Biliana Alexandrova-Kabadjova[1] at Banco de México.

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Contents

1 Introduction **1**
1.1 Literature Review . . . 2
1.2 Financial Market Infrastructures . . . 4
1.2.1 Payment Systems . . . 6
1.2.2 Mexican Payment System: SPEI . . . 7
1.2.3 Liquidity Provision Mechanisms . . . 8

2 Objectives **10**

3 Methodology **11**
3.1 Distressed Liquidity Scenario Simulations . . . 11
3.1.1 Normal Liquidity . . . 11
3.1.2 Extraordinary Liquidity . . . 13
3.2 Individuation of the Systemically Important Institutions . . . 14

4 Results **16**
4.1 Extraordinary Liquidity . . . 16
4.2 Systemic Impact Index . . . 23

5 Conclusion **29**

References **31**

A Network Measures **35**
A.1 Topological Measures . . . 35
A.2 Centrality Measures . . . 36
A.3 SinkRank . . . 37

B Tables **39**

C Codes **42**
C.1 Python code . . . 42
C.2 Stata code . . . 46

1 Introduction

Ever since the 2008 global financial crisis, it has been clear how systemic risk had to be regarded as one of the major risk factor in the banking sector, especially for the severity of the consequences that a contagion could trigger. Despite that, regulations on this subject are still a grey area, mostly because of the authorities' lack of a common agreement on how to define this risk class (Acemoglu et al., 2015). This inadequacy has lead to the flourishing of a wide range of studies that tackled the issue of measuring the stability of financial systems.

Research about contagion has been undertaken with different approaches, e.g. probabilistic (Bae et al., 2003; Rodriguez, 2007), epidemiological (Tovainen, 2012). In the last decades, though, it privileged the use of graph models which rely on network theory to characterise the structures under examination. In particular, attention has been paid on identifying the topological features of observed networks, in order to be able to infer the resilience to contagion of a particular system from its own structure. The extent of literature on financial networks is such that it makes it impossible to give credit to all the contributions to this line of research, therefore, we will only give account of the papers that somehow inspired or dealt with similar topics to the one treated in this work.

The rest of the report is structured as follows. Section 1.1 reviews the existent literature, whereas section 1.2 briefly introduces to the framework of Financial Market Infrastructures, and in particular to that of Payment Systems. It also provide a description of the Mexican Payment System SPEI. Section 2 outlines the objectives of this work, whereas section 3 describes the methodology followed throughout this project in order to perform the stress-testing of SPEI and to construct the Systemic Impact Index. Section 4 shows the results obtained from the simulations of distressed liquidity scenarios and, eventually, section 5 wraps up the conclusions drawn from the analysis of the results.

1.1 Literature Review

The first attempt to model financial systems using networks can be traced back to the early Noughties, but one of the milestones for this branch of research it is widely considered to be (Allen & Gale, 2000). In this work the authors model the propagation of liquidity shocks in the interbank loan market and measure the dependency of the severity of contagion on the completeness of the network. The same approach has been followed in many other studies such as (Amini et al., 2013; Gai & Kapadia, 2010; Alfarano & Milakovíc, 2008). While the first one provides a theoretical estimate of the default fraction starting from network characteristics, the others focus on the simulation of random networks. In particular, in (Gai & Kapadia, 2010), the authors analyse the dependence of the severity of contagion on the network structure and what emerges from the study is a *robust-yet-fragile* tendency: the probability of a contagion is, in general, low, but, in case there is an outbreak, the effects can be extensive. In (Alfarano & Milakovíc, 2008), instead, the attention is drawn onto the topology of random networks and how it influences the results of simulations.

The general unavailability of information about interbank claims has forced the researchers to run simulations based on randomly generated graphs. The choice of the topology is generally restricted to the easier regular and binomial graphs or to the more complex small-world and scale-free models introduced respectively in (Watts & Strogatz, 1998) and (Albert & Barabási, 2002). The use of such models provides a useful insight to the topic of financial contagion, but the results can arguably be assumed to be correct estimates of real scenarios. In (Mistrulli, 2008), the author compares the results of previous simulations based on randomly generated networks with the ones obtained from the observed bilateral exposures in the Italian interbank market. It emerges that simulations relying on the *maximum entropy*[2] approach tend to underestimate the actual severity of contagion.

[2]In absence of real data about the bilateral exposures, it is common practice to assume that banks evenly spread their lending, maximising the entropy of their linkages.

Although the vast majority of studies focused on the interbank market, recently other types of financial networks came under the spotlight. If, on the one hand, the recent turmoils that involved colossi such as A.I.G and General Motors drew the attention to the private companies sector (Elliott et al., 2014), on the other hand, Central Banks have started carrying out studies about the network structure of Payment Systems. Examples of these studies are (Rordam & Bech, 2009; Bech & Atalay, 2008; Becher, Millard, & Soramäki, 2008; Pröpper et al., 2008; Soramäki et al., 2006). All of them mainly analise the network topology of national Payment Systems i.e. the Denmark Kronos, the United Kingdom CHAP, the Netherlands TOP and the United States FedWire. In particular, the Bank of England study (Becher, Millard, & Soramäki, 2008) also provides a useful analysis of how the network characteristics changed during a day in which an *operational outage* affecting one bank was observed.

The concern about the effects of failures in payment and settlement systems has previously inspired studies that aimed at measuring the potential effects of disruptions in financial market infrastructures. Examples of those are (Humphrey, 1986) and (Angelini et al., 1996). In both the cases, the authors run simulations on a *DNS*[3] system, namely the United States CHIPS and the Italian BI-COMP, the results, though, are substantially different. In the first case, Humphrey shows that the failure of a major participant would affect approximately 37% of the participants in CHIPS. In the Italian case, instead, the domino effect is limited to a much smaller share of the participants (less than 1%). In (Angelini et al., 1996), the authors suggest that this difference could be due to the diverse scales of the fund flows in the two systems. A similar approach is used in (Hellqvist & Snellman, 2007). In this case, the authors use real transactional data to perform stress tests of the Finnish securities settlement system (SSS) HEXClear. The main finding is that, unless the disruption lasts for more than one operational day, a contagion is unlikely to occur and the failure has no significant impact.

[3]A Deferred Net Settlement system is a Payment System in which the transactions are settled at specic times during the day and the relative positions are netted out before the settlement.

A more recent series of studies carried out by Central Banks focuses on the analysis of the reaction of Payments Systems to distressed liquidity conditions. Notably examples are (Mazars & Woelfel, 2005; Enge & Øverli, 2006; Lublóy & Tanai, 2009). A common feature of these studies is that they base their simulations on data provided by payment simulators. In the first case, Mazars and Woelfel analyse the impact of a technical default (i.e. the inability to send payments do to technical incidents) onto the French PNS[4]. The results show that the technical default of a major institution could cause up to 10% of the payments by non-defulating participants to be rejected. In the latter studies the authors use the payment simulator developed by the Bank of Finland (Leinonen & Soramäki, 2003) to simulate transactional data for the national RTGS systems (the Norvegian NOB and the Hungarian VIBER). The results presented in (Enge & Øverli, 2006) are of particular interest since they distinguish between a Lower and an Upper Bound for the liquidity requirements entailed in case of a participant failure. The two measures, introduced in (Koponen & Soramäki, 1998), represent respectively the liquidity that participating institutions need to hold in order to be able to cover their net positions at the end of the day and the amount needed to settle every payment as soon as it is entered in the system. In (Lublóy & Tanai, 2009), instead, the authors show that, in case the participants are allowed to react to the liquidity shock, the disturbance to the system is greatly lessened.

1.2 Financial Market Infrastructures

The scope of this project is limited to the study of a specific type of systemic risk in the context of the Mexican Payment System SPEI®. Therefore, before describing the features of such system, it seemed appropriate to briefly introduce the topic of Financial Market Infrastructures (FMIs).

As defined by the Committee on Payment and Settlement System, a FMI is a *multilateral system among participating institutions, including the operator of the system, used for the purpose of* ***clearing***, ***settling*** *or* ***recording*** *payments, securities, derivatives, or*

[4]PNS (Paris Net Settlement) is a Large-Value Settlement System that operates on a Real-Time Net Settlement basis, although only payments above the threshold of EUR 1 million are considered for netting.

other financial transactions (CPSS, 2012).

This definition gives a clear idea of the objectives for which these infrastructures have been designed, which are coping with operational problems, restarting operations promptly and avoiding the loss of information due to such disruptions (Diehl, 2016). The very structure of FMIs is such that they are to be considered pivotal in the Financial System they operate in. On the one hand, they provide three levels of links: to their participants (e.g. banks), to other financial market infrastructures, and to external service providers (Berndsen, 2011, 2012). On the other hand, especially as far as Payment Systems are concerned, the type of activities they offer is, to some extent, unique and, therefore, the effects of outages could have an unpredictably vast extent (Chouinard & Ens, 2013).

Because of their own nature, the variety of FMIs is extensive: they differ for function, organisation, propriety and non-profit status. For the sake of simplicity, and according to distinction made in (CPSS, 2012), we will identify five main types of FMIs.

1. **Payment Systems:** *set of instruments, procedures, and rules for the transfer of funds between or among participants;*

2. **Central Securities Depositories:** *provide securities accounts, central safekeeping services, and asset services, which may include the administration of corporate actions and redemptions, and play an important role in helping to ensure the integrity of securities issues;*

3. **Securities Settlement Systems:** *enable securities to be transferred and settled by book entry according to a set of predetermined multilateral rules;*

4. **Central Counterparties:** *interpose themselves between counterparties to contracts traded in one or more financial markets, becoming the buyer to every seller and the seller to every buyer and thereby ensuring the performance of open contracts;*

5. **Trade Repositories:** *entities that maintain a centralised electronic record of transaction data.*

Accordingly to the purpose of this work, we will discuss in more details only the first type of Infrastructure: the Payment Systems.

1.2.1 Payment Systems

Payments System (PS) are those Infrastructures that channel the transfer of funds between the institutions that have accounts in the system. The way in which transfers are arranged is based on agreements between the participants and the operator of the infrastructure (CPSS, 2012). As mentioned above, the variety of FMIs is extensive. As far as PS are concerned, there are several distinctions that can be applied. The first one relates to the type of payment settled. Thus, there are *Retail* (RPS) and *Large-Value Payment Systems* (LVPS). In RPS the value of transactions is, generally, lower but it is balanced out by a much higher volume. LVPS, instead, are used to perform large value payments, they deal with a lower number of operations and they are usually run by National Central Banks (Bech & Hobijn, 2007). It has to be pointed out that recently some countries have introduced LVPS that also settle retail payments, as is the case of the Mexican SPEI.

A further classification of PS can be done according to the way in which payments are settled. Table 1 summarises all the possible cases. Of the four categories, Real-Time Gross Settlement and Deferred Net Settlement systems are the most widespread. Recent studies, though, showed that RTGS are more resilient to systemic risk, and, therefore, are to be preferred for LVPS (CPSS, 1997; Leinonen & Soramäki, 2005). This is due to the fact that, despite settling each transaction submitted to the system without any netting mechanism, by processing the operations on a continuous base, they avoid the accumulation of payments that could result in an excessive demand of liquidity to the sender, and, eventually, to its failure.

