Risk Measurement with High-frequency Data – Value-at-Risk and Scaling Law Methods

by

Jun QI

A thesis submitted for degree of Doctor of Philosophy

in Computational Finance

Center for Computational Finance and Economic Agents

University of Essex

March 2011

Acknowledgements

I would like to express my sincere gratitude to all those who have helped and encouraged me through the whole studying period. I also apologise to those I can not mention personally, but are always in my heart.

First and foremost, I would like to thank my supervisor Dr. Wing Lon for guiding me and supporting me during the past three years. It has been a great academic and personal privilege to work with him. I would also thank him for his selflessness and treating me as his own child.

I am also extremely grateful to my second supervisor Dr. Richard Olsen for giving me valuable advice and inspirational comments which significantly contributed to this thesis.

I am proud of being a student in CCFEA. I would like to thank Prof. Sheri Markose, Prof. Edward Tsang, Kyriakos Chourdakis, Lynda Triolo and Julie Peirson for their help and support.

My gratitude also goes to my friends Evi, Rafael, Konstantinos, Jin, Jian, Zijian, Fei, Wenwen, Jiming and Xiaoxiao. Thank you all for the love and care during these years.

I would like to thank my husband, Bin, for being there with me day after day, for his patience, for giving me strength during crisis periods, for taking care of me during my illness.

Lastly, and most importantly, I owe my sincere gratitude to my parents, Xiangdong and Jihong, to whom I dedicate this thesis. They raised me, supported me and loved me. Without their unconditional support, this PHD would have never been completed.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

The theory of risk measurement has evolved immensely since the work of Harry Markowitz in the 1950s. In today's environment, which is marked by intensifying competition, traditional risk measurement tools are no longer sufficient. The core subject of this thesis is the measurement of financial risk with high frequency data. This introductory chapter provides a general theoretical background to risk management and a brief overview of risk measurement is presented in Section 1.1. In Section 1.2, essential concepts of market risk quantitative models are introduced and weaknesses of the Value at Risk (VaR) model are addressed. Section 1.3 discusses the importance of high frequency analysis that influences the risk measurement. The main objectives and contributions are summarised in Section 1.4. The final section in this chapter outlines the structure of the whole thesis.

1.1 Risk and Risk Management

Understanding the concept of risk is a crucial condition for creating a good risk management strategy. Risk can be defined as the uncertainty of outcomes. In the financial literature, risk is the likelihood of losses resulting from unexpected events related to movements in the financial market (Jorion (2006)). Extreme events (like crashes) may have a low probability of occurring but also may cause a big loss. These low probability events are more intractable because they are usually hard to anticipate (Horcher (2005)).

Companies are exposed to different types of risk which could be generally categorised into two main types: business risk and financial risk.

1. Business risks *"are those which the corporation willing assumes to create a competitive advantage and add value for shareholders"* (Jorion (2006), p. 4). Business risk pertains to the risk a company faces solely due to their presence in the product market. Activities such as technological innovations, product design and marketing are related to this kind of risk.

2. Financial risk, which arises through changes in financial variables and transactions in financial markets, is classified into various subcategories, such as market risk, credit risk, liquidity risk, operational risk, and legal risk. Market risks are the unexpected changes in financial asset prices (such as equity price, commodity price, interest rate and foreign exchange rate). Credit risk is the risk of loss due to the counterparty's inability to fulfill its obligations or other credit event. In the

banking industry, credit risk is the most common reason for bankruptcy. Liquidity risk can be classified into market (or asset) liquidity risk and funding liquidity risk. Market liquidity risk normally arises from the higher cost and difficulty of executing the trade which is caused by an illiquid market. Funding risk is driven by the inability to meet obligations with immediacy due to the unsecured funding sources. Operational risk is due to human and technical errors, such as inadequate systems, management failures or fraud. Finally, legal risk normally occurs when a counterparty has failed to provide the legal enforcement to engage in a transaction.

Risk management has gained much attention over the past two decades, especially in the current financial crisis. Traditional risk control models are challenged by today's financial environment. Beyond the lessons from financial disasters, regulation is an important factor defining the behaviour of financial institutions in how they deal with risk. The Group of Thirty (1993)(G30) provided 20 best-practice price risk management recommendations and four recommendations for regulators. The G30 emphasised the importance of consistent risk measurement which stresses the VaR method. The Basel Committee on Banking Supervision published Basel II [1] to create an international standard regulation. The famous "three pillars" framework proposed in Basel II is: (1) minimum capital requirement, (2) supervisory review process and (3) market discipline requirement. The new Basel III also

[1] Basel II are the second Basel Accords, which were initially published in 2004 and are updated every year by the Basel Committee on Banking Supervision (BCBS). The BCBS promote greater consistency and improve the quality of banking supervision across national borders (see http://www.bis.org).

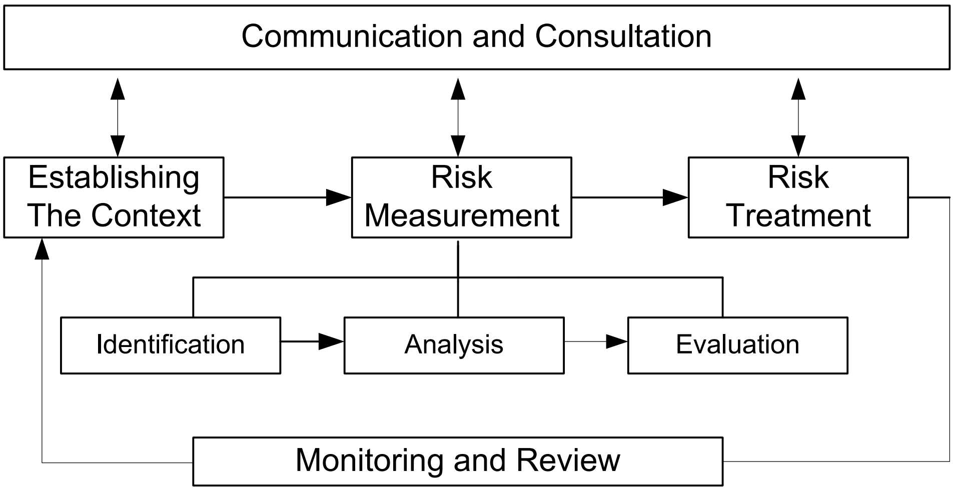


Figure 1.1: Risk Management Process (adapted from: ISO 31000 (2009), page 8).

strengthens the "three pillars" and increases the capital requirements based on the original Basel II.[2]

Risk management includes five components: (1) risk identification, (2) risk measurement, (3) model evaluation, (4)model selection and (5) monitoring the consequence. The ISO 31000 (2009)[3] provides a standard process of risk management which is illustrated in Figure 1.1. The necessary condition for successful risk management requires a method to accurately measure risk. Value at Risk (VaR) as a traditional method of measuring market risk is introduced in the next section.

[2]Basel III are the third Basel Accords, which were recently published by the Basel Committee on Banking Supervision (BCBS). (see http://www.bis.org).

[3]ISO 31000 (Risk Management Principles and Guidelines on Implementation) is codified by the International Organisation for Standardisation which provides standard principles and guidelines on risk management.

1.2 Traditional Market Risk Assessment Method

The theory of risk measurement has evolved since the work of Harry Markowitz in the 1950s (see Markowitz (1952) and Markowitz (1959)). In this section, we first introduce the background to conventional risk measurements and discuss the limitations of the VaR model.

1.2.1 Value at Risk

Value at Risk (VaR) was initially developed to solve market risk, which arises from the changes in the prices of financial assets and liabilities. The VaR model measures the worst loss at a certain probability over a specific time period:

$$Pr(\triangle W < -VaR) = 1 - c = \alpha, \tag{1.1}$$

where $\triangle W$ is the change of wealth of the asset over the holding period, c is the confidence level and α is the probability that the expected loss exceeds the VaR value (see Dowd (2002), page 19 and Jorion (2006), page 157). In other words, VaR answers the question: how much can I lose with $\alpha\%$ probability over a specific time horizon? Three factors need to be kept in mind when measuring the VaR of an asset. First, we need to know the initial value of the asset. The second factor is the holding period. Thirdly, the confidence level is required to ascertain the likelihood that we will get an outcome no worse than our VaR. Figure 1.2 shows an illustration of VaR with a confidence level of 99% and 95%. The VaR computation methods will be introduced in Chapter 2.

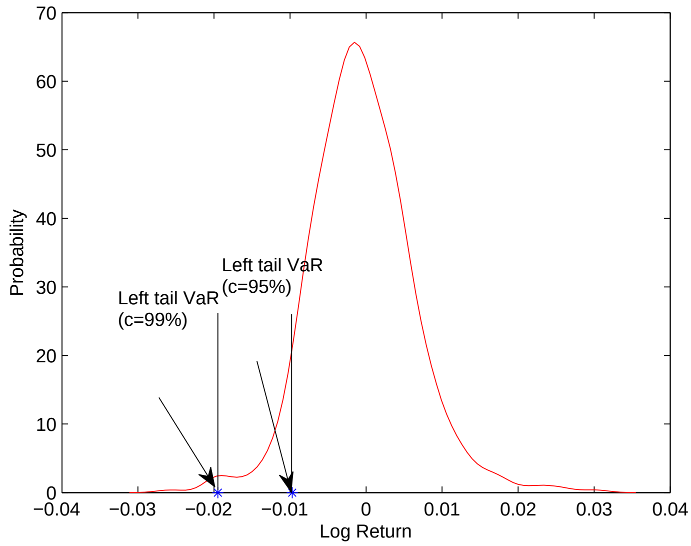


Figure 1.2: Illustration of Value at Risk. The red line displays the empirical density of daily log returns over two years on the FTSE100 (07/2007-07/2009). With a 95% confidence level, we need to cut-off the lower 5% tail to obtain the VaR.

Over the last decade, VaR has become a standard tool for evaluating market risk used by banks, trading firms and other financial institutions. The main advantage of VaR as a risk measure is that it is very simple and easy to apply: it can be used to summarise the risk of individual positions, or portfolios of positions of large multinational financial institutions, such as the large dealer-banks. VaR was adopted because of its simplicity for regulatory purposes. Figure 1.3 shows the relationship between a risk management tool and a regulation (Vasanta (2004)). VaR is commonly used to measure potential losses, but can also be used as a regulatory measure to determine the capital adequacy requirements, for example, by supervisory authorities.