In general, Retail Payment Systems can be either DNS or RTGS since in this type of PS the size of payments is limited and so is the liquidity need of the participants involved. On the contrary, LVPS tend to be RTGS and, in order to reduce the systemic risk, they often implement Liquidity Saving Mechanisms (Davey & Gray, 2014). These procedures consist of algorithms that look for group of offsetting payments, match and settle them simultaneously.

The next subsection is devoted to the description of the Mexican Payment System SPEI, which belongs to the category of RTGS.

1.2.2 Mexican Payment System: SPEI

As mentioned above, SPEI®[6] (Sistema de Pagos Electrónicos Interbancarios) is a LVPS, which operates on a RTGS regime, run by the Mexican Central Bank. Every institution in the Mexican Financial System that wishes to perform Electronic Payments needs to have an account in SPEI in order to transfer funds. Such institutions include Commercial Banks, Public Development Banks, Brokerages, Insurance Companies, Pension Funds and other financial entities. According to the distinction made by the system, we will divide the institutions into two groups: Banks (including the first two classes) and Others (including all non-bank financial institutions). During the reference year, i.e. 2013, there were fifty Banks and forty-five Other institutions operating in SPEI. In addition to those, Banco de México and two other Financial Market Infrastructures (the Securities Settlement System and the Continuous Linked Settlement system that allows for Foreigh-Exchange settlements) had accounts in SPEI (Alexandrova-Kabadjova et al., 2014).

Despite working on a real-time base, SPEI operations are restarted every day. This means that the system opens at 6 pm of the previous day and closes at 6 pm of the following day (during 2013, the year from which the historical data is taken, the daily schedule was reduced, with the system opening at 7pm of the previous day and closing at 5:30 pm of the following day). During operation hours, participants send payments to the system according to their needs or on behalf of their customers. SPEI does not settle payments as soon as they are posted, but works on a Settlement Process (SP) base, instead. The

[6] All the information about Mexican Financial Market Infrastructures can be found on the Bank of Mexico website: http://www.banxico.org.mx/sistemas-de-pago/

system runs frequent SPs (on the time scale of seconds, therefor the Real-Time regime) and, in each process, only the transactions that fulfill certain criteria are settled. Those conditions include the total number of payments per SP, which must be greater than 300 (alternatively, if after five seconds the threshold is not reached, the payments are settled anyway), and the restriction to overdrafts on SPEI accounts, which must always be positive. If a payment does not fulfill the conditions, it is kept in queue until the following SP but, at the end of the operational day, the queue is emptied and every payment in it canceled. During 2013, the average number of daily transactions settled was 895,000, of which, 93% had a value below 10,000 USD and just 0.5% above 1,000,000 USD[7].

A feature that distinguishes SPEI from the majority of other PS is that it stores the log of transactions, including the time at which they were settled. This allows the analysis of intra-day dynamics using real historical data which is not feasible otherwise (many of the Central Bank studies cited in section 1.1 had to rely on simulated data). In particular, this availability of information allowed for a detailed description of the network topology of SPEI (Martínez-Jaramillo et al., 2012) (from which the authors derived a criterion for detecting Systemically Important institutions), and also the distinction between the networks composed of participant-to-participant and customer-to-customer transactions (Alexandrova-Kabadjova & Garcia-Ochoa, 2015).

1.2.3 Liquidity Provision Mechanisms

Given the nature of Payment Systems, the main risk entailed is the sudden lack of liquidity that could cause a participant to fail to fulfill its obligations. This is why it is paramount to understand how institutions provide themselves with the funds required, and to measure the magnitude of the needs in case some disruption occurs, which is the purpose of this work.

Whenever a participant needs to send a payment, and lacks the funds to settle it,

[7]Banco de México statistics

there are two actions it can take. First, it can borrow the amount needed, and second, it can delay the payment waiting for enough incoming funds to cover the obligation. The trade-off between these two strategies defines the borrowing behaviour of the participant and has been thoroughly studied with approaches ranging from game theory (Bech, 2008) to simulations (Leinonen & Soramäki, 2005), from real data analysis (Becher, Galbiati, & Tudela, 2008) to simulated data analysis (Massarenti et al., 2012).

This type of study has already been performed on SPEI, and the outcomes showed that the majority of participants (Banks and Other institutions) is capable of *recycling* an important share incoming payments (up to 90%) in order to cover its obligations (Alexandrova-Kabadjova et al., 2016). Despite that, there is evidence of a heterogeneity in participants' behaviours, especially when different size participants are taken into consideration (Alexandrova-Kabadjova & Solís-Robleda, 2013).

In both the studies cited above, the authors distinguish between incoming payments and external funds. In order to fully comprehend the risks associated with sudden liquidity shortages, it is useful to point out what can be the sources of external funds. As mentioned in section 1.2.2, every participant must have an account in SPEI. In addition to that, Banks must also have an account in SIAC[6]. SIAC is a system run by Banco de México that allows institutions to incur collateralised overdrafts. The collateral is composed of the Monetary Regulation Deposit (DRM) held by banks at the Central Bank (the ovedrafts are capped by the DRM). Participants that wish to borrow money, can transfer funds from their accounts in SIAC to their accounts in SPEI. At the end of each operational day, since the accounts in SPEI must be zero, all the remaining funds must be transfered back to SIAC. A further source of external liquidity is DALÍ[6]. DALÍ is the Mexican Security Settlement System, run by a private company (INDEVAL), that operates in a Delivery versus Payment regime. In DALÍ both Banks and Other institutions can perform repos, and the transfer of liquidity is done through the account that INDEVAL has in SPEI. Figure 1 shows the overall fund flows.

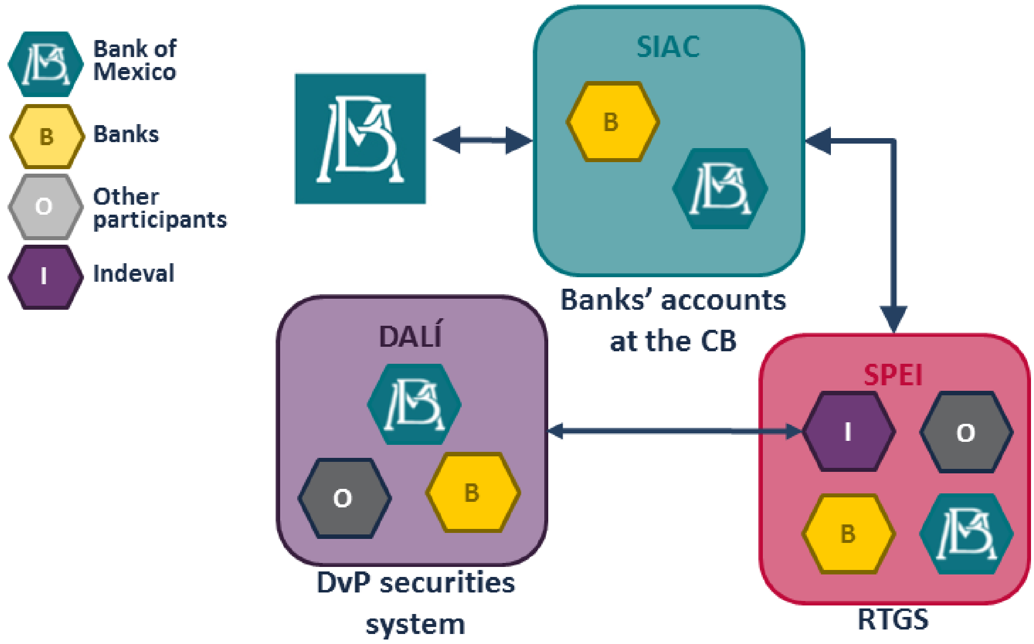


Figure 1: Scheme of the connections between the FMIs in Mexico. Source: Banco de México.

2 Objectives

The purpose of this work is to assess the impact of the operational failure of a participant in SPEI. The impact is measured in terms of the extra funds (from now on we will refer to it as Extraordinary Liquidity or EL) that non-defaulting institution will require to gather in order to be able to continue operating normally. This amount, which has been labelled Upper Bound in (Koponen & Soramäki, 1998), is the difference between the levels of liquidity needed in normal times and in distressed conditions, and has to be considered an estimate of the worst-case scenario.

Furthermore, this works aims at constructing a statistical index capable of ranking the participants to SPEI according to their potential Systemic Impact. The purpose of such index is to provide a proxy for the amount of funds needed by the whole system, should a target institution fail to send payments, starting from its network properties.

3 Methodology

In this section we describe the methodology followed in order to perform the measure the Upper Bound level of liquidity in distressed conditions, EL, and to construct the statistical index.

3.1 Distressed Liquidity Scenario Simulations

The objective of estimating the amount of liquidity needed in the worst-case scenario is to assess whether an institution would be able to borrow those extra funds, should it be willing to send all the payments scheduled, or be forced to.
The data used for this purpose is the daily historical data spanning from January 2 2013 to December 30 2013. Since the typical operational day is composed of a variable number of transactions, ranging from hundreds of thousands to millions, each simulation is computationally demanding. Therefore, we decided to extract from the sample a subset of 152 days containing all the Mondays, Wednesdays and Fridays of that year. The choice of the days aimed at capturing possible intra-week patterns by considering the beginning, the middle and the end of the week. For each of the 152 days we run the simulation of operational failures.

In order to measure the EL needed by every participant in a given scenario, it is first necessary to know the amount of liquidity originally borrowed during the given day. Hence, for every day in the data set we calculated the Normal Liquidity, and then, starting from that, we derived the Extraordinary Liquidity.

3.1.1 Normal Liquidity

The calculation of Normal Liquidity (NL) has been carried out using an algorithm that has already been exploited for the same purpose in (Alexandrova-Kabadjova et al., 2016). In particular, this algorithm relies on some simple hypotheses:

1. **Accounts starts from zero:** since, at the end of each operational day, the accounts

in SPEI must be zero, it is reasonable to assume that every participant starts the new day with no money.

2. **There are just two sources of liquidity:** as mentioned in section 1.2.2, accounts is SPEI cannot incur overdraft, therefore, the liquidity used to send payments must come either from any reserve accumulated thanks to previous incoming payments, or from an external source, be it SIAC or DALÍ[8].

3. **Payments within a SP are offset:** despite SPEI being a RTGS, some sort of offsetting takes place within every SP. The payment selection algorithm in SPEI must ensure that participant accounts do not go negative. At the same time, though, it must guarantee that enough liquidity flows in the system. Therefore, even if an outgoing payment would make the sender account negative, if another participant is sending enough funds to make the sender account positive, the algorithm allows both the transactions to be settled. Note that both the gross operations are settled, not just their net difference.

Provided that those hypotheses hold true, the algorithm calculates the NL in the following way. Let I and T bet the set of participants and Settlement Processes in a given day. Let P_{it}^r and P_{it}^s be respectively the total funds received and sent by participant i during cycle t. Then, $\forall i \in I, \forall t \in T$, we define A_{it}, the net amount received, S_{it}, the reserve of liquidity hoarded, and F_{it}, the cumulative amount borrowed from the external resource. Given that, at $t = 0$, the three quantities are zero for all the participants, during each cycle they are updated as follows:

$$A_{it} = P_{it}^r - P_{it}^s \tag{1a}$$

$$F_{it} = F_{it-1} - (S_{it-1} + A_{it}), S_{it} = 0 \qquad \Leftrightarrow (S_{it-1} + A_{it}) < 0 \tag{1b}$$

$$F_{it} = F_{it-1}, \qquad S_{it} = (S_{it-1} + A_{it}) \Leftrightarrow (S_{it-1} + A_{it}) \geq 0 \tag{1c}$$

[8]It has to be noted that money transfer from participant accounts in SIAC or DALÍ to the one of the same participant in SPEI are not recorded in the payment transaction log. Therefore, these liquidity flows are not observed in SPEI

Normal Liquidity for institution i at the end of the day is then represented by F_{iT}.

3.1.2 Extraordinary Liquidity

Once the NL for every participant is known, it possible to proceed to the simulation to calculate the EL. The model used to perform the simulation relies on the following assumptions, common in contagion literature:

1. **Shocks are exogenous:** the cause of the failure is external to the system. This means that, originally, only the failing institution is affected by the default.
2. **Failing institution does not recover:** for the sake of simplicity, we assume that when a participant faces an outage, the problem is not fixed until the next day.
3. **Counterparties cannot react:** since we want to calculate an Upper Bound, the only action that counterpaties to the failing institution can take is borrowing the missing liquidity.
4. **Liquidity is not capped:** there is no limit to the liquidity that counterparties can borrow to cover their obligations.

Whilst the first two hypotheses are plausible since usually the source of the disruption is an operational problem within the systems of a single institution, and the extension in time of the outage depends on the severity of the problem, the last two are unlikely to be be fulfilled. Despite that, they were necessary in order to reduce the spectrum of possible scenarios that would require an agent-based model to be explored. However, the EL levels obtained have been compared with the size of the DRM for every institution in order to check the validity of hypothesis 4.

Differently from the studies cited in section 1.1, we decided to simulate the failure of each and every participant, one at a time, disregarding their size or degree of connectivity. The motivation of this choice lies in the fact that only 98 institutions have an account in SPEI, and therefore, it is possible to obtain a complete overview of the scenarios without it being too computationally demanding. Additionally, we decided to simulate defaults happening at different hours: 6, 8, 10 and 11 am, and 12, 1, 2, 4, and 5:30 pm. The rationale of this choice is linked to the distribution of the volume of transactions (Figure 2). Before 6-8 am only a small number of low value payments is usually settled (typically fund transfers from Banco de México and CLS). Between 10 am and 4 pm the big bulk of operations is performed while, although at 5:30 pm there are still some large-value payments to be settled, the majority of transactions already took place and the liquidity has been allocated.

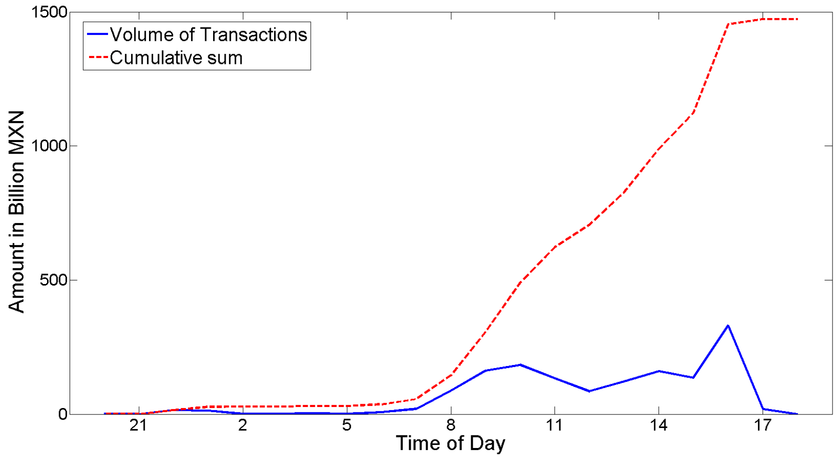


Figure 2: Intraday distribution of the Volume of Payments

The simulation of a default was performed in two steps. First, all the payments sent by the chosen institution after the failing hour were set to zero (which is equivalent to removing them). Subsequently, the same algorithm presented in section 3.1.1 was applied to the new list of transactions. The EL have been obtained from the results provided by the algorithm by subtracting the NL calculated beforehand.

3.2 Individuation of the Systemically Important Institutions

The final step of this work consisted in the construction of the statistical index to be used for ranking SPEI participants according to their Systemic Impact. The Systemic Impact (SI) is defined as the sum of the Extraordinary Liquidity that the counterparties to the failing institution would require in case a given participant failed at a specific time of the day. In practical terms, the SI measures the extent of the liquidity shortage triggered in a given scenario.

The rationale of this index is correlating the network properties of participants with their importance in terms of the severity of the consequences their outage could spark. Since the objective was to produce an easy-to-use synthetic measure to provide a proxy for the Systemic Impact, we decided to keep it as simple as possible, i.e. not to include too many indicators.
Before proceeding to describe the methodology followed at this stage, we wish to point out that it is not within the scope of this work to provide a theoretical structure for the index, nor to attempt to link it to the complex dynamics underlying the observed phenomenon.

The index has been constructed by using a simple linear model. The dependent variable is the Systemic Impact of institution i, calculated at hour h of day d. The explanatory variables are the network characteristics of institution i during day d[9]. Since the SI proved to be strongly dependent on the time of the day in which it is calculated, we decided to include a constant term and to let all the coefficients and constant to be a function of the failure hour. The estimation of the coefficients has been performed with the Ordinary Least Square method. Inasmuch we were regressing data concerning a population, in order for the model to be consistent with the assumptions of the OLS method (errors must be orthogonal to the explanatory variables), we had to account for *fixed effects*, i.e. unobserved constant contributions different for every institution in the sample. Although this addition was required when estimating the coefficients, we decided not to include it in the model when calculating the ranking of the participants. The model used is then:

$$y_{ih} = \delta_h + \mathbf{x_i}^T \boldsymbol{\beta_h}, \tag{2}$$

where $\mathbf{x_i}$ is the vector containing all the network properties of institution i.

[9] We considered to calculate the network properties at different hours of the day but, in addition to being computational intensive, it did not improve the result of the regression

In literature there are many examples of methods to rank institutions by their systemic importance (BCBS, 2011; Martínez-Jaramillo et al., 2012; Chouinard & Ens, 2013). Despite not agreeing on the methodology to be followed to produce the ranking, they all set out some criteria that Systemically Important Institutions must fulfill. Those are **Size**, **Interconnectedness**, and **Substitutability**. In order to capture those features, we selected three typical network measures: **Strength**, **Degree**, and **Centrality**[10]. Alongside with those, we included a further measure specifically designed to describe the importance of a participant in a Payment System: **SinkRank** (Soramäki & Cook, 2013).
In order to construct our index, we performed the regression with combinations of the above indicators in order to identify the subset of them that produced the best fit for the Systemic Impact.

4 Results

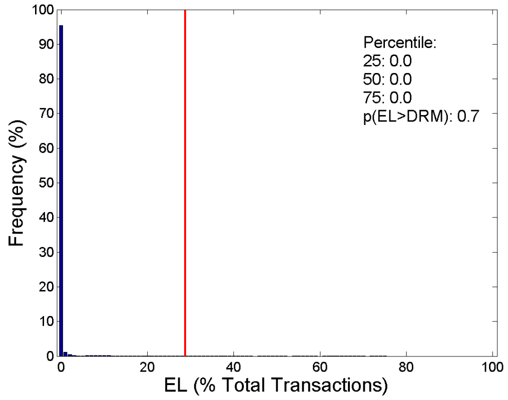
This section is devoted to the presentation the results obtained from the simulation of the distressed liquidity scenarios and the Systemic Impact Index.

4.1 Extraordinary Liquidity

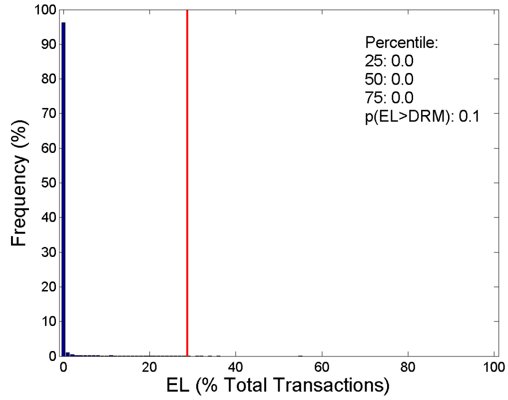
In order to simulate the failures, we used a Python code, part of which is shown is appendix C.1. The output of the program is a matrix containing the Extraordinary Liquidity needed by each institution, depending on which participant failed, in a given day and at a given time. Having a reasonable number of available days, and 97 different scenarios for each day (one for every failing institution) allowed us to draw the distribution of the EL for every participant. Note that, in order to be able to compare distributions related to differently sized participants, we had to normalise the results obtained. Thus, we chose to use as normalisation the total payments sent during the day by the target participant.

[10]Among all the centrality measures, we selected Betweenness, Closeness and PageRank Centrality. The description of the network measures can be found in appendix A.

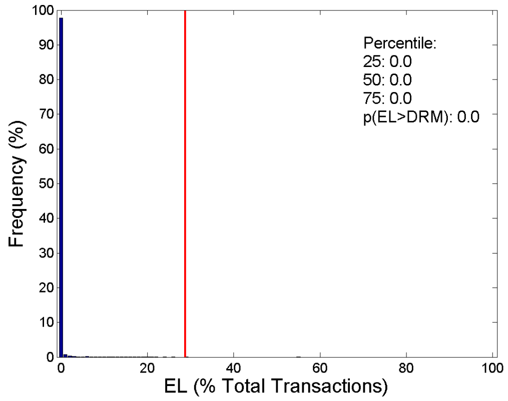
Here it follows the overview of the results.