Although VaR is a simple concept, the accuracy of its computation is a very

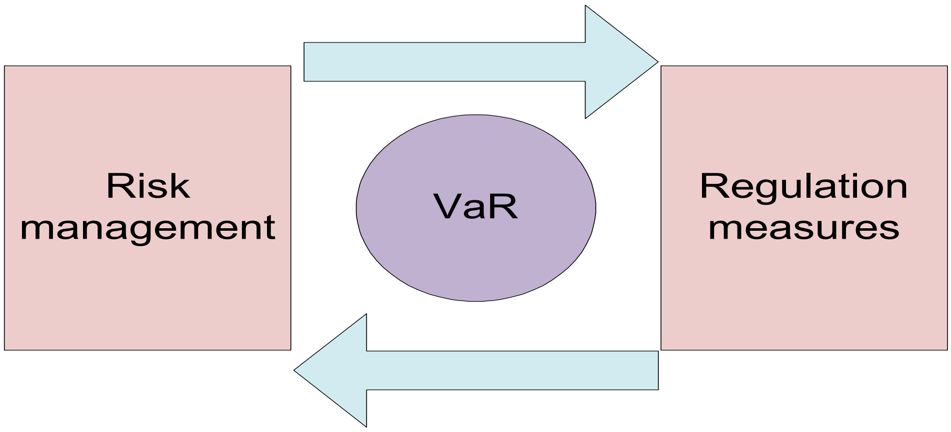


Figure 1.3: The relationship between risk management and regulatory measures (adopted from: Vasanta (2004)).

important and challenging problem in risk management. The risk measures from the various VaR methodologies on the same date could generate different result of VaR and various degrees of accuracy for the same portfolio. If there is an error in estimating the risk position, it may affect the whole investment strategy, or, in extreme cases lead to bankruptcy. Many notable financial disasters occurred in many large institutions due to the miscalculation of risk, such as Orange County (1994), Baring Bank (1995), Long Term Capital Management (1998), or Lehman Brothers (2008).

1.2.2 Limitations of VaR

As a risk measurement tool, VaR is subject to some common drawbacks, such as lack of estimation accuracy and model risk. In this section, we address some of the limitations of VaR.

1. The most distinctive limitation of VaR is that it only provides an es-

timation of the size of the loss at some confidence level and neglects valuable information about the distribution. If an extreme event occurs, VaR does not provide information on the size of the expected loss.

2. VaR is not coherent. According to Artzner, Eber, and Heath (1999), a coherent risk measure is a risk measure $\rho(.)$ that satisfies the following properties (see also Wilmott (2007)):

   - Sub-additivity: $\rho(Z_1) + \rho(Z_2) \leq \rho(Z_1 + Z_2)$.

     Sub-additivity means that the sum of individual risks is equal to or greater than the overall risk.

   - Homogeneity: $\rho(aZ) = a\rho(Z)$ if $a \geq 0$.

     Homogeneity implies that the position size does not influence the risk measure.

   - Monotonicity: $\rho(Z_1) \geq \rho(Z_2)$, if $Z_2 \geq Z_1$.

     Monotonicity means that the increased future payoff should lower the downside risk of a position.

   - Translation invariance: $\rho(Z + a) = \rho(Z) + a$, if $a \in R$ (risk-free condition).

     Translation invariance ensures that the addition amount a to one's position will decrease the risk by the same amount.

Acerbi and Scandolo (2008) investigate liquidity risk and coherent measures of risk from a pure risk theoretical point of view. As a new axiom

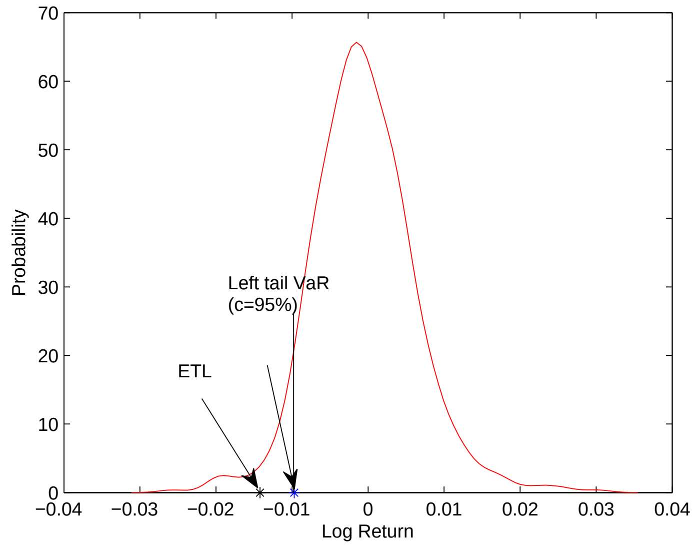


Figure 1.4: Comparison of Expected Tail Loss and VaR.

in the literature, the coherent portfolio risk measure is proposed based on the the original coherent risk measures.

Artzner, Eber, and Heath (1999) have proven that the Expected Tail Loss (ETL) is a coherent measure which overcomes the inadequacies of VaR. ETL is the expected value of loss (L) given that the loss exceeds the VaR level:

$$ETL_{\alpha} = E(L \mid L < VaR_{\alpha}). \tag{1.2}$$

Figure 1.4 shows the ETL, based on daily data over two years (07/2007-07/2009) on the FTSE100. The VaR is 0.0097 and the ETL is 0.0142 with a 95% confidence level. Both VaR and ETL depend on the distribution of the underlying asset.

ETL is an alternative method to VaR which gives a more intuitive view of extreme losses. We also present this method as a standard risk measurement for performance comparison in Chapter 4.

Artzner, Eber, and Heath (1999) criticise VaR, because is not a coherent measurement as it fails to satisfy the sub-additivity condition. The VaR only satisfies sub-additivity condition if price changes follow the normal distribution assumption. If the risk measure does not satisfy the sub-additivity condition, it will underestimate the combined risk by adding the individual risk together. Furthermore, if a non-sub-additivity risk measure is used by regulators to set capital requirement, the financial institutions can set up subsidiaries to reduce their regulatory capital requirements (see Dowd (2002)).

3. The conventional VaR model measures the likelihood of losses on the underlying asset subject to the market risk. This definition has two flaws. First, most VaR research focuses on the left tail of the distribution which neglects the "upside risk". Second, most VaR measures concentrate on the impact of price and disregard other risk factors (such as impact of volume) when measuring the market risk. This will lead to an underestimation of the risk. The best solution is to explore the conventional VaR model by considering other related risk factors. This issue is addressed in Chapter 3.

4. In the recent financial crisis, financial experts complained that VaR is not a sufficient measure in a crisis. Nocera (2009) investigated the performance of VaR in the financial crisis of 2007-2008 after interviewing top risk managers. The limited performance of VaR in a bearish

market is sometimes described as *"an air bag that works all the time, except when you have a car accident"* (Nocera 2009). The VaR model is suggested as a useful tool, but it is dangerous to rely on when an extreme event happens. This problem is investigated in Chapter 4.

1.3 High Frequency Finance

For decades, researchers used low frequency and regularly spaced data in financial analysis. Low frequency data misses valuable information between the data points. This issue is addressed by Engle and Russell (2006): "*Like the view from the airplane above, classic asset pricing research assumes only that prices eventually reach their equilibrium value, the route taken and speed of achieving equilibrium is not specified*". Analogous to watching the traffic from a street (rather than from a plane), high frequency data can provide more details on the price adjustment compared to analysing daily data.

High frequency data is time series data of events that includes every quote or transaction price in real-time. These time series are inhomogeneous, which means that the data series are irregularly spaced in time (see Dacorogna, Gençay, Muller, Olsen, and Pictet (2001)). Table 1.1 shows a sequence of time points $t_1, t_2, ..., t_N$, where transactions occur with unequal time intervals and occasionally several transactions occur simultaneously. High frequency data (depending on the source) includes information such as time stamp, trade-price and trade-size and have irregular time intervals.

Low frequency data is usually taken on a daily or weekly basis at equally

Table 1.1: Sample snap-shot of transaction high-frequency data for Glaxo Smith Kline

Time	**Trade-price(in Pence)**	**Trade size**
01.03.2007 08:03:59	1417	21506
01.03.2007 08:03:59	1417	7104
01.03.2007 08:04:00	1416	2814
01.03.2007 08:04:00	1417	986
01.03.2007 08:04:00	1416	886
01.03.2007 08:04:00	1415	7455
01.03.2007 08:04:00	1417	4641
01.03.2007 08:04:00	1415	2814
01.03.2007 08:04:00	1415	2769
01.03.2007 08:04:01	1415	1622
01.03.2007 08:04:01	1417	2814
01.03.2007 08:04:03	1417	3019
01.03.2007 08:04:05	1417	4500
01.03.2007 08:04:06	1417	2955

spaced intervals in business time. Compared with low frequency data, the time horizon of high frequency data can be as short as a few seconds, allowing it to reveal more detailed information for the corresponding time window.

There are some key features associated with high-frequency data:

1. Strong intraday seasonality. For most stock markets, volatility, volume and the frequency of trades all present an intraday "U-shape" pattern. For example, the quantity of transactions is more numerous at the open and close of every day and fewer at lunch time (see Goodhart and O'Hara (1997)). Figure 1.5 shows the intraday volume pattern for 30 minute returns for the DSG International stock. The sample period ranges from 1st March 2007 to 30th March 2007. We can observe that the trading activity is not constant over the trading day and follows a

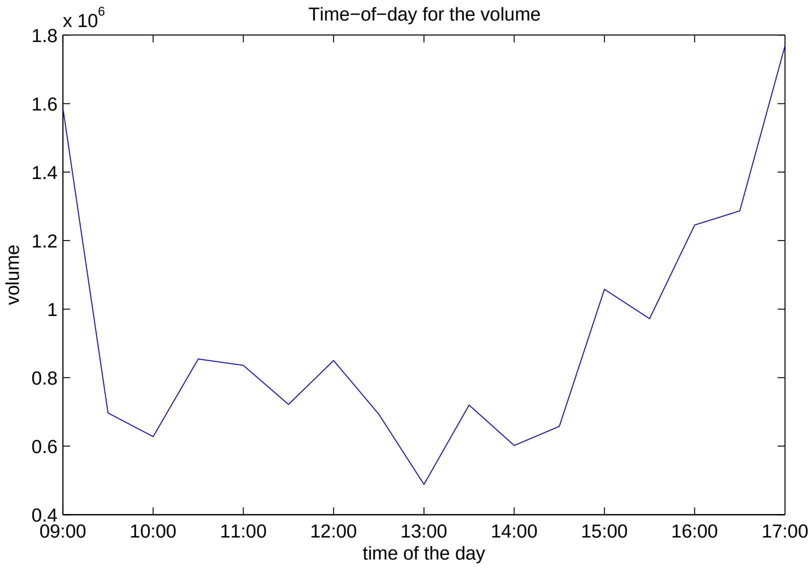


Figure 1.5: Intraday U-shape pattern of trading volume. The graph shows the intraday volume pattern for 30 minute returns for the DSG International stock. The sample period ranges from 01 Mar, 2007 to 30 Mar, 2007.