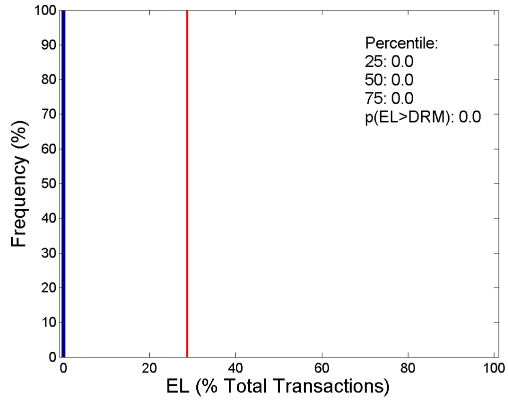
(a) 6:00 am



(b) 10:00 am



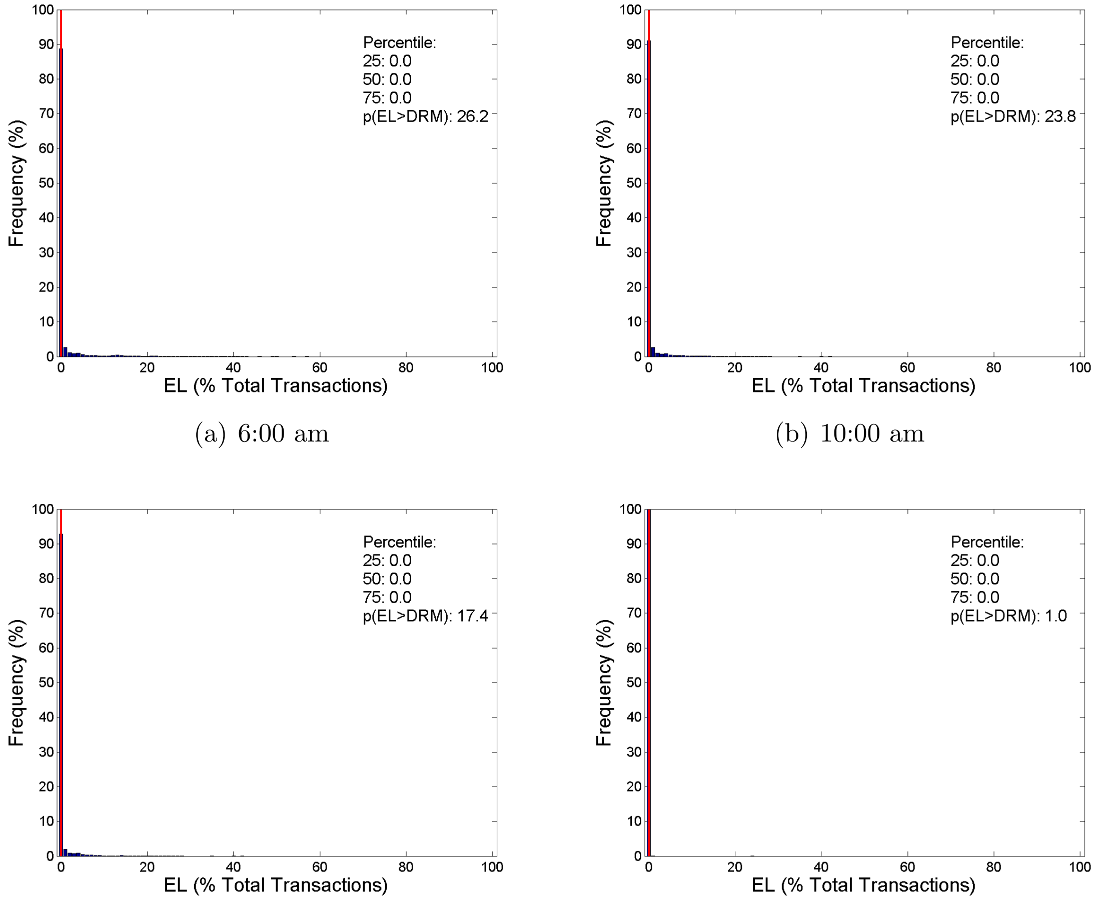
(c) 2:00 pm



(d) 5:30 pm

Figure 3: Unconditional distribution of the EL of a core participant for failures happening at different hours.

Figures 3 and 4 show the distributions of Extraordinary Liquidity of two participants for failures taking place at 6, and 10 am and 2, and 5:30pm. As mentioned in section 1.2.3, in every graph we added the limit represented by the DRM in order to compare the magnitude of the extra funds needed with the cap to the overdrafts. (Note that only Banks can incur overdrafts. Other institutions, though, as well as Banks, can still access Central Bank liquidity via repos in DALÍ). Before commenting on the results shown in figure 3 and 4, we wish to point out that, in the network of SPEI, it is possible to identify two types of participants according to their level of connectivity: **core** and **periphery** (Alexandrova-Kabadjova et al., 2014). The core is represented by the subset of nodes that are almost completely interconnected with each other, whereas the scarcely-connected participants belong to the periphery.



(a) 6:00 am

(b) 10:00 am

(c) 2:00 pm

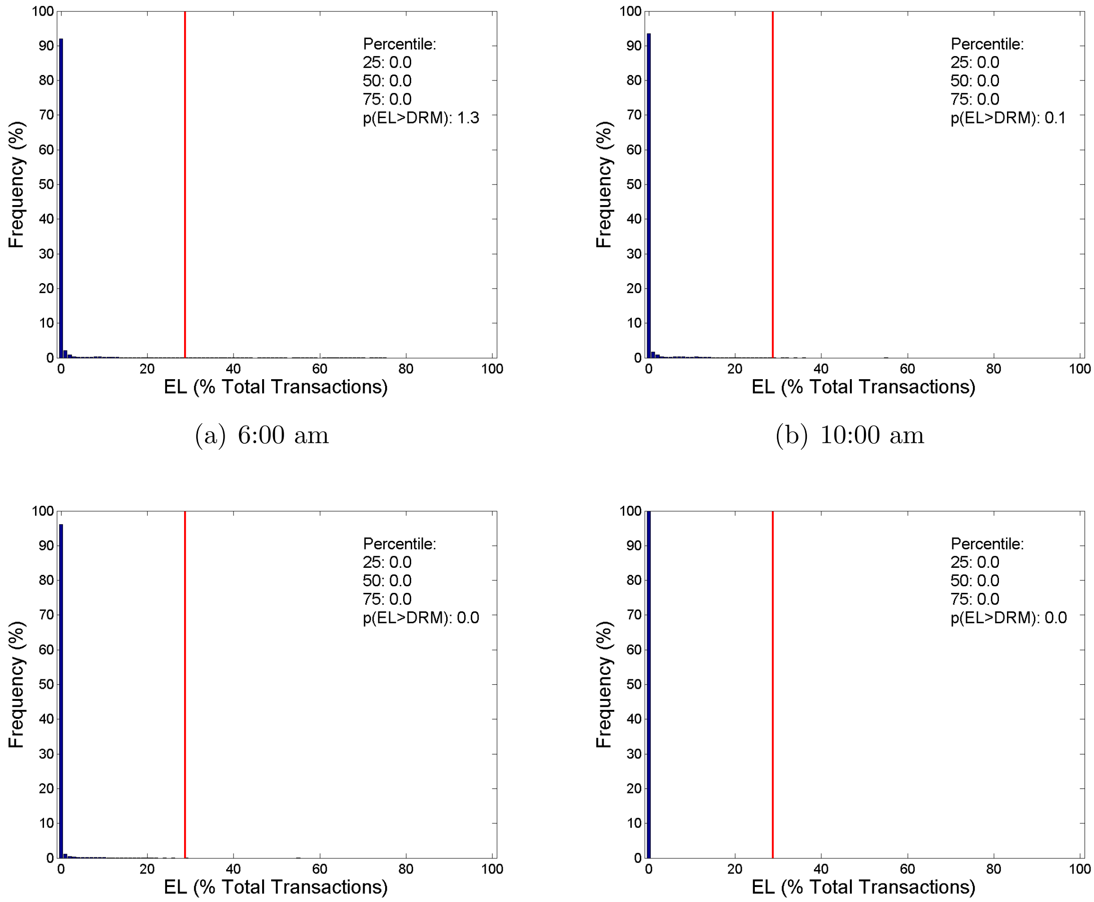
(d) 5:30 pm

Figure 4: Unconditional distribution of the EL of a periphery participant for failures happening at different hours.

Bearing this distinction in mind, we can look at the distributions. Figure 3 shows the results for a core institution. What is immediately clear is that, independently from the time of failure, the core participant will, almost always (in more than 99% of the cases), be able to cover all its obligations using only the liquidity provided by SIAC. Moreover, we can see that, even in the worst scenario, i.e. when the failure takes place at 6 am, it is unlikely (the probability is lower than 5%) that more funds will be required, compared to normal times.

Figure 4, instead, displays the results for a periphery participant. In this case, since the institution has no deposit at the Central Bank, it will not be able to incur overdrafts.

Nevertheless, in still nearly 75% of the cases the institution will not need to gather more funds, since the defaults will not trigger any Extraordinary Liquidity. Furthermore, the probability of the EL exceeding the DRM (in this case it is equivalent to say EL being greater than zero) decreases as the hour of failure increases. This is in line with our expectations since, as the failure is "delayed", the amount of the outstanding payments is reduced, and so is the the liquidity needed by the participant.



(a) 6:00 am

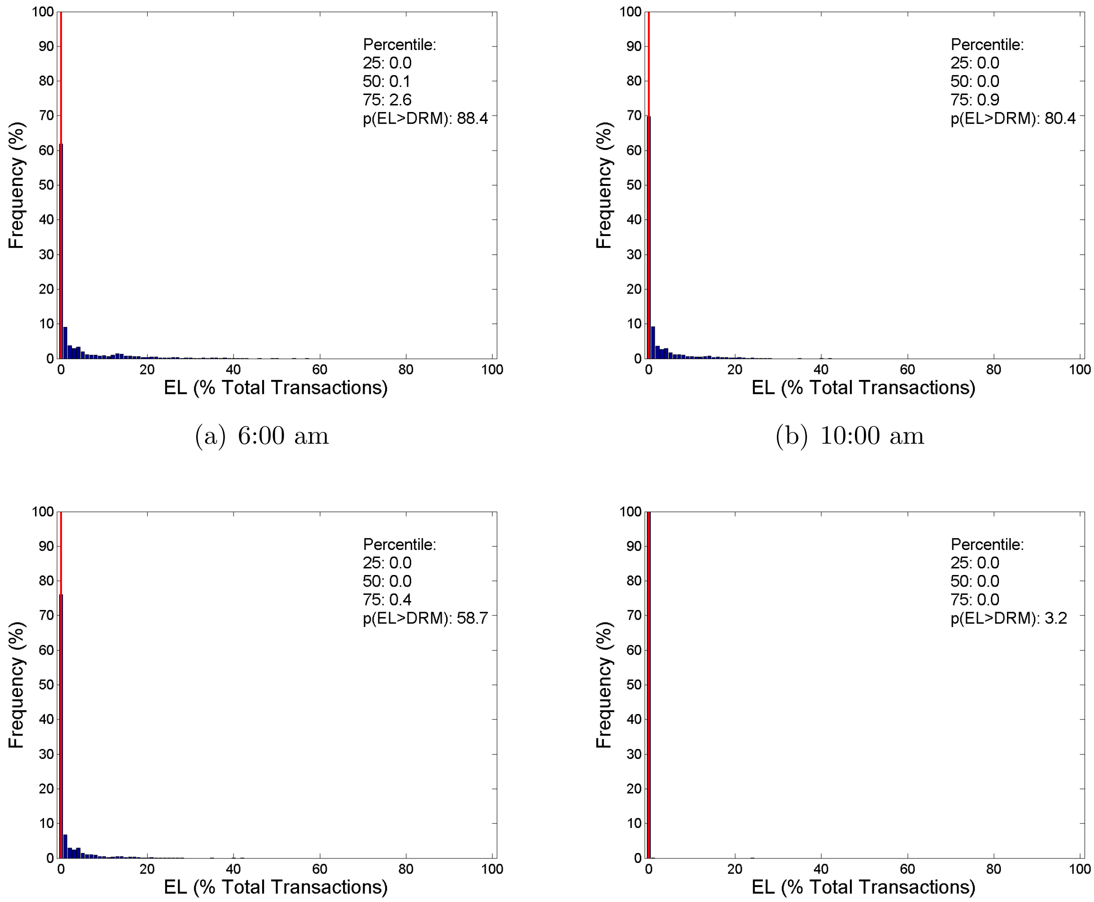
(b) 10:00 am

(c) 2:00 pm

(d) 5:30 pm

Figure 5: Conditional distribution of the EL of a core participant for failures happening at different hours.

Given the difference in nature of the core and the periphery, we decided to look at the results from a further perspective. Since the two types of participants differ for the number of counterparties they have, we chose to draw the distributions of EL, conditional on the failing institution being a counterparty of the participant. Figure 5 and 6 show such distributions.



(a) 6:00 am

(b) 10:00 am

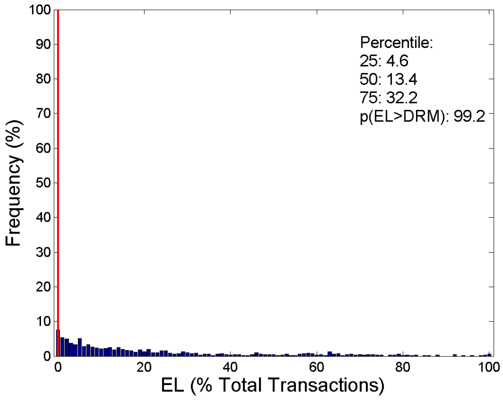
(c) 2:00 pm

(d) 5:30 pm

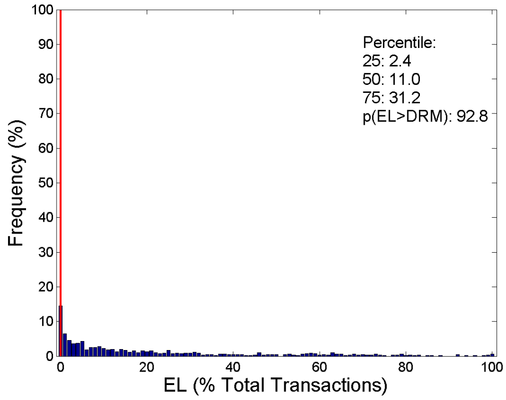
Figure 6: Conditional distribution of the EL of a periphery participant for failures happening at different hours.

As expected, the conditional and unconditional distributions for the core participant (Figure 5) do not differ much since the core tends to have as counterparties the majority of other participants. The probability of not suffering EL is always still above 90%, and, despite being double compared to the unconditional case, the probability of the EL exceeding the DRM is still just around 1%.

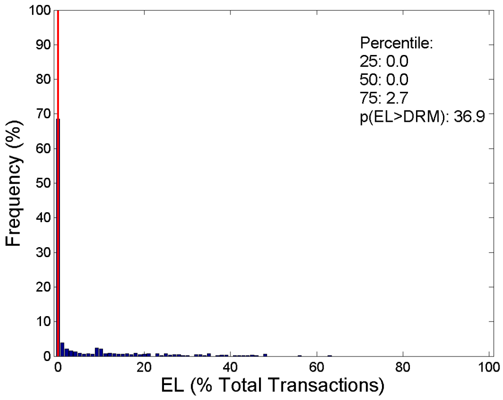
The comparison of the results for the periphery proves to be of more interest. By looking at figure 6, we can see that the distribution exhibits a much fatter tail. While in the unconditional case the probability of the participant needing extra funds was just above 25%, when looking only at the failures of the counterparties, this amount reaches nearly 90%. Moreover, we can observe that, in both the cases, the frequency by which the EL falls beyond the DRM remains stable in the first two scenarios (6 and 10 am), it reduces for failures happening at 2pm, and then it substantially drops for disruptions starting at 5:30pm.



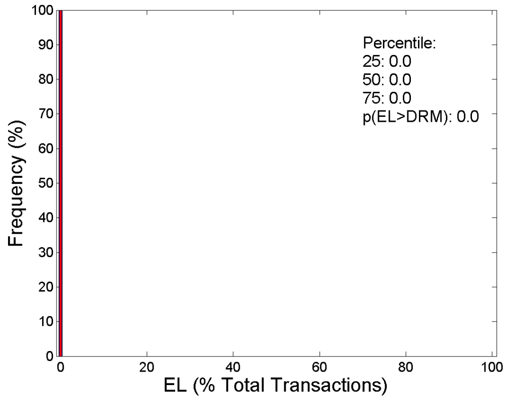
(a) 6:00 am



(b) 10:00 am



(c) 2:00 pm



(d) 5:30 pm

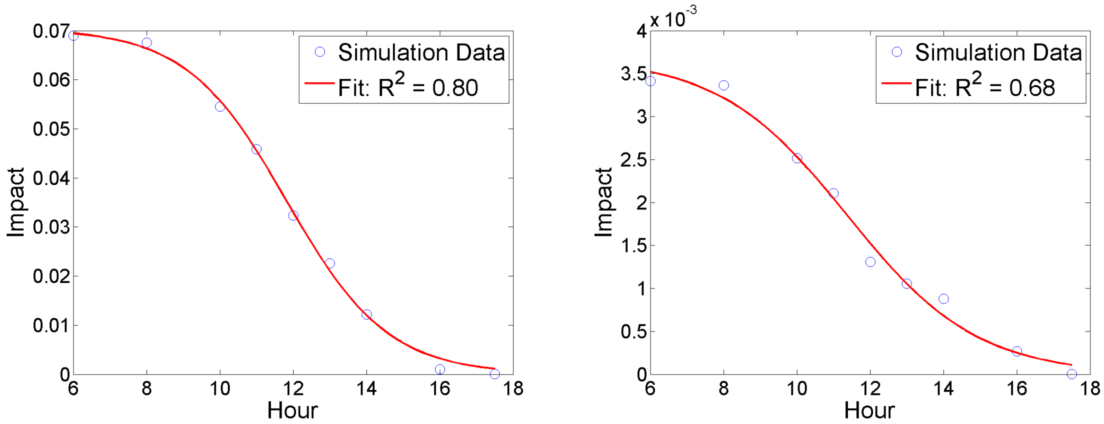
Figure 7: Conditional distribution of the EL of a periphery Other participant for failures happening at different hours.

We wish to remark that participants tend to show different behaviours according to the category they belong to. While in the core there are only Banks, in the periphery there is an heterogeneous mixture of Banks and Other institutions. This causes the periphery participants to manifest a range of different EL profiles. While figures from 3 to 6 refers to Banks, figure 7 represents the distribution for a Non-Bank Financial Institution. As we can see, the typical initial peak exhibited in the previous cases is absent (at least in the first two plots), and the probability of the EL to be greater than the DRM reaches 99%. Moreover, we notice that, for disruptions starting at 6 - 10 am, the magnitude of the impact on this institution can reach up to 100% of the total payments sent during the day. This feature is common among the NBFIs and can be explained in the following way. The majority of the firms that fall under the category of Other institutions have accounts at the some Bank in SPEI. It happens that, instead of providing themselves with liquidity via repos, they require those Banks to send them the funds they need by the means of payments. If those counterparties fail to send the payments, they will need to gather extra funds to cover all the outstanding payments scheduled for the day. In other terms, this means that their behaviour will be overturned from prefect *recycler* to *free rider*.

Once again, in figure 7, we can observe the same pattern for the probability of the EL exceeding the DRM. It seems to be stable from 6 to 10 am, it decreases from 10 am to 2 pm and then it plummets to zero from 2 to 5:30 pm. This behaviour suggested us to try to look at the impact of failures, rather than the EL, and how it changes over time. Therefore, we defined the Systemic Impact of an institution as the sum of all the Extraordinary Liquidities triggered by the failure of that particular institution. In this case, for the normalisation we have chosen the total volume of transactions settled during the day. Figure 8 shows the Impact of two participants, a core and a periphery one.

The first thing that we notice is the different scale of the Impact of the periphery institution compared to the core one: the consequences triggered by the core are several times more severe. Despite that, they both show the same monotonically decreasing trend. Given the level of regularity exhibited by the trends of the majority of the institutions, we tried to fit them using a *logistic function*[11]. The result of the fit is shown in figure 8. So far, we have not found any explanation for this trend. Although we expected it to be monotonically decreasing, we did not expect it to have a regular shape, since it is the result of the trade-off between the reduction of the number of incoming and outgoing payments, and the behaviour of each participant in terms of payment timing.

[11]The logistic curve is a function often used to describe the evolution of a population. It is derived from the *Verhulst equation:* $\frac{dP}{dt} = rP(1 - \frac{P}{K})$, from which we find that $P(t) = \frac{K}{1+qe^{-rt}}$, where $q = \frac{K-P_0}{P_0}$



(a) core (b) periphery

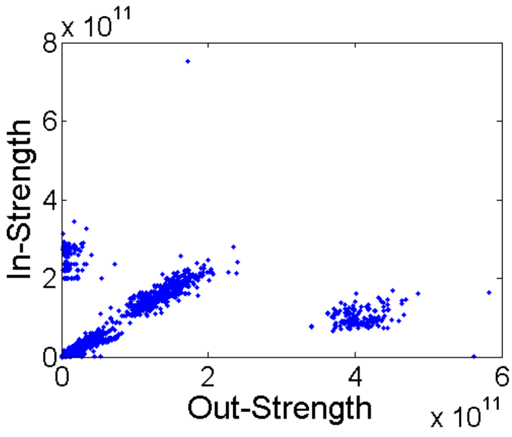
Figure 8: Time behaviour of the Impact of a core and a periphery participant in SPEI

4.2 Systemic Impact Index

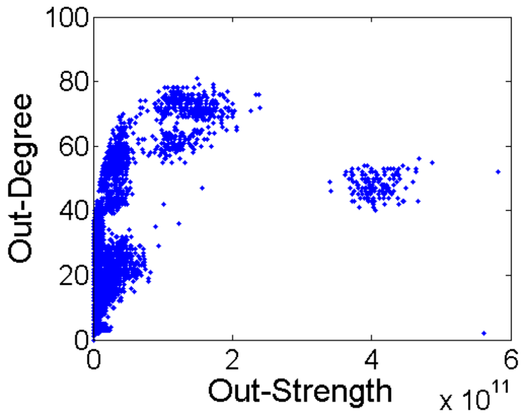
For the construction of the Systemic Index we used the Stata code that can be found in appendix C.2. The Systemic Impact is the same as the one defined in the previous section. The indicators selected for the regression are:

- In-Strength and Out-Strength;
- Out-Degree;
- Betweenness Centrality, Closeness Centrality and PageRank;
- and SinkRank.