"U-shape" pattern.

The plots of the intraday seasonal component for volatility for different days of the week are shown in Figure 1.6, illustrating the day-of-the-week effect[4]. The intraday volatility is computed by 10 minute returns for the DSG International stock and the sample period is from 1st March 2007 to 30th March 2007. Higher volatility and seasonality on Mondays and Fridays are visible in the plots.

Andersen and Bollerslev (1999) investigated the relationship between the seasonality and persistence of volatility, and proved that if the seasonality of the data is removed, the persistence decreases.

[4]Day-of-the-week effect (also known as the Monday effect), refers to the tendency of stocks to have a higher return and volatility on Mondays and Fridays (see Kiymaz and Berument (2003) and Dacorogna, Gençay, Muller, Olsen, and Pictet (2001), page.160-170).

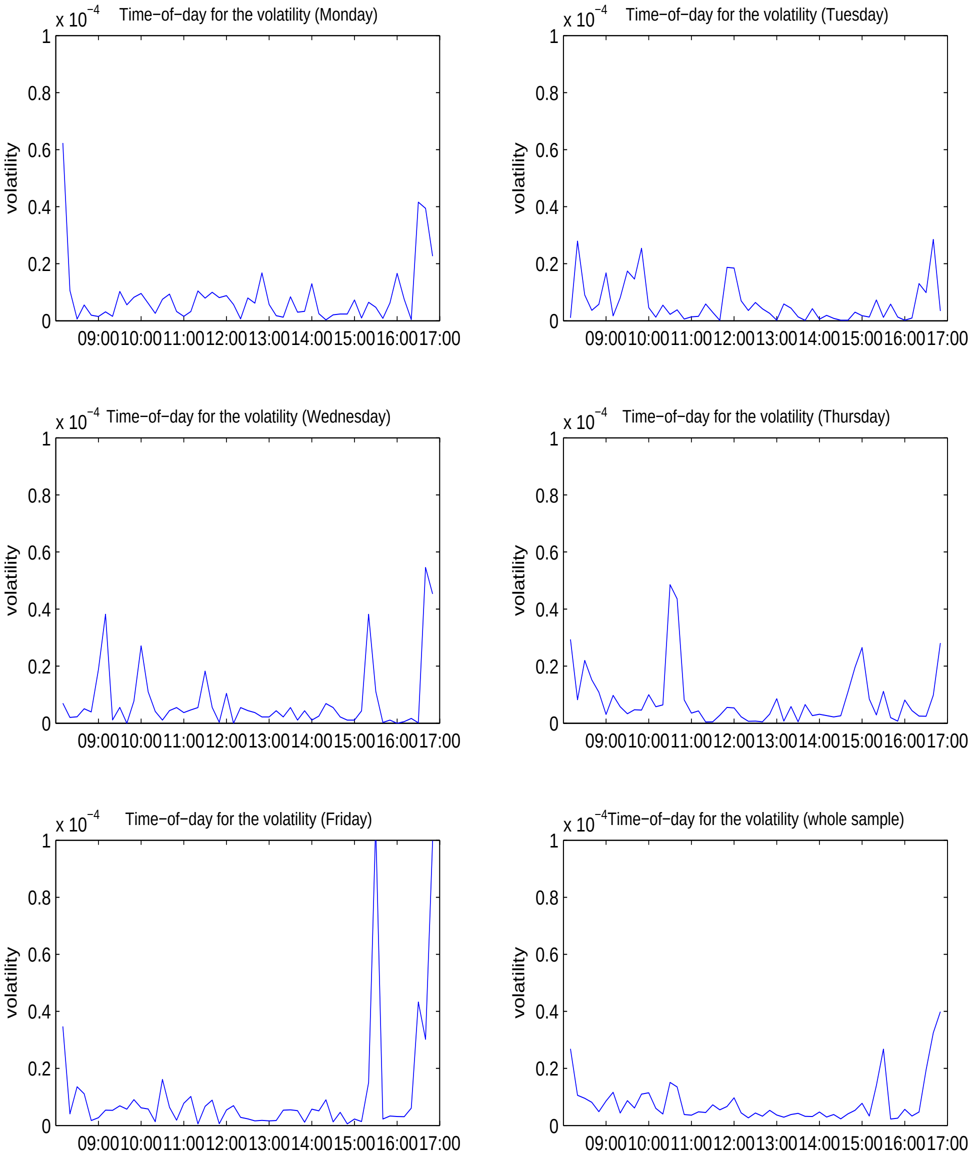


Figure 1.6: Intraday seasonality pattern of volatility. The graph shows the intraday seasonal component for volatility for different days of the week for the DSG International stock and the sample period is from 1st March 2007 to 30th March 2007.

2. Extreme high kurtosis is a typical fact of high frequency data. Andersen and Bollerslev (1998) proved that the kurtosis of data increases with the frequency level. For one minute frequency data, the kurtosis is higher than normal.

High-frequency data (tick data) provide detailed information on aspects of financial market activities recorded at a given time indicated by a "time stamp". It is challenging and complicated to analyse the high frequency data sets which contain tens of thousands of transactions in a single day at irregular time intervals.

According to the efficient market hypothesis, asset prices should either reflect all known information or instantly change to reflect new information. In fact, price evolution in financial communities is far more complex, and especially in path dependent markets, it is important to track prices on a tick-by-tick basis. Research on the data collected from the market is the best way to understand the information behind the market.

Risk management previously was used to analyse the low-frequency (daily or weekly) price data. Improvements in computer and information technologies open up opportunities for new risk management methodologies. In the development of financial markets, algorithmic trading and high-frequency trading have acquired important roles in today's financial trading system (see Kissell and Malamut (2006) and Aldridge (2010)). It is no longer sufficient to only rely on the risk control model based on low frequency data. New risk management techniques are needed, which can help us to get clear insight on what it is telling us in real-time.

1.4 Objectives and Contributions of the Thesis

The core task of this thesis is to develop reliable measures of market risk using high frequency data. There are three main objectives in the rest of this thesis:

1. The measurement for empirical intraday VaR using a non-parametric model and five different parametric models.
2. Incorporating liquidity risk to compute the actual intraday VaR.
3. Applying a new empirical scaling law method based on ultra-high frequency data to measure and forecast the market risk.

We will now briefly highlight the contributions of three core chapters in this thesis. In Chapter 2, we quantify intraday market risk using one non-parametric and five parametric models. The main findings are: firstly, our results confirm that the Historical Simulation as a popular method for measuring daily VaR, also has satisfied performance in intraday risk measurements. Secondly, so far in the literature, the Student-t distribution is popular and used to capture fat tails in empirical data. However, we find that the GARCH models based on the normal distribution outperformed Student-t distribution. Finally, our results show that the intraday volatility persistence exists and the MRS-GARCH model is a good candidate for intraday market risk measurement.

Chapter 3 deals with the difficulty of incorporating both endogenous and exogenous liquidity risk in to IVaR measurement.[5] We estimate the one-step-ahead liquidity adjusts IVaR of both market sides in order to quantify their real risk position. Furthermore, our results show that there is an asymmetry in up and down movements in liquidity adjustments of the equity market. Downward movements typically have a higher magnitude than upward movements.

Chapter 4 proceeds into the analysis based on multiple time scale analysis. In this chapter, we propose three new scaling law methods in risk measuring and forecasting. The empirical results provide evidence that the new exponential moving average maximal price change (EMAMPC) scaling law outperform all other models. Furthermore, the forecasting errors are smaller when shorter in-sample data is used. The scaling law methods with one month data provide good forecasting on the maximum loss within ten days.

[5] Endogenous and exogenous liquidity risk are introduced in Chapter 3, Section 2.2.

1.5 Structure of the Thesis

The structure of the remaining four chapters of this thesis is briefly outlined as follows. Chapter 2 investigates the market risk measurement by using high-frequency data. The deseasonal high-frequency data used in the experiments are for three stocks, namely Northern Rock (NR), Royal Bank of Scotland (RBS) and Hong Kong and Shanghai Banking Corporation (HSBC). All models are estimated on a sub-sample (estimation sample) and compared the performance with the remaining data set (forecast sample). The intraday Value at Risk (IVaR) analyse are based on three short sample intervals (1 minute, 5 minutes and 10 minutes), and estimated using Historical Simulation with different rolling windows and five parametric models called normal GARCH, Student-t GARCH, Normal EGARCH, Student-t EGARCH and MRS-GARCH. Empirical estimation results for three stocks are presented and the performance of different models are backtested by using Kupiec's test method.

The traditional VaR is a very popular tool for measuring market risk, but it does not incorporate liquidity risk. In reality, the assumption of mid-price execution which does not hold, in particular when investors execute large trades. Chapter 3 proposes an extended VaR model to take account of liquidity risk in intraday trading strategies when analysing high frequency order book data. In this chapter, we address the importance of liquidity risk and introduce the one-step-ahead liquidity adjusted intraday VaR (LAIVaR) for both bid and ask positions, considering several threshold trading sizes. The asymmetric liquidity risk premia are quantified by comparing our result with the standard VaR approach.

Chapter 4 introduces a multiple time scale based on scaling law methods for risk measurement and forecasting. So far, the literature on the application of scaling law methodology to risk measurement is very limited. A new empirical framework for the measurement of financial risk is proposed where empirical scaling laws based on the maximum price change are applied. The empirical analysis is carried out on different time scales of ultra-high frequency data for five FX pairs which considers all the dynamic information of the market. The data sample covers the time interval from 1st January 2006 to 31st December 2008 which is provided by Olsen Financial Technology. Traditional risk measurements and a comparison of their forecasting performance are also presented.

Finally, Chapter 5 presents a summary of the work in previous chapters and the conclusion of the whole thesis.