The choice of including both In- and Out-Strength but only the Out-Degree is motivated by the recycling behaviour of SPEI participants and their network properties. If on the one hand the high level of recycled funds would suggest that the total amount of incoming liquidity should be correlated with the total amount of outgoing liquidity, on the other hand we can still observe net sender or receiver of funds, as we can see in figure 9(a). While the central region represents the *"recyclers"*, the top left one demonstrates the presence of net receivers and the bottom right that of net senders. These three different "behaviours" led us to consider the combination of In- and Out-Strength more explanatory rather than just either one of them.



(a) In-Strength



(b) Out-Degree

Figure 9: Correlation between Out-Strength and In-Strength and Out-Degree

Analogously, we could have expected Out-Degree and Out-Strength to be correlated, i.e. participants tend to evenly spread their payments (maximum entropy). But, once again the real practice proved to differ from expectations. As we can see from figure 9(b), the majority of the institutions are concentrated in a narrow vertical band to the left hand side of the plot. This tells us that to a higher number of links does not necessarily mean a higher volume of funds sent. Hence, given this lack of correlation, we decided to include them both. As far as the relation between In- and Out-Degree is concerned, we relied on the results presented in (Martínez-Jaramillo et al., 2012). The paper shows that the network of SPEI exhibits a high reciprocity, which means that, in more than 80% of the cases, for each outgoing link between two nodes there is also an incoming link. Thus, we assumed the two metrics to be correlated, and, therefore, it was redundant to include them both.

According to what prescribed by (BCBS, 2011) and (Chouinard & Ens, 2013), we initially selected three centrality measures to account for substitutability. What turned out, is that they were not adding any extra information to the Index because of their high correlation with other topological measures (figure 10), or between themselves, (figures 11). Therefore, we decided to drop all of the centrality measures.

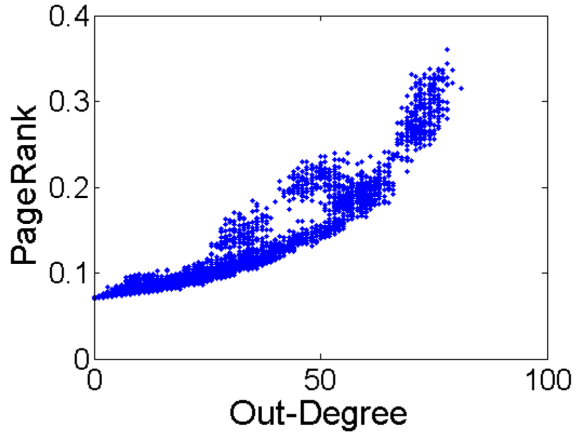
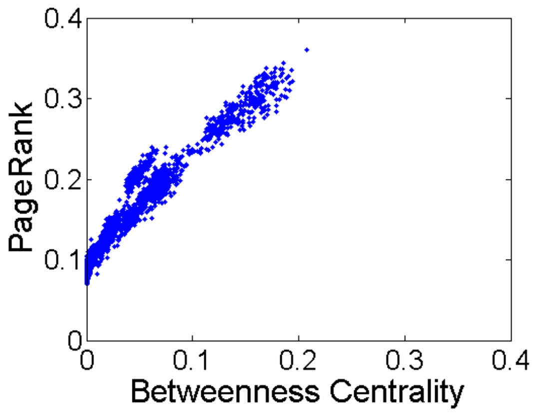
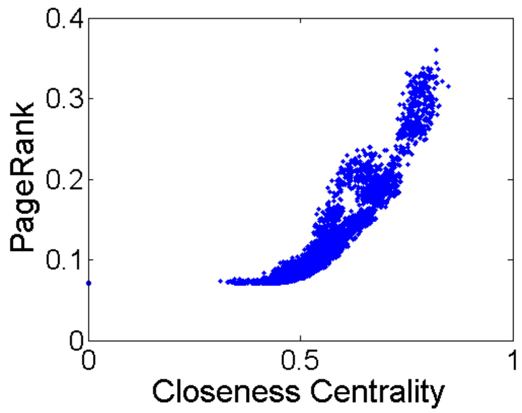


Figure 10: Correlation between Out-degree and PageRank



(a) Betweenness Centrality

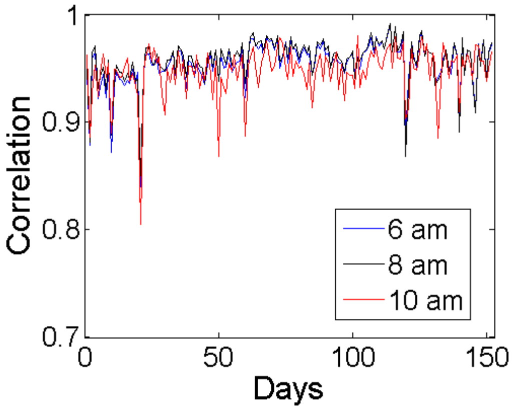


(b) Closeness Centrality

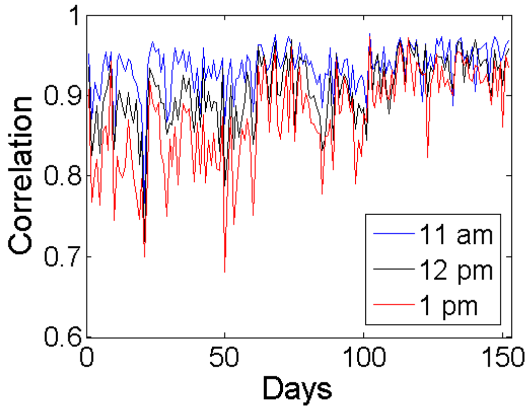
Figure 11: Correlation between PageRank and Betweenness and Closeness Centrality.

As far as SinkRank is concerned, although it has been designed to predict the effect of the failure of a participant to a Payment System, it proved to be reliable only for disruption starting during early hours (from 6 to 11 am). As we can see in figure 12, the correlation between this measure and the simulated systemic impact plunges, or becomes unstable when the failure takes place later on during the day. Nevertheless, this was the only measure that, if used as sole indicator in the Index, produced a reasonable fit of the Impact (R^2 above 0.7, see table 2).

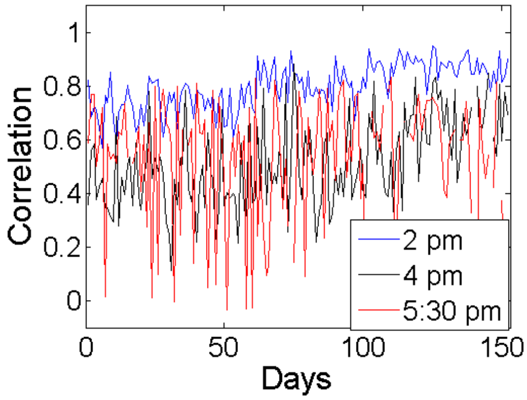
After running the regression with the four indicators (SinkRank, Out-Degree, Out-Strength and In-Strength), we noticed that the quality of the fit did not substantially improve compared to SinkRank-only case. Therefore we decided to change the model in



(a) 6, 8, and 10 am



(b) 11 am, 12, and 1 pm



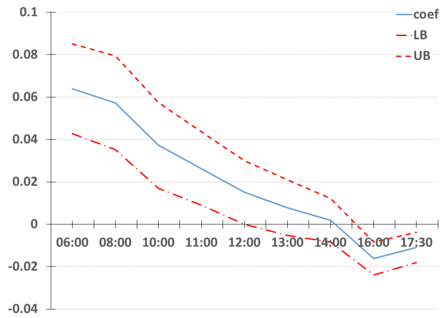
(c) 2, 4, and 5:30 pm

Figure 12: Correlation between SinkRank and the Systemic Impact at different hours of the day

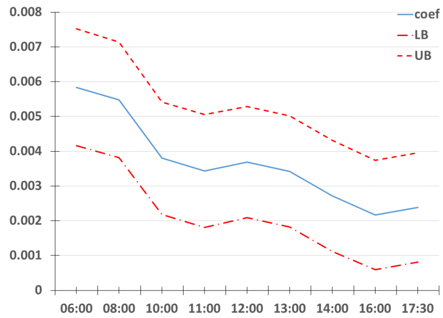
equation 2, in order for it to account for interactions between the indicators. What we found to be the most significant interaction, in terms of improvement of the regression, was the one between In- and Out-Strength. Therefore, final form of the Systemic Impact Index is:

$$y_{ih} = \delta_h + \beta_h^{SR} SR + \beta_h^{OD} OD + \beta_h^{OS} OS + \beta_h^{IS} + \beta_h^{IS \cdot OD} IS \cdot OD. \tag{3}$$

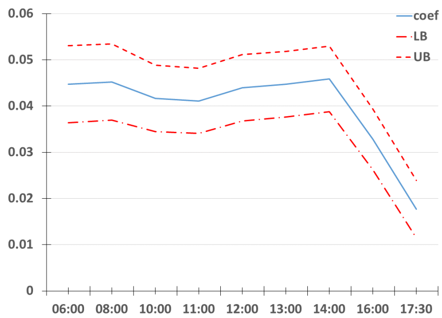
The results of the OLS regression are displayed in table 3, and are organised as follows. The first column shows the constant δ, and the β coefficients for all the indicators, for failures happening at 6 am. What follows are the corrections that must be applied to the first column in order to obtain the β coefficient for the later hours. The asterisks indicate the $p-$value of the coefficient or the correction. It is worth noting that, whilst the values found for 6 am are all statistically significant, the same does not apply to the corrections, especially for the case of 8 and 10 am. Here it follows the plot of the behaviour of the coefficients throughout the day.