Chapter 2

Intraday Value at Risk with High Frequency Data

In this chapter, we quantify market risk with small intraday time horizons (1 minute, 5 minutes and 10 minutes) using one nonparametric (Historical Simulation (HS)) and five parametric models (Normal GARCH, Student-t GARCH, Normal EGARCH, Student-t EGARCH and Markove regime switching (MRS) GARCH models)[1]. The six existed models are applied to deseaonalised intraday data for three stocks traded on the London Stock Exchange (LSE). Moveover, the model performance comparisons are assessed by two backtesting methods.

The empirical findings show that the Intraday VaR (IVaR) estimated by HS provide an accurate performance for all stocks and time horizons. For conditional VaR method, the GARCH type models based on normal distribution

[1]The details of the models are introduced in Section 2.2 and Section 2.3.

are outperformed, especially for Normal GARCH and MRS-GARCH models. The Student-t distribution is widely used to capture the fatness of the tails of the distributions of stock return. Surprisingly, the models with Student-t innovations perform poorly. This finding is different with many existing literatures we reference which recognised the important of the fat tails of daily or intraday returns.

The chapter is organised in the following way. Section 2.1 motivates estimating intraday market risk with small time horizons. Section 2.2 reviews the common VaR calculation methods. Sections 2.3 and 2.4 discuss the GARCH type volatility models used in this chapter and the IVaR. Section 2.5 provides the empirical results of IVaR. Finally, Section 2.6 summarises the main findings.

2.1 Introduction

In recent years, corporations are in the business of managing risks. Since the 1970s, due to the increased volatility of the financial market there has been a growth in demand for the risk management industry. Some huge financial institutions and multinational companies have experienced bankruptcy, disasters, and big losses have been caused as a result of ineffective risk management.

Risk management has truly experienced a revolution in the last few years. Value at Risk (VaR) has become a standard tool for evaluating market risk used by banks, trading firms and others. Due to the important status of VaR in the financial world, plenty of research has been explored in this field. One of the most interesting and challenging developments is to evaluate intraday VaR using high frequency data (see Dacorogna, Gençay, Muller, Olsen, and Pictet (2001), Dionne, Duchesne, and Pacurar (2009)). In the financial literature most of the studies published have dealt only with "low-frequency" and regularly spaced data such as monthly or daily data.

Nowadays intraday data, high frequency data for the stock prices and other financial assets, are widely available. The conventional way of computing VaR has been challenged by the current trading environment (Dionne, Duchesne, and Pacurar (2009)). In empirical finance, high-frequency finance has become a very important field in recent years. However, analysis of risk measurement based on high frequency data has only just begun fairly recently. Andrea and Claudio (2001) compared the computation of VaR with daily and with half hour high frequency data. Giot (2005) has provided market risk models for intraday data and calculated intraday VaR (IVaR) using 15 minute and 30 minute intervals of three stocks. He also studied the performance of the one-step-ahead Value at Risk predicted by normal GARCH, Student-t GARCH, RiskMetrics and Log-ACD models and found that the Student-t GARCH model performs best. Dionne, Duchesne, and Pacurar (2009) proposed a method based on the Log-ACD-ARMA-EGARCH model to estimate intraday VaR using tick-by-tick data.

The use of Markov-switching models to capture the volatility dynamics of financial time series has grown considerably during the past few years, in part because they give rise to a plausible interpretation of nonlinearities.

So far in the literature, single regime GARCH type models are normally used to model time-varying volatility which have priority of capturing the volatility clusters (see Akgiray (1989), McMillan, Speight, and Gwilym (2000) and Poon and Granger (2003)). However, the high persistence of a standard GARCH model and structural changes in the variance process are claimed by Hamilton and Susmel (1994), Gray (1996), Klaassen (2002) and Abramson and Cohen (2007). Researchers find that the volatility estimations based on the standard single regime GARCH models suffer from an upward bias of parameters' persistence which could end up with a conservative VaR evaluation. To solve this problem, one possible way is using Markov Regime Switching (MRS) GARCH models to capture the volatility dynamics of financial time series depending on different regimes (high or low) of volatility persistence. Marcucci (2005) compares the forecast ability of a set of GARCH models within MRS-GARCH frameworks. The empirical results indicate that MRS-GARCH models outperform standard GARCH models in forecasting volatility. However, this study only focuses on the a low frequency context. In this chapter, we extend the current literature by applying the MRS-GARCH model to measure IVaR with high frequency data.

The VaR model based on high frequency data is challenging and meaningful for the following reasons:

1. The volatility model is an essential component influencing the accuracy of the VaR measurement. Using high frequency data to estimate volatility has been demonstrated to provide much more accuracy (see details in Giot (2000) and Koopman, Jungbacker, and Hol (2005)).

2. Today's trading system forces firms to continuously build their own strategies to beat the market. It is not sufficient enough to analyse VaR based on daily data as was the conventional method (Ulrich (2000)). High frequency data contains more information about the market. We need to develop a better risk measurement method based on high-frequency data to surpass the limitation of the conventional method. IVaR is an effective tool of risk evaluation for short-term traders.

3. Intraday price movement has been observed more and more frequently nowadays. For daily traders, it is a normal day when the opening price is fairly close to the closing price. However, there is a big difference for short-term trader, if the stock price has some great variation during the day. It is not sufficient that we use only one set of data to characterise the market activity of an entire day. In the current trading environment, research on IVaR can benefit short-term traders and quantify real-time market risk for people participating in algorithmic trading and high frequency trading. VaR is a popular tool for risk management, so we need to consider its practicability. The conventional standard daily data estimation assumes a long term time horizon, which might be too long for some short-term traders. The IVaR can be used to set limits for short-term traders and to assess the risks to different financial products before decisions are made.

In this chapter high frequency data is used for risk measurement, which is quite different from usual VaR models. In the empirical study, the Historical

Simulation method and standard GARCH models are used to compute IVaR. In order to capture the structure change in an intraday variance process, the IVaR estimated by the MRS-GARCH model will also be provided. To our best knowledge, the application that use MRS-GARCH model to estimate IVaR, has never been addressed in the literature. The different models are assessed by the forecasting performance with three short time horizons (1 minute, 5 minutes and 10 minutes). Illustrations of these techniques are presented for three actively traded stocks of the London Stocks Exchange. Backtesting results are also provided for performance comparison.

2.2 Review of VaR Calculation Methods

The calculation of VaR measures has become of paramount importance in risk management. Existing conventional models for measuring VaR are based on analysing daily or weekly data and they can be summarised in three common basic categories which calculate VaR based on correlations, distribution patterns and volatilities. These three categories are: Historical Simulation (non-parametric), Monte Carlo Simulation (parametric) and the Variance-Covariance method (semi-parametric) (see Dowd (2002) and Jorion (2006)).

2.2.1 Historical Simulation

Historical Simulation is the most common and simplest estimation of VaR.[2] The main advantage of the Historical Simulation approach is that it is non-

[2] The Historical Simulation performed in Section 2.5.2 is empirical.

parametric which means the data does not have to follow a specific distribution. However, Historical Simulation assumes that the changes in the financial market conditions from today to tomorrow are the same as the changes that took place some time ago in the past.

The Historical Simulation method can be applied in the following way (Dowd (2002)):

1. Obtain the historical market price of a chosen time period.

2. Calculate the asset returns and then sort in ascending order.

3. Select the α-th quantile of the observation.

4. Estimate the VaR based on the current price.

The most important concept in the Historical Simulation method is the rolling window. The size of the window can affect the estimation results of the VaR. For example, Van Den Goorbergh and Vlaar (1999) use various VaR techniques applied to the Dutch stock market index AEX and to the Dow Jones Industrial Average. They use different lengths of rolling window over a 15 year period to backtest the daily data on the AEX Dutch equity index and they found that the failure rate[3] often exceeds the corresponding left tail probabilities. They found that the Historical Simulation produced satisfactory results only when there is a large enough sample. Brooks and Persand (2000) investigated the sensitivity of VaR models to changes in the sample size and weighting methods. They used a set of national equity indices, bond futures and FX rates and found strong evidence that VaR mod-

[3]The failure rate is the proportion of actual losses that exceed the VaR.

els could provide very inaccurate estimates when the "wrong" historical data sample length is selected.

An important limitation of the Historical Simulation method is the assumption that the distribution of the portfolio returns within the rolling windows does not change. In practice, however, this can cause a lot of problems. On the one hand, the size of the window must be large enough to keep the consistency of the empirical quantile estimator. For example, at least 100 observations are needed when estimating VaR at a 99% confidence level. On the other hand, if the window size is too large, the observations could be taken from outside the current volatility bunches, in other words, the VaR will not be adequately responsive to the most recent returns (Jorion (2006)).

2.2.2 Monte Carlo Simulation

The Monte Carlo Simulation method covers a wide range of possible values in financial variables and fully accounts for correlation. We can draw a large number of scenarios randomly and price the portfolio for each one. Monte Carlo Simulation is generally used to compute VaR for portfolios containing securities with non-linear returns (e.g. options). The main difference between Monte Carlo Simulation and Historical Simulation is that instead of using historical observations, Monte Carlo Simulation chooses a statistical model that is believed to adequately capture or approximate the possible changes in the market (see Linsmeier and Pearson (1996)).

The general steps of the Monte Carlo method are as follows (Dowd 1998):

1. Identify or select a suitable model for the changes in market factors.
2. Estimate the parameters (means, standard deviations and correlations).
3. Generate n random variables to obtain the simulated changes in market factors over a time of horizon.
4. Select the α-th quantile of the observation.
5. Estimate the VaR based on the current price.

2.2.3 Variance-Covariance Method

The Variance-Covariance model (or delta-normal) is based on the assumption that the returns are normally distributed and on a linear approximation of the portfolio. The VaR is then given by

$$VaR_\alpha = Z_\alpha \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{i,j}} \quad , \tag{2.1}$$

where w_i and w_j denote the weight of assets i and j in the portfolio; $\sigma_{i,j}$ denotes the covariance and Z_α denotes the one-side confidence level for cumulative normal distribution of the portfolio returns. For example, under the normal distribution, the quantile $Z_\alpha = 1.65$ if $\alpha = 95\%$.

The covariance between two assets is

$$\sigma_{i,j} = \rho_{i,j} \sigma_i \sigma_j \quad , \tag{2.2}$$

where $\rho_{i,j}$ denotes the correlation and σ_i the volatility of asset i.

One of the most famous Variance-Covariance models is RiskMetrics by J.P. Morgan (1996). The advantage of the Variance-Covariance approach is that it simplifies VaR calculations because all percentiles are assumed to be known multiples of the standard deviation. However the assumption that the data is normally distributed is not consistent with real-world observations, which could lead to an underestimation of the VaR (see Hull and White (1997)).

After reviewing the three generous categories of method to compute VaR, we will introduce volatility models in the next section.

2.3 Volatility Models

As a matter of fact, the volatility model is an essential component influencing the accuracy of the VaR measurement. In this section, we will introduce volatility models of single regime (in Section 2.3.1) and multiple regimes (in Section 2.3.2).

2.3.1 Single Regime Models

Volatility models have been used in a wide variety of applications. Even for the same financial asset, a remarkable difference in the computation of VaR can be obtained according to different volatility models. Nowadays, a number of research papers mainly focus on three kinds of volatility models.

The first type is the historical volatility model (HV) which is based on historical return data for which the time scales are normally long (daily, weekly or monthly). Typical historical volatility models are the Autoregressive Conditional Heteroskedastic (ARCH) model (Engle (1982)), the Generalised Autoregressive Conditional Heteroskedasticity (GARCH) (Bollerslev (1986)) and the Stochastic Volatility model (Taylor (1986)).

The second type is the implied volatility model (IV) which is based on option pricing data. Becker (1981) pointed out that the implied volatility is a good predictor when compared with the historical standard deviation. Giot and Laurent (2004) report a good predictive power of the implied volatility model.

The third type is the realised volatility (RV) model which was first introduced in the literature by Taylor and Xu (1997) and Andersen and Bollerslev (1998). The daily realised volatility is constructed as an aggregated measure of volatility by intraday returns. Assuming that a day can be divided into n equidistant periods and if $y_{i,t}$ denotes the intraday return of the ith interval of day t, it follows that the daily volatility for day t can be written as

$$\left[\sum_{i=1}^{n} y_{i,t}\right]^2 = \sum_{i=1}^{n} y_{i,t}^2 + 2\sum_{i=1}^{n}\sum_{j=1}^{n} y_{i,t}y_{j-i,t} \tag{2.3}$$

(Andersen, Bollerslev, Diebold, and Ebens (2001)). Compared with the conventional squared daily volatility, the RV model can dramatically decrease the noise and error. Moreover, using high frequency data in the RV model can lower the measurement error.

Does the RV model work well in forecasting market risk? There are no consistent conclusions in the recent literature on realised volatility, however, the broader literature indicates that the RV model can improve the accuracy of volatility measurement. Giot and Laurent (2004) measure VaR for two stock indexes and two exchange rates using a daily ARCH type model (which uses daily returns) and a model based on the daily realised volatility (which uses intraday returns). Their results show that the performance of the two models is equivalent. Koopman, Jungbacker, and Hol (2005) instead have a totally different view. They compared the accuracy of volatility forecasting using seven years of tick-by-tick data of the S&P100 and show that the realised volatility models produce much more accurate volatility forecasts compared to models based on daily returns.

The Autoregressive Conditional Heteroskedastic (ARCH) model was introduced by Engle (1982) to accommodate the dynamics of conditional heteroskedasticity. The ARCH model defines the conditional variance σ_t^2 of the return as a function of past innovations. Bollerslev (1986) proposed the GARCH(p,q) (Generalised Auto Regressive Conditional Heteroskedasticity) model:

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{p} \beta_p \sigma_{t-p}^2 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 \quad , \tag{2.4}$$

where the parameters p and q in parentheses are standard notations, p refers to how many autoregressive lags appear in the variance equation, while the parameter q refers to how many lags are included in the ARCH term.

The GARCH (1,1) is the simplest and most robust of the family of volatility models and is defined as

$$\sigma_t^2 = \alpha_0 + \beta_1 \sigma_{t-1}^2 + \alpha_1 \varepsilon_{t-1}^2 \quad , \tag{2.5}$$

where $\varepsilon_t \mid I_t \sim N(0,1)$ and I_t denotes the information set before time t, $\alpha_0 > 0, \alpha_1 > 0$, and $\beta_1 > 0$.

The parameters α_1 and β find sum up close to 1 when GARCH model is estimated using longer period samples or higher frequency data samples.

Empirical evidence has shown that the GARCH(p,q) model can capture several stylised facts of financial time series such as volatility clusters. However, the GARCH(p,q) model neglects leverage effect of stock market price which is first discussed by Black (1976).[4] In order to improve the original model, Nelson (1991) introduced the Exponential GARCH process (EGARCH) to allow asymmetric volatility shocks in the innovation term. The EGARCH(1,1) model without the volume term in its conditional variance can be written as

$$log(\sigma_t^2) = \omega + \alpha_1 \varepsilon_{t-1} + \gamma \left(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} - E\left(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} \right) \right) + \beta_1 log(\sigma_{t-1}^2), \tag{2.6}$$

where $\beta_1 < 1$ and

$$E\left(\frac{|\varepsilon_{t-1}|}{\sigma_{t-1}} \right) = \begin{cases} \sqrt{\frac{2}{\pi}}, & Gaussian \\ \sqrt{\frac{\nu-2}{\pi}} \frac{\Gamma(\frac{\nu-1}{2})}{\Gamma(\frac{\nu}{2})}, & Student\text{-}t \end{cases} .$$

[4]The leverage effect is referred to as the negative correlation between the changes in stock returns and changes in the returns' volatility.

Empirical studies conducted on daily data using Normal EGARCH specifications usually conclude that negative shocks have a greater impact on volatility, i.e. that α is negative (Engle and Ng (1993)).

Maximum likelihood estimation is the most common method to estimate GARCH type models. If we assume the ε_t term is normally distributed, the parameter estimates can be obtained by maximising

$$\begin{aligned} L_n(\theta) &= L_n(y_1, ...y_n; \sigma_0^2; \theta) \\ &= \frac{1}{n}\sum_{t=1}^{n} l_t(\theta); \\ &= -\frac{n}{2}ln(2\pi) - \frac{1}{2}\sum_{i=1}^{n}(ln(\sigma_t^2) + \frac{\varepsilon_t^2}{\sigma_t^2}). \end{aligned} \tag{2.7}$$

Here, n refers to the number of observations, and θ is the vector of parameters to be estimated. Alternatively we take account for "fat tails" by using a Student-t GARCH model with a different log likelihood function:

$$\begin{aligned} L_n(\theta) = log\left[\Gamma(\frac{\nu+1}{2})\right] - log\left[\Gamma(\frac{\nu}{2})\right] - \frac{1}{2}log[\pi(\nu-2)] - \frac{1}{2} \\ -\sum_{t=1}^{n}\left[log\sigma_t^2 + (1+\nu)log\left(1+\frac{\varepsilon_t^2}{\sigma_t^2(\nu+2)}\right)\right], \end{aligned} \tag{2.8}$$

where ν is the degree of freedom of the distribution and $2 < \nu \leq \infty$, and $\Gamma(.)$ is the Gamma function. The lower the degree of freedom, the fatter the tail is (Alberg, Shalit, and Yosef (2008)).

2.3.2 Markov Regime Switching GARCH Models

Standard GARCH type models are widely used to model time-varying volatility. However, single regime GARCH type models are claimed that imply high persistence of individual shocks in volatility (see Hamilton and Susmel (1994) and Mikosch and Starica (2004)) for daily returns. In this chapter, we consider to extend this issue into a high frequency context and use a two regime switching model to capture high and low volatility persistence.

The standard GARCH models are unable to capture the structure changes in the variance process and suffer from an upward bias of parameters which could end up with a conservative VaR estimation (Sajjad, Coakley, and Nankervis (2008)). In order to solve this problem, several models based on combining Regime Switching (RS) models introduced by Hamilton and Susmel (1994) and GARCH models have been proposed (see Gray (1996), Klaassen (2002) and Abramson and Cohen (2007)). The main feature of regime-switching models is to allow the process switches across different regimes with certain transition probabilities, and the process of unobservable state variable s_t follows a first-order Markov chain.

For a two states process, Hamilton (1994) assumes the existence of unobserved variable s_t to have two regimes: $s_t = 1$ denotes low volatility persistence and $s_t = 2$ denotes high volatility persistence. The matrix of Markov transition probabilities between the two states P is expressed as:

$$P = \begin{bmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} p & (1-q) \\ (1-p) & q \end{bmatrix} \quad ,$$

where the transition probability $p_{ij} = Pr(S_{t+1} = j|S_t = i)$ indicates the probability that regime i at time t will be followed by regime j at time $t+1$.

In this chapter, we follow the MRS-GARCH model proposed by Klaassen (2002). The main advantage of this model is the high flexibility to capture the persistence of shocks to volatility (Marcucci (2005)). The conditional deseasonal return of the MRS-GARCH model given by (Klaassen (2002)):

$$y_t = \mu_t^{(i)} + \varepsilon_t \tag{2.9}$$

where $\mu_t^{(i)}$ denotes the mean of return at time t with $i = 1, 2$. The conditional variance of an MRS-GARCH(1,1) can be written as:

$$\sigma_t^{(i)2} = \alpha_0^{(i)} + \alpha_1^{(i)}\varepsilon_{t-1}^2 + \beta_1^{(i)}E_{t-1}\left(\sigma_{t-1}^{(i)2}|S_t\right) \quad , \tag{2.10}$$

where the expectation is

$$E_{t-1}\left(\sigma_{t-1}^{(i)2}|S_t]\right) = \sum_{j=1}^{2}\widetilde{p}_{ji,t-1}\left((\mu_{t-1}^{(j)})^2 + \sigma_{t-1}^{(j)2}\right) - \left(\sum_{j=1}^{2}\widetilde{p}_{ji,t-1}\mu_{t-1}^{(j)}\right)^2 \quad , \tag{2.11}$$

and the probabilities are computed as

$$\widetilde{p}_{ji,t-1} = Pr(s_t = j|s_{t+1} = i, I_{t-1}) = \frac{p_{ji}Pr(s_t = j|I_{t-1})}{Pr(s_{t+1} = i|I_{t-1})} = \frac{p_{ji}p_{j,t}}{p_{i,t+1}} \tag{2.12}$$

with $i, j = 1, 2$. I_{t-1} denotes the information set at time $t-1$.