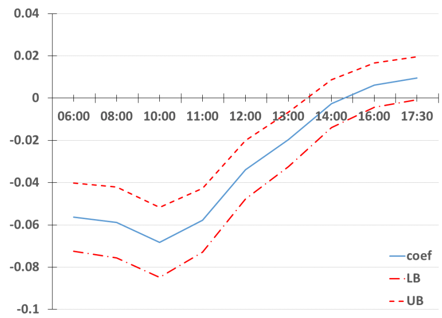
(a) SR



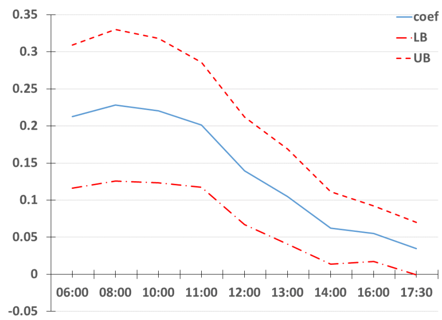
(b) OD



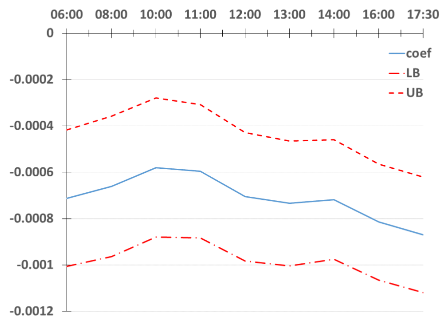
(c) OS



(d) IS



(e) IS·OD

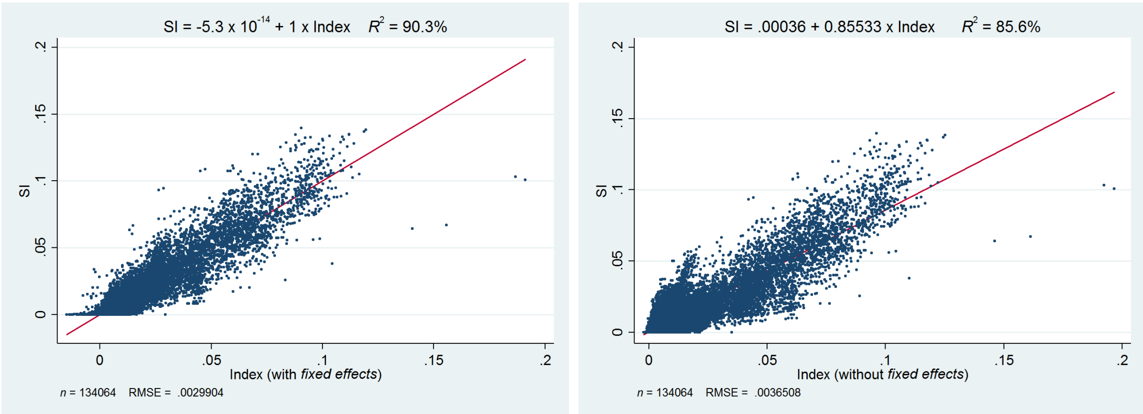


(f) δ

Figure 13: Behaviour of the coefficient during the day

By looking at figures 13, we can distinguish three different behaviours. In 13(a), 13(b), 13(d), and 13(e), we see that the coefficient gradually converge to zero, as the hour of disruption is moved ahead. In figure 13(c), we see that the magnitude of the coefficient remains stable until 2pm and then plummets at 4pm and 5:30 pm, and eventually, in 13(f), we see that δ tends to be constant during all the day. We wish to remark that what is observed for SinkRank, Out-Degree, In-Strenght and the interaction factor is in accordance with the trend observed for the Systemic Impact: it reamains stable between 6 and 8 am, it undergoes a big drop between 10am and 2pm, and then it reaches zero at the end of the day.

Eventually, in order to double check the results of the regression, we plotted the Index as it was constructed against the Systemic Effect and we performed a linear regression.



(a) Index with *fixed effects*

(b) Index without *fixed effects*

Figure 14: Comparison between the Systemic Impact and the Index with and without the fixed effects.

As we can see in figure 14, in both the cases the Index seems to accurately reproduce the Systemic Impact. As expected, the model performs better when it includes the fixed effects. As mentioned in section 3.2, though, we would recommend not to include them in the calculation of the Index, as they were introduced only for the model to be consistent with the the regression, and we are not capable of explaining their role in the Index.

5 Conclusion

The main findings of this work are the following.

- We constructed the distributions of Extraordinary Liquidity for every institution and for disruptions starting at nine different hours. This allowed us to have an insight of the magnitude of the consequences that the participants will need to face, depending on the scenario that is realised. What we observed is that, even in the worst scenario, in 90% of the cases, institutions will not need to gather extra funds compared to normal times. Despite that, when we looked at the effects of the failure of countarparties only, we spotted differences according to the category of the institution involved.
  - Core Banks proved to be more resilient and they hardly required to borrow more than 50% of their total payments sent, in addition to the liquidity they normally need to provide themselves with;
  - Periphery Banks demonstrated an intermediate behaviour, with some resembling their core counterparts, and some resembling Other institutions. In particular, we observed that their distributions still exhibit the initial peak for null EL, but the thickness of the tail varies within the sample. The maximum value of the EL ranges from 40 to 100% of the total payments sent;
  - Periphery Non-Bank Financial Institutions are the ones most affected by disruptions. Their distributions rarely show the initial peak, meaning that they will require extra funds in most of the cases, if one of their counterparties fails. Additionally we observe that, in the case of the least connected institutions, their worst scenario distribution resembles a uniform distribution, often reaching 100% of the total payments, and with average of 50% or above.
- We studied the dependence of the Systemic Impact of the participants on the hour of failure. What we observed is that, disregarding the type of institution, they almost always show the same behaviour. The fit of the trend with a logistic function proved

to be accurate. Although we cannot explain the dynamics that lead to this shape, they will be the topic of further investigation for their power to reconstruct the whole Systemic Impact curve given some sample data.

- We constructed a statistical Index that, starting from some simple network measures, could capture the Systemic Impact of any participant for failure happening at specific hours of the day. The power of the Index lies in the fact that the network measures used seems to be stable over time (Martínez-Jaramillo et al., 2012), and therefore, it is not necessary to update them frequently in order to obtain an accurate estimate of the Systemic Impact.

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A Network Measures

Here it follows a brief description of the network measures used for the construction of the statistical index.

We will use in this section the same notation that can be found in many papers dealing with Network Theory. A network is defined as a set of vertices (or nodes) and edges, where and edge is a link between two vertices, and it is indicated as (V, E). To every network it is associated an *adjacency matrix*, $A = [a_{ij}]\, \forall i, j \in V$, and the elements of A are defined as follows:

$$a_{ij} = \begin{cases} 1 & if\, \exists e_{ij} \in E \\ 0 & otherwise \end{cases},$$

where e_{ij} is the edge from the node i to the node j (given the framework we are working in, we will only deal with *directed* graphs).

A.1 Topological Measures

These category of measures is calculated looking at the nodes as single entities within the network.

Degree

The degree of a node is simply the number of links which the node is connected to. It is possible to distinguish between in- and out- degree, depending on whether only incoming or outgoing links are taken into consideration. Formally, the in- and out-degree of a node are defined as follows:

$$d_{in}(i) = \sum_{j \in V} a_{ji}$$

$$d_{out}(i) = \sum_{j \in V} a_{ij}$$

Strength

The strength of a node is somehow related to its degree. Indeed, it is defined as the sum of the weights of the links connected to the node. Analogously, we will have in- and out-strength, defined as follows:

$$s_{in}(i) = \sum_{j \in V} a_{ji} w_{ji}$$

$$s_{out}(i) \sum_{j \in V} a_{ij} w_{ij}$$

where w_{ij} is the weight associated with the edge e_{ij}. In our case, the weights represents the amount that participant i sends to participant j, therefore, the easiest interpretation of the in-(out-)strength of institution i, is the total amount that is received (sent) by that institution.

A.2 Centrality Measures

This set of measures aims at characterising the importance of a node as member of the whole network.

Betweenness Centrality

Betweenness centrality, as the name suggests, measures the grade of betweenness of a node, i.e. the fraction of shortest paths that passes through the node. Formally,

$$b(i) = \sum_{j \neq i \neq k \in V} \frac{\sigma_{jk}(i)}{\sigma_{jk}},$$

where σ_{jk} is the total number of shortest paths from j to k, and $\sigma_{jk}(i)$ the number of such paths that passes through i.

Closeness Centrality

Closeness centrality aims at measuring the average distance of a node from all the others in the network. It is therefore defined as

$$c(i) = \frac{1}{\sum_{V \ni j \neq i} d(i,j)},$$

where $d(i,j)$ is the distance between i and j, i.e. the number of links that are required to connect i to j. Clearly this definition applies only to connected graphs (where every node can be reached from any other). For disconnected graphs the definition of closeness centrality becomes

$$c(i) = \sum_{V \ni j \neq i} \frac{1}{d(i,j)},$$

where by definition $d(i,j) = \infty$, if j cannot be reached from i.

PageRank

PageRank uses a completely different approach to measure centrality. It was first proposed in (Page et al., 1998) as a method to rank websites. Its rationale is very simple: the centrality score of a node is equal to the sum of the scores of all its neighbours. Computationally, this translates into finding an eigenvector of the adjacency matrix. In the original definition, Page and Brin, instead of using the adjacency matrix, they used a "normalised" version of it, M, obtained by dividing each element a_{ij} by the number of outgoing links from node i. They also introduced a damping factor d that multiplies the matrix M, and shifts the score of all the nodes up by the amount $\frac{1-d}{N}$, where N is the number of nodes. If $\mathbf{P}$ is the vector containing all the scores of the nodes, the definition of PageRank becomes

$$\mathbf{P} = \left(d\mathbf{M} + \frac{1-d}{N}\mathbf{E} \right) P := \hat{\mathbf{M}}\mathbf{P}$$

A.3 SinkRank

SinkRank is a network measure that differs from all the previous ones for the purpose for which it has been designed. The aim of this metric is assessing the potential disruption that the failure of a participant to a Payment System would cause in the network. Given the matrix of weights $\mathbf{W} = [w_{ij}]$, we define the transition matrix, $\mathbf{P} = \left[\frac{w_{ij}}{\sum_j w_{ij}}\right]$, whose elements represent the probabilities of going from node i to node j. To calculate the SinkRank of node i, we remove from matrix $\mathbf{P}$ the i^{th} row and column and we define the matrix left $\mathbf{S}$. From $\mathbf{S}$, we calculate the fundamental matrix $\mathbf{Q} = (\mathbb{I} - \mathbf{S})^{-1}$ and finally, the SinkRank is defined as follows

$$SR(i) = \frac{n-1}{\sum_{j,k} q_{jk}}.$$

B Tables

Table 1: Classification of Payments Systems according to the Settlement Characteristics. Source:(CPSS, 1997)

Settlement Characteristics	**Gross**	**Net**
Designated-time (Defered)	Designated-time Gross Settlement	Deferred Net Settlement **DNS**
Continuous (Real-Time)	Real-Time Gross Settlement **RTGS**	(not applicable)[5]

[5] Although by definition netting involves accumulating transactions in a queue, and this is incompatible with continuous settlement, some systems, such as the French PNS, are to be considered hybrid between a Net Settlement System and an RTGS. This implies that settlements are performed on a continuous base (or at least with a frequency of the order of seconds) and, in every settlement cycle, positions are netted before being settled.