The log-likelihood function is:

$$L(\theta) = \sum_{t=1}^{T} log \left(p_{1,t} f(r_t | s_t = 1) + ((1 - p_{1,t}) f(r_t | s_t = 2) \right). \qquad (2.13)$$

where $f(\cdot | s_t = i)$ denotes the conditional distribution of state i at time t.

The MRS-GARCH model has multiple parameters across regimes in capturing the persistence of jumps to volatility and allows the estimation of multi-step-ahead volatility forecast (Sajjad, Coakley, and Nankervis (2008)). Moreover, the MRS-GARCH model not only captures the volatility persistence, but can also explain the pressure-relief effect of large shocks.[5]

2.4 Intraday VaR

As an important method for financial risk measurement, VaR has been widely implemented by international financial institutions. In previous research, the computation of VaR and the analysis of the persistence of VaR are based on low-frequency data, which ignore the intraday volatility of the financial market. High frequency data is increasingly available in the markets, which pushes more and more economists to focus on intraday data in order to improve the risk management system, especially for active market participants such as high frequency traders and market makers. In this chapter, we focus on the computation of Intraday VaR (IVaR) and the one-step-ahead IVaR is defined as

[5] According to Klaassen (2002), some large shocks are followed by a period of low volatility which is called as the pressure-relieve effect.

$$Pr(y_t < -IVaR_t | I_{t-1}) = \alpha, \tag{2.14}$$

where y_i is the asset log-return in resembled regular time space at time t. I_{t-1} denotes the information set before time $t-1$.

Andrea and Claudio (2001) compared the computation of VaR with daily and with high frequency data. They mainly focus on using the stochastic volatility models (GARCH and FIGARCH) to estimate the VaR. But they found the VaR computed using half-hour data is too conservative.

Giot (2005) introduced five parametric models (Normal GARCH, Student-t GARCH, RiskMetrics and Log-ACD) and two non-parametric models (empirical quantile and extreme distribution models) for intraday VaR. Furthermore, Giot uses short time horizon (15 and 30 minute) data to compute the intraday VaR which is different from traditional VaR models.

The ACD (Autoregressive Conditional Duration) model has previously been reported as unsatisfactory for measuring IVaR with fixed time intervals or shorter time intervals. Giot (2005) used an Log-ACD (Autoregressive Conditional Duration) model applied to price duration to estimate 15 minutes and 30 minutes IVaR and the backtesting results are not satisfied. Dionne, Duchesne, and Pacurar (2009) proposed a method based on a Log-ACD-ARMA-EGARCH model to estimate intraday VaR using tick-by-tick data. The backtest interval length l (number of resampled intervals) are selected as: 15, 25, 35, 45 and 90 (observations), instead of the calendar units.[6] The

[6]The interval length l corresponds different time intervals for each stock, depending on the trading intensity in sampling periods.

model is rejected when l is small (or smaller time intervals) for all stocks (for instance, the interval $l = 15$ is on average seven minutes minutes for the Royal Bank of Canada (RY)). Colletaz, Hurlin, and Tokpavi (2007) use ACD models and a non-parametric quantile estimation to quantify intraday market risk. The difference with Giot (2005) and Dionne, Duchesne, and Pacurar (2009), is that they proposed an irregularly spaced intraday VaR (ISIVaR) model defined in price event time instead of calendar time. The ISIVaR then corresponds to the maximum loss at a given confidence level at the next *price event.* This chapter subjected to estimate the IVaR with a short fixed-time horizon of ten minutes or less.

There is a mass of literatures investigating the comparison of parametric models or non-parametric models separately. The comparison of historical simulation with GARCH type models are discussed less. Aussenegg and Miazhynskaia (2006) evaluates a set of non-parametric and parametric daily VaR models. They report the non-parametric historical simulation models have acceptable failure rates for all cases. The parametric Normal GARCH and Student-t GARCH models have good performance in most cases, but are rejected for several stocks. In order to extend the literature, we compare the VaR in high frequency context with six different models: one of them is a non-parametric IVaR model (Historical Simulation), and the others are parametric IVaR models (three single regime and one two-regime GARCH models). The empirical results discussed in the next section show the comparison of different models and their performance.

2.5 Empirical Analysis

In this section, non-parametric and parametric models are applied to estimate IVaR and the outline is as follows: Section 2.5.1 provides the data description. Section 2.5.2 and Section 2.5.3 show the empirical result of IVaR by HS and GARCH type models, respectively. The last subsection presents the backtesting results.

2.5.1 Data Description

The empirical order book data is taken from the the London Stock Exchange's "Stock Exchange Trading System" (SETS). The SETS is a modern fully electronic market offering a trading platform for the constituents of the FTSE All Share Index, Exchange Traded Funds and Exchange Traded Commodities. Order submission and execution in the SETS is continuous during opening time and is following the rule of the so-called continuous double auction concept. The trading system records all submitted orders and order changes. The matching of buy and sell orders is entirely computerised, based on the widely applied price-time order priority. The historical order book data provides full market details recording all activities on the trading platform (limit orders, market orders, iceberg orders, cancelations, changes, full/partial executions) and their matching outcomes.

In this Section, we apply the proposed volatility models on three actively traded stocks on the London Stock Exchange (LSE), Northern Rock (NR), Royal Bank of Scotland (RBS) and Hong Kong and Shanghai Banking Corporation (HSBC). The daily trading begins after the opening auction at 8 am and finishes at 4.30 pm with the daily closing auction. The data sample covers the period from 1st March 2007 to 30th March 2007, excluding the weekends.[7] The original data set contains several hundred thousand data points of irregularly spaced trades and quotes. When the method of regular time-spaced filtering is used, three total numbers of 11,220, 2,244 and 1,122 observations are selected as the high-frequency data whose time interval is 1 minute, 5 minutes and 10 minutes, respectively. Then, we split the selected data set into two parts. The first period is from 1st March 2007 to 21st March 2007 and is used for estimation (estimation sample). The second period is from 22nd March 2007 to 31st March 2007 and is used for backtesting purposes (forecasting sample).

1. The intraday return Y_t is defined as a log return:

$$Y_t = log(P_t) - log(P_{t-1}) \quad , \tag{2.15}$$

   where P_t is the stock price.

2. Seasonal adjustment

As noted by Engle and Russell (1998), high frequency data displays very strong intraday seasonality[8]. To remove the seasonal property of high frequency data, Giot (2000) assumed a deterministic seasonality in the intraday volatility, and defined the deseasonalised return y_t as

[7] One month data is too short for day-to-day analysis, but it is acceptable from a short-term trader's point of view. For instance, we have 1,122 observations in a 10 minute data sample which is roughly equivalent five years of daily data samples.

[8] The seasonality of intraday data has been explained in the Introduction, Chapter 1.

$$y_t = \frac{Y_t}{\sqrt{\phi(t_d)}} \quad , \tag{2.16}$$

where Y_t is the raw intraday return and $\phi(t_d)$ is the intraday seasonal component of the volatility.[9] Following Giot (2000), the cubic spline method is used to smooth the squared returns.

Table 2.1: Descriptive statistics for deseasonalised high-frequency returns

1 minute	**NR**	**RBS**	**HSBC**
Mean	1.3E-3	-1.6E-3	-4.3E-3
Standard deviation	1.4240	1.1267	1.2189
Skewness	0.9926	-0.6316	-0.0730
Kurtosis	49.0865	58.2823	93.5683
$Q_{(20)}$	664.2073 (0.00)	432.4120 (0.00)	1377.8200 (0.00)
5 minutes			
Mean	0.092	-0.0178	-0.0064
Standard deviation	1.2020	1.4983	1.1843
Skewness	-0.8011	-1.2085	1.6207
Kurtosis	21.8729	46.3882	33.6623
$Q_{(20)}$	122.9631 (0.00)	182.2299 (0.00)	376.7764 (0.00)
10 minutes			
Mean	0.0166	-0.0183	-0.0157
Standard deviation	1.1762	1.2296	1.3249
Skewness	-1.0101	-1.3703	0.9180
Kurtosis	16.9721	22.8280	8.5333
$Q_{(20)}$	51.4876 (0.00)	41.4294 (0.00)	123.0578 (0.00)

Note: $Q_{(20)}$ denotes the Ljung-Box Q-statistic for the first 20 lags of autocorrelations (with the corresponding p-values in the bracket.

[9]The component $\phi(t_d)$ is computed by averaging the square of raw return Y_t^2 on half hour time intervals for each day d of the week (Giot (2005)).

Table 2.1 reports the descriptive statistics and Ljung-Box Q-statistic test results of deseasonalised return. As shown in this table, the mean of the intraday deseasonalised return is extremely small for all three stocks. Moreover, the kurtosis increases with the frequency of data sampling, because higher kurtosis exhibits fat tails. The Ljung-Box Q-statistic test (Ljung and Box (1978)) results are rejected, which means that there is still a significant autocorrelation in the deseasonalised return sample.

2.5.2 Intraday VaR by Historical Simulation

The first method we are presenting here is Historical Simulation. Historical Simulation is an unconditional method which can capture the features of the market, regardless of whether the features are normal or not. As a most common method to compute daily VaR, Historical Simulation method is worth while to test its forecast ability in a high frequency context.

The sample of returns is split up into a number of different window lengths. The window size is a very important factor of the historical method. If the whole sample size is n, and the window size is n_1, we construct $(n - n_1 + 1)$ subsamples (rolling windows) and pick the confidence level of each subsample ($\alpha = 1-$confidence level). The IVaR is estimated for three different rolling window sizes with $\alpha = 5\%$.

Table 2.2 gives the failure rates in an estimation sample of historical (95%) IVaR with different window sizes for three different stocks. We can observe that the selected window size n_1 does effect the accuracy of the historical

Table 2.2: The failure rate of historical IVaR in an estimation sample

Stocks	n_1	**1 minute**	n_1	**5 minutes**	n_1	**10 minutes**
	500	0.0483	100	0.0505	40	0.0477
NR	1000	0.0490*	300	0.0499*	100	0.0504*
	3000	0.0470	600	0.0476	200	0.0488
	500	0.0518	100	0.0517	40	0.0531
RBS	1000	0.0506*	300	0.0499*	100	0.0477*
	3000	0.0486	600	0.0436	200	0.0445
	500	0.0507	100	0.0517	40	0.0510
HSBC	1000	0.0502*	300	0.0516*	100	0.0466*
	3000	0.0457	600	0.0459	200	0.0412

The table is presenting failure rates of HS within an estimation sample period for three stocks. The corresponding IVaR is better when the failure rate is closer to the theoretical counterpart. n_1 denotes the window size. A figure with [*] indicates the closest failure rate of IVaR to the theoretical counterpart (5%).

IVaR result. If a larger window size had been chosen, the risk tends to be overestimated. However, the historical IVaR tends to underestimate the risk when a smaller window size is selected. For example, in the ten minute frequency case, the failure rate is 0.0445 and 0.0531 for historical IVaR of RBS with window size 200 and 40, respectively. Therefore, we select the proper window size (marked with $*$) which seems to provide the closest result to the theoretical counterpart (5%).

The deseasonal return series of an estimation sample and forecasted historical IVaR for three stocks are shown in Figures 2.1, 2.2 and 2.3. We can observe that the blue line (IVaR with smallest window size) fluctuated the most, and the red line (IVaR with largest window size) is more stable. We can observe that the larger the window size the flatter the lines are which can lead to sustained periods of constant IVaR predictions. On the other hand, using a small window size for IVaR prediction gives more fluctuation in the results. This indicates that using a smaller window size will obtain more sensitive VaR prediction. So choosing a proper window size is quite important to get the accurate historical IVaR estimation.

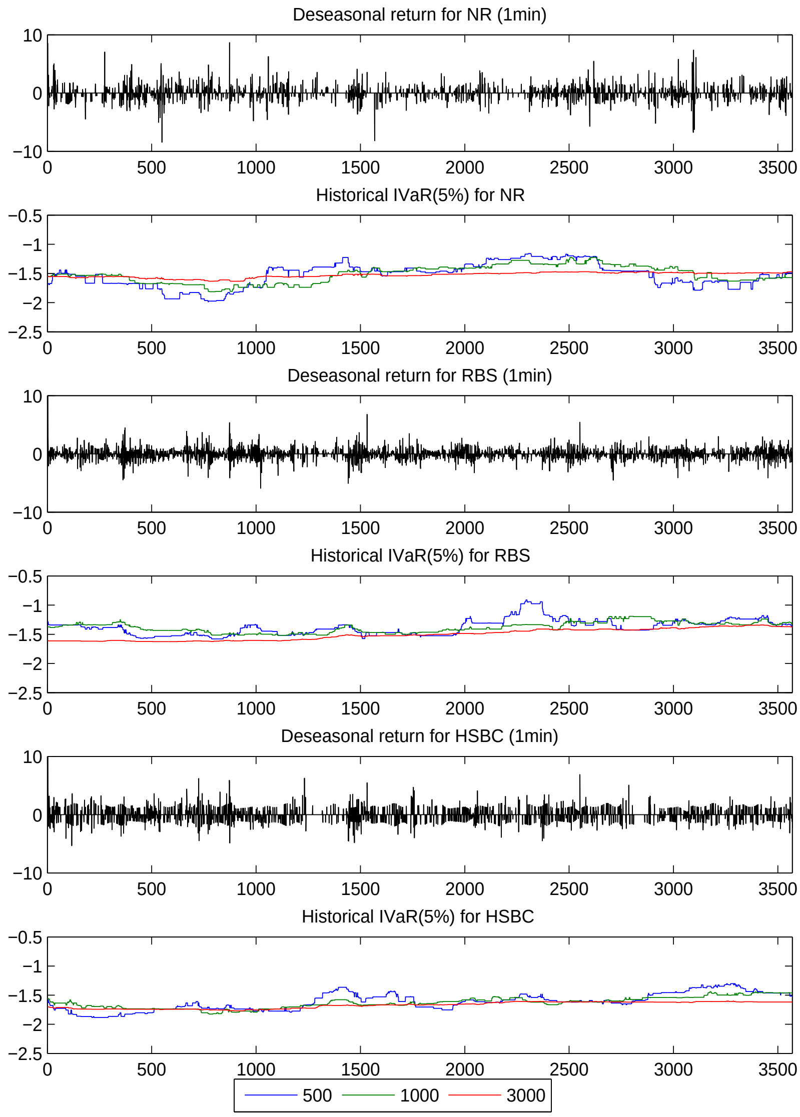


Figure 2.1: Deseasonalised one minute return of estimation sample and historical IVaR for three stocks with three different window sizes. The one minute deseasonalised return (black line) of estimation sampling period is plotted for three stocks. The corresponding historical one minute IVaR with three representative window sizes are shown in three colours: 1. blue line displays the IVaR with smallest window size; 2. green line displays the IVaR with middle window size; 3. red line shows the largest window size.

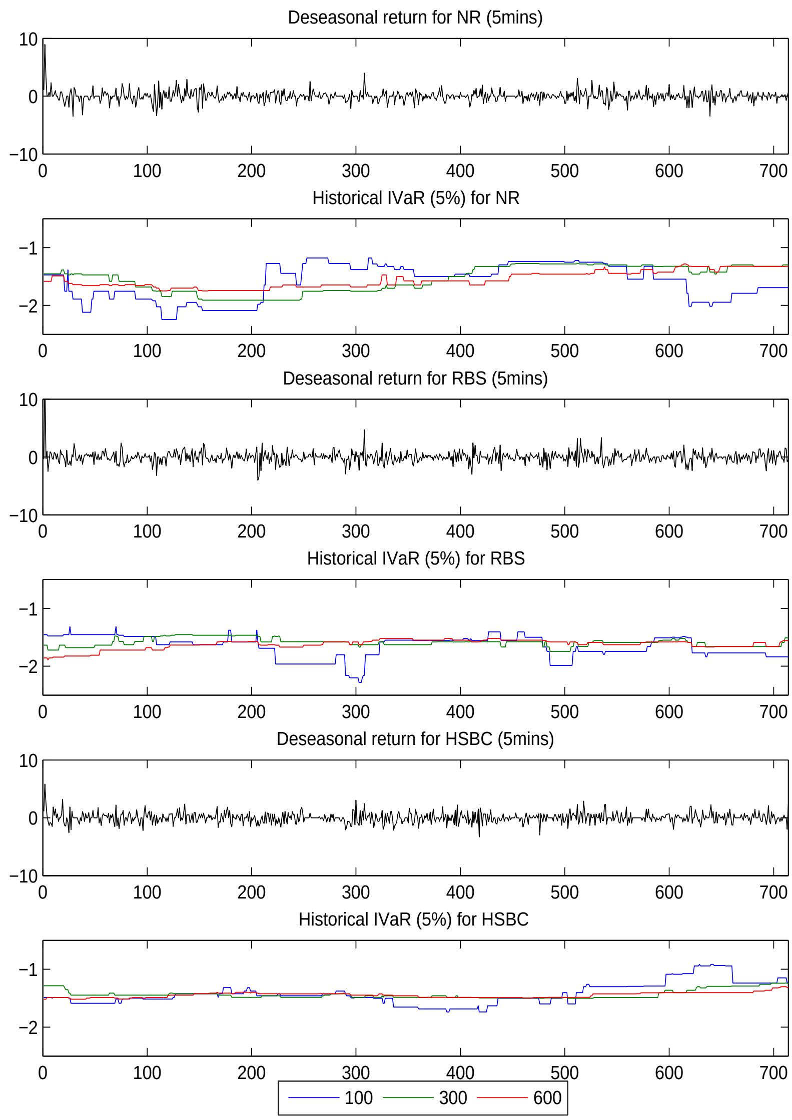


Figure 2.2: Deseasonalised five minute return of estimation sample and historical IVaR for three stocks with three different window sizes. The five minute deseasonalised return (black line) of estimation sampling period is plotted for three stocks. The corresponding historical five minute IVaR with three representative window sizes are shown in three colours: 1. blue line displays the IVaR with smallest window size; 2. green line displays the IVaR with middle window size; 3. red line shows the largest window size.

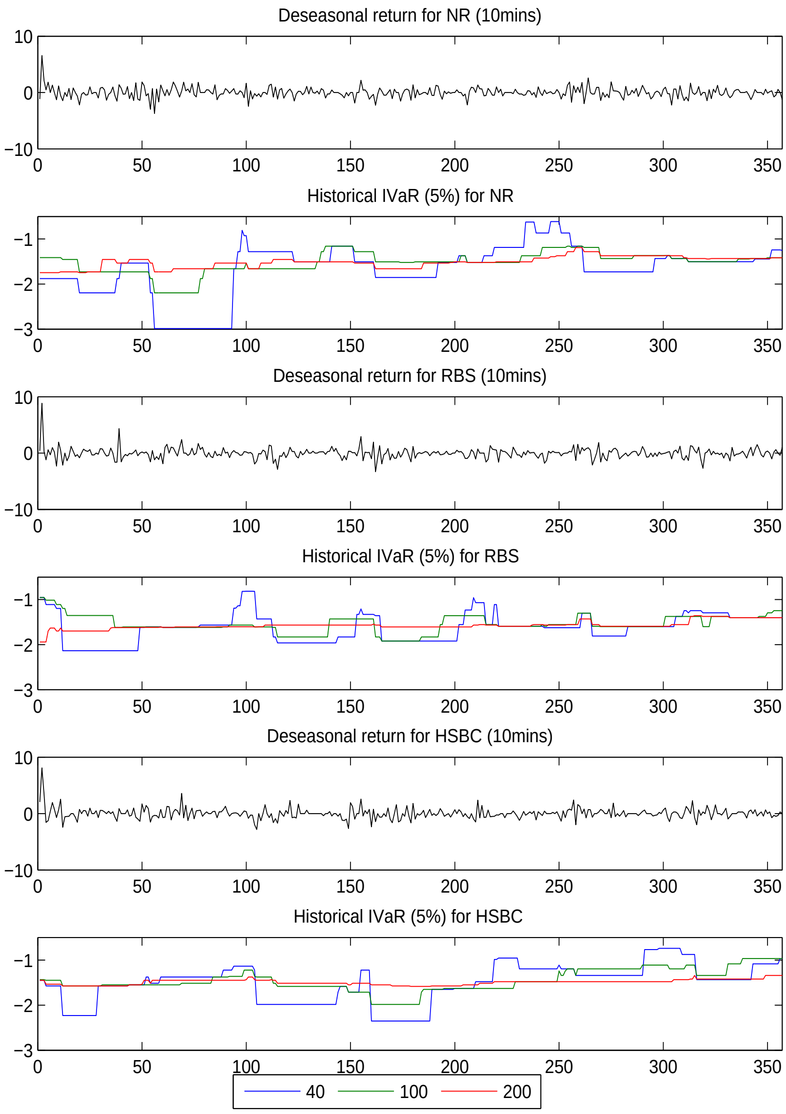


Figure 2.3: Deseasonalised ten minute return of estimation sample and historical IVaR for three stocks with three different window sizes. The ten minute deseasonalised return (black line) of estimation sampling period is plotted for three stocks. The corresponding historical ten minute IVaR with three representative window sizes are shown in three colours: 1. blue line displays the IVaR with smallest window size; 2. green line displays the IVaR with middle window size; 3. red line shows the largest window size.