Table 2: Summary of the coefficients obtained from the OLS regression

		HOUR CORRECTION							
COEFF.	6am	8am	10am	11 am	12pm	1pm	2pm	4pm	5:30pm
δ	0.00104***	-3.34e-05	-0.000217***	-0.000233***	-0.000143***	-0.000119***	-9.59e-05***	-0.000276***	-0.000296***
	(7.59e-05)	(2.94e-05)	(2.97e-05)	(2.88e-05)	(2.56e-05)	(2.42e-05)	(2.30e-05)	(2.14e-05)	(2.10e-05)
SR	0.0976***	-0.00362	-0.0324***	-0.0455***	-0.0633***	-0.0748***	-0.0864***	-0.110***	-0.115***
	(0.00272)	(0.00300)	(0.00273)	(0.00263)	(0.00248)	(0.00239)	(0.00229)	(0.00218)	(0.00215)
Observations					134,064				
R^2					0.719				

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

Table 3: Summary of the coefficients obtained from the OLS regression

		HOUR CORRECTION							
COEFF.	6am	8am	10am	11 am	12pm	1pm	2pm	4pm	5:30pm
δ	-0.000711***	5.09e-05	0.000132	0.000117	6.12e-06	-2.25e-05	-5.98e-06	-0.000103	0.000158**
	(0.000151)	(0.000102)	(0.000100)	(9.45e-05)	(8.99e-05)	(8.62e-05)	(8.08e-05)	(7.54e-05)	(7.42e-05)
β_{SR}	0.0639***	-0.00662	-0.0266*	-0.0376***	-0.0489***	-0.0561***	-0.0620***	-0.0800***	-0.0748***
	(0.0108)	(0.0157)	(0.0151)	(0.0140)	(0.0133)	(0.0128)	(0.0120)	(0.0115)	(0.0114)
β_{OD}	0.00584***	-0.000364	-0.00204***	-0.00241***	-0.00216***	-0.00242***	-0.00313***	-0.00367***	-0.00346***
	(0.000860)	(0.000538)	(0.000503)	(0.000485)	(0.000451)	(0.000434)	(0.000423)	(0.000401)	(0.000395)
β_{OS}	0.0447***	0.000494	-0.00305	-0.00363	-0.000767	-2.59e-05	0.00115	-0.0119***	-0.0271***
	(0.00424)	(0.00395)	(0.00346)	(0.00336)	(0.00334)	(0.00327)	(0.00321)	(0.00301)	(0.00286)
β_{IS}	-0.0562***	-0.00256	-0.0121	-0.00152	0.0224**	0.0367***	0.0535***	0.0624***	0.0657***
	(0.00825)	(0.0103)	(0.0100)	(0.00949)	(0.00893)	(0.00857)	(0.00798)	(0.00761)	(0.00750)
$\beta_{OD \cdot IS}$	0.212***	0.0155	0.00803	-0.0110	-0.0728	-0.108*	-0.150***	-0.158***	-0.178***
	(0.0492)	(0.0679)	(0.0659)	(0.0615)	(0.0581)	(0.0557)	(0.0519)	(0.0495)	(0.0491)

Observations	134,064
R^2	0.773

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

C Codes

C.1 Python code

Here it follows a list of the functions defined in order to be able to perform the simulations. For the sake of simplicity, for the support function, only the header has been included in the report, the original source code will be available in the attachments.

```python
#------------------- SUPPORT FUNCTIONS------------------------------------------------------------
def get_matrix(fail_time):
    '''
    Returns the adjacency matrix of a simulated graph
    '''
    return np.matrix(matrix, dtype = float)

def next_time(hour_list, minute_list, second_list, time, idx):
    '''
    Returns the next time stamp
    '''
    return time, idx

def conversion(num):
    '''
    Convert num into a string of at least 2 characters
    '''
    return converted
#------------------- PARSING FUNCTION ------------------------------------------------------------
def get_payment_history(only_time = False ):
    '''
    Retrieve and parse data
    '''
            return sender, receiver, amount, hour, minute, second

def calculate_total_payments(sender, amount):
    '''
    Sum all the payments sent by the participants during the day
    '''
    return

def get_contemporary_transactions(sender, receiver, amount, hour_list, minute_list,
    second_list, time):
    '''
    Return all the transactions that are performed at at the specified 'time'
    '''
    return nSender, nReceiver, nAmount, nHour, nMinute, nSecond

def write_csv(local, time):
    '''
    Write the adjacency matrix of the payment cycle
    '''
    return

def get_cycle_from_matrix(time):
    '''
    Retrieve the transaction settled during a single cycle
    '''
    return sender, receiver, amount

def break_in_blocks():
    '''
    Break the list of transactions in payment cycles, calculate total
    payments and the Normal Liquidity
    '''
    return
```

The ones that follow are the functions used for simulating the stress scenarios. First, we have the function used to implement the liquidity algorithm showed in section 3.1.1

```
def check_if_borrow(A,S,F):
    '''
    Check if the participants have enough reserve to sent the payments
    '''
    for i in range(99):
        #if there the reserve is not enough increase the amount borrowed
        if A[i]+S[i] < 0:
            F[i] = F[i] - (A[i]+S[i])
            S[i] = 0
        #otherwise reduce the reserve
        else:
            S[i] = S[i]+A[i]

    return S,F
```

Then we have the function that calculates the amount of liquidity needed by every participant during days in which no disruptions are observed.

```
def NL():
    '''
    Calculate the liquidity needed by each participant
    '''
    #retrieve the transaction data
    hour, minute, second = get_payment_history(only_time=True)

    time = [hour[0], minute[0], second[0]]
    idx = 0

    A = np.zeros(99)
    S = np.zeros(99)
    F = np.zeros(99)

    while not time == 0:
        #retrieve the transactions that are settled in the current cycle
        sender, receiver, amount = get_cycle_from_matrix(time)

        P_rec = np.zeros(99)
        P_sent = np.zeros(99)
        #calculate the amount sent and received
        for j in range(len(sender)):
                P_sent[sender[j]] = amount[j]+P_sent[sender[j]]
                P_rec[receiver[j]] = amount[j]+P_rec[receiver[j]]
        #net the positions
        A = [P_rec[j]-P_sent[j] for j in range(99)]
        #calculate the new reserve and amount borrowed
        S,F = check_if_borrow(A,S,F)

        time, idx = next_time(hour, minute, second, time,idx)
    #write the results
    dir_name = 'EL/'+date
    if not os.path.isdir(dir_name):
        os.mkdir(dir_name)

    file1 = open(dir_name +'/ordinary_liquidity.txt', 'w')
    for number in F:
        file1.write(str(number)+'\n')

    file1.close()

    return
```

The next function determines the effects of the failure of an institution in terms of missing payments.

```
def failure(bank, fail_time, sender, amount, time):
    '''
    Calculate the effects of a failure
    '''
    #check if the current cycle follows the failure time
    if type(fail_time) is list:
        condition = fail_time <= [float(time[0]), float(time[1])] < [19,0]
    else:
        condition = fail_time<=float(time[0])<19
    #if it does, wipe out the amount sent by the target institution
    if condition:
        for i in range(len(sender)):
                if sender[i] == bank:
                    amount[i]=0

    return amount
```

Finally, we have the function that simulates the failure of a participant and calculates the Upper Bound level of liquidity required to settle all the payments originally scheduled.

```
def EL(bank, fail_time):
    '''
    Simulate the failure of the target institution
    '''
    #retrieve the transaction data
    hour, minute, second = get_payment_history(only_time = True)
    time = [hour[0], minute[0], second[0]]
    idx = 0

    A = np.zeros(99)
    S = np.zeros(99)
    F = np.zeros(99)
    #retrive the cycles and calculate the effects of the failure
    while not time == 0:
        sender, receiver, amount = get_cycle_from_matrix(time)
        amount = failure(bank, fail_time, sender, amount, time)

        P_rec = np.zeros(99)
        P_sent = np.zeros(99)

        for j in range(len(sender)):
                #calculate the amount sent and received
                P_sent[sender[j]] = amount[j]+P_sent[sender[j]]
                P_rec[receiver[j]] = amount[j]+P_rec[receiver[j]]
        #net the positions
        A = [P_rec[j]-P_sent[j] for j in range(99)]
        #calculate the new reserve and amount borrowed
        S,F = check_if_borrow(A,S,F)

        time, idx = next_time(hour, minute, second, time,idx)
    #write the results
    if type(fail_time) is list:
        fail_time = str(fail_time[0])+'-'+str(fail_time[1])
    dir_name = 'EL/'+date+'/'+str(fail_time)
    if not os.path.isdir(dir_name):
        os.mkdir(dir_name)

    title = dir_name+'/EL-fail-'+str(bank)+'time-'+str(fail_time)+'.txt'
    file1 = open(title, 'w')

    for number in F:
        file1.write(str(number)+'\n')

    file1.close()

    return
```

The simulations are then performed for every institution participating to the Payment System and for the 152 days of the data set.

```
def create_distribution(fail_time):
    '''
    Compute the extraordinary liquidity for the failure of every participant
    '''
    for bank in range(1,99):
        print bank
        EL(bank, fail_time)

    return

def write_matrix(fail_time):
    '''
    Store the result of the simulations of the 98 scenarios given a date and a
    fail_time
    '''
    return

def normalise_matrix(fail_time):
    '''
    Normalise the EL with each institution's total sent payment
    '''
    return

date_list = [date[-9:-4] for date in os.listdir('transaction')]

for date in date_list:
    if not os.path.isdir('matrices/matrices-'+date):
        print date
        break_in_blocks()
    for hour in [6,8,10,11,12,13,14,16, [17,30]]:
        if hour == [17,30]:
            check = '17-30'
        else:
            check = str(hour)

        if not os.path.isfile('DISTRIBUTIONS/'+check+'/NORMALISED/matrix-normal-'+date+'-'+check+'.csv'):
            print date, 'time:', str(hour)
            create_distribution(hour)
            write_matrix(hour)
            normalise_matrix(hour)
```

C.2 Stata code

```
clear all
set more off
//import variables
import delimited "C:\Users\T41896\Documents\Mis archivos recibidos\data_stata.csv"
//rename variables
rename v1 day
rename v2 hour
rename v3 id
rename v4 SE
rename v5 SR
rename v6 PR
rename v7 OD
rename v8 OS
rename v9 IS
rename v10 BC
rename v11 CC
rename v12 core
//generate the time-group variables
egen timeid = group(day hour)
//generate the ids
xtset id timeid
//normalise variables
egen maxSR = max(SR)
replace SR = SR/maxSR

egen maxOD = max(OD)
replace OD = OD/maxOD

egen maxOS = max(OS)
replace OS = OS/maxOS

egen maxIS = max(IS)
replace IS = IS/maxIS

egen maxBC = max(BC)
replace BC = BC/maxBC

egen maxCC = max(CC)
replace CC = CC/maxCC
//linear regression with fixed effects
xtreg SE i.hour##(c.SR c.OS c.OD##c.IS), fe
outreg2 using "C:\Users\T41896\Documents\Mis archivos recibidos\day_stata_regression.tex", tex(pr) replace
//evaluate the fitted values
predict xbuhat, xbu
//scatter plot with linear fit
aaplot SE xbuhat
//generate the interaction between Out-Degree and In-Strength
gen IN = OD*IS
//generate the hour-depending variables
xi i.hour*SR i.hour*OD i.hour*OS i.hour*IS i.hour*IN
//linear regression with fixed effects and standard errors
xtscc SE _Ihour_* SR _IhouXSR_* OD _IhouXOD_* OS _IhouXOS_* IS _IhouXIS_* IN _IhouXIN_*, fe lag(0)
outreg2 using "C:\Users\T41896\Documents\Mis archivos recibidos\day_stata_regression.tex", tex(pr) append
//evaluate the fitted values without fixed effects
predict xbhat, xb
//scatter plot with linear fit
aaplot SE xbhat
```