2.5.3 Intraday VaR by GARCH Type Models

The main challenge of VaR computation is to obtain a good forecast of volatility for parametric models. In this section, the IVaR estimation with parametric volatility models are provided.

Single Regime IVaR

To take account of the correlation in returns, the conditional return of AR(1)-GARCH(1,1) model given by:

$$y_t = \mu + \delta y_{(t-1)} + \varepsilon_t \tag{2.17}$$

By assuming two different density functions for ε_t: the normal and Student-t, we estimated the parameters by in-sample data in four conditional volatility models with conditional mean (AR) which are AR(1)-GARCH(1,1), AR(1)-GARCH-T(1,1), AR(1)-EGARCH(1,1) and AR(1)-EGARCH-T(1,1) for different frequencies. The estimated parameters of four selected models are presented in Table A.1, Table A.2 and Table A.3 in the Appendix A.

For forecasting purposes, the one-step-ahead IVaR are generated recursively by moving forward the estimation window, considering the dynamic time-vary structure of data in different time horizons. The one-step-ahead conditional variance is used in order to obtain the IVaR. In the case of the

GARCH(1,1) model, the one-step-ahead conditional variance is

$$\hat{\sigma}_t^2 = \alpha_{0,t} + \alpha_{1,t}\varepsilon_{t-1}^2 + \beta_{1,t}\sigma_{t-1}^2 \quad , \tag{2.18}$$

For the one minute frequency case, the number observations for each stock is 11,880. The estimation sample data set has 9,480 data points and the rest belong to the forecast sample set. For every parametric model the first 9,480 returns (in-sample) are used to obtain a one-step-ahead IVaR forecast for the next minute. For the 9,482 minute one-step-ahead IVaR, we use the one minute frequency return data from the second minute to 9,481 minute and so on. Therefore, each model has to be re-estimated forward by the length of time frequency till the end of the whole data set.

Markov Regimes Switching IVaR

Apart from the single regime GARCH type model, we adopt the two regimes switching model with Normal GARCH(1,1) to calculate the volatility as in Equation 2.10, and the estimated parameters are presented in Table A.4 in Appendix A. Similar to Klaassen (2002), high and low volatility regimes are assumed in the MRS-GARCH model to capture the volatility clusters of intraday returns with three time frequencies. Table 2.3 reports the unconditional variances, $E\sigma_i$, and the persistence $p_{(i)}$ for MRS-GARCH model. The different unconditional variances indicate the existence of two regimes. The difference of unconditional variances between high and low volatility regimes is larger the one minute frequency case. The two regimes are also characterised by the persistence $p_{(i)} = \alpha_1^{(i)} + \beta_1^{(i)}$. In general, the low volatility regimes generally manifest as a lower persistence of the shocks. Contrarily, the high volatility regime reveals a higher volatility which is characterised by a higher persistence. In Table 2.3, the persistence of the high volatility regime p_2 is always higher than 0.95 and the persistence of the low volatility regime p_1 is never higher than 0.8231.

Table 2.3: MRS-GARCH properties of three stocks.

Stock	Parameters		1 minute		5 minutes		10 minutes	
NR	$E\sigma_1$	$E\sigma_2$	0.7705	8.9261	0.0308	2.4831	0.0141	2.4993
	p_1	p_2	0.5514	0.9954	0.4466	0.9937	0.4584	0.9994
RBS	$E\sigma_1$	$E\sigma_2$	0.1905	5.1646	0.1267	4.9489	0.1098	6.9657
	p_1	p_2	0.6822	0.9631	0.5393	0.9998	0.4676	0.9998
HSBC	$E\sigma_1$	$E\sigma_2$	0.2348	7.4337	0.0225	4.5950	0.0102	2.1519
	p_1	p_2	0.8231	0.9997	0.6550	0.9582	0.8022	0.9997

Unconditional variances and the persistence for the MRS-GARCH model of three stocks. Note: $E\sigma_i$ denotes the unconditional variances; and $p_{(i)}$ denotes the persistence for the MRS-GARCH model.

2.5.4 Backtesting Results

Backtesting is a process which refers to applying a model or strategy to historical data to evaluate the performance during the specified time period (Kupiec (1995)). The main purpose of this section is to compare the performance of six different IVaR models (Historical simulation, Normal GARCH, Student-t GARCH, Normal EGARCH, Student-t EGARCH and MRS-GARCH). The model performance is evaluated by using two common backtesting approaches: failure rate and Kupiec's test (Kupiec (1995)).

Failure rate test examine the frequency of VaR exceptions, which is set up as a regulatory backtesting method by the Basel Committee (1996).[10] The observed failure rate should be close to the confidence level. For example, if VaR parameters are measured at 95% confidence level with 100 time horizons, the expected VaR violations would be 5 within this period.

In this chapter, all methodologies of IVaR are tested with quantile $\alpha = 5\%$ and the performance is assessed by the failure rate for each single stock. If the IVaR model is accurate or has good forecasting performance, the failure rate should be equal to the present IVaR level. Table 2.4 reports the failure rates of forecasting sample for different IVaR (95%) models and sampling frequencies. Based on the results of failure rates, the Historical Simulation, Normal GARCH, and MRS-GARCH models perform better. The Normal EGARCH model has an acceptable performance of IVaR prediction, but slightly overestimates the risk in most cases. We can also observe that the single regime GARCH and EGARCH model with Student-t innovation did not improve on the IVaR forecast performance. In fact, the failure rates of IVaR estimated by the Student-t GARCH model and Student-t EGARCH model are far less than the expected value at 5% which indicates that the risk is consistently overestimated. The failure rate results of MRS-GARCH IVaR for all three stocks are fairly close to the theoretical confidence level.

In addition, the overall failure rates are computed to investigate that how a model performed subject to all stocks and sample frequencies.[11] In Table

[10] Failure rate is the proportion of VaR violations of the return which equals V/N, where V is the aggregated violation of stock and N is the size of sample.

[11] The overall failure rate equals: $\sum_{a=1}^{n}\sum_{b=1}^{m} V_{ab} / \sum_{a=1}^{n}\sum_{b=1}^{m} N_{ab}$, where V_{ab} is the aggregated violations of stock a and time frequency b; N_{ab} is the size of forecast sample of

Table 2.4: The failure rate result of backtesting for IVaR at 5% quantile for the three stocks

Stocks	**Model**	**1 minute**	**5 minutes**	**10 minutes**
NR	HS	0.0493	0.0476	0.0532
	Normal GARCH	0.0485	0.0463	0.0478
	Student-t GARCH	0.0163	0.0266	0.0225
	Normal EGARCH	0.0471	0.0415	0.0449
	Student-t EGARCH	0.0266	0.0013	0.0197
	MRS-GARCH	0.0504	0.0504	0.0503
RBS	HS	0.0510	0.0504	0.0504
	Normal GARCH	0.0437	0.0392	0.0421
	Student-t GARCH	0.0180	0.0070	0.0197
	Normal EGARCH	0.0429	0.0350	0.0421
	Student-t EGARCH	0.0175	0.0089	0.0197
	MRS-GARCH	0.0482	0.0478	0.0506
HSBC	HS	0.0499	0.0448	0.0392
	Normal GARCH	0.0479	0.0462	0.0449
	Student-t GARCH	0.0202	0.0224	0.0225
	Normal EGARCH	0.0424	0.0420	0.0393
	Student-t EGARCH	0.0122	0.0168	0.0197
	MRS-GARCH	0.0488	0.0462	0.0478
	Overall failure rate			
	HS	0.0489	Normal GARCH	0.0465
	Student-t GARCH	0.0187	Normal EGARCH	0.0427
	Student-t EGARCH	0.0164	MRS-GARCH	0.0488

Failure rates and overall failure rates for HS, the normal GARCH, Student-t GARCH, normal EGARCH, Student-t EGARCH and MRS-GARCH measures for intraday VaR. Failure rate is the proportion of VaR violations of the return which equals V/N, where V is the aggregated violation of stock and N is the size of sample. The overall failure rate equals: $\sum_{a=1}^{n}\sum_{b=1}^{m} V_{ab}/\sum_{a=1}^{n}\sum_{b=1}^{m} N_{ab}$, where V_{ab} is the aggregated violations of stock a and time frequency b; N_{ab} is the size of forecast sample of stock a and time frequency b; n is the number of stocks and m is the number of different time frequencies.

2.4, the overall failure rates are 0.0489, 0.0465, and 0.0488 for HS model, Normal GARCH model and MRS-GARCH model, respectively, suggesting that those three models are good alternatives in modeling volatility and in

stock a and time frequency b; n is the number of stocks and m is the number of different time frequencies.

estimating IVaR.

Apart from the failure rate, the Kupiec's test (Kupiec (1995)) is used to further analyse the performance of all IVaR models. Kupiec's test checks whether the observed failure rate is consistent with the frequency of exceptions predicted by the IVaR model.

The hit sequence (Hit_t) is defined as

$$Hit_t = \begin{cases} 1, & if \quad R_t < -VaR_t \\ 0, & if \quad R_t \geq -VaR_t \quad . \end{cases} \tag{2.19}$$

Under the null hypothesis that the model is 'good', Hit_t follows a Bernoulli distribution as with H_0: $Hit_t \sim Bernoulli(p)$. The Kupiec's likelihood ratio (LR) statistic is written as

$$LR = 2ln\left((\hat{\alpha}^m(1-\hat{\alpha})^n/(\alpha^m(1-\alpha)^n)\right) \sim \chi^2(1); \tag{2.20}$$

where m is the sum of number when $Hit_t = 1$ and n is the sum of number when $Hit_t = 0$. Under the null hypothesis, the LR is distributed as a $\chi^2(1)$.

The Kupiec test results for all methodologies are presented in Table 2.5. The bold numbers (p-values) denote a failure of the IVaR models at a 95% confidence level. The results indicate that the HS, Normal GARCH and MRS-GARCH models lead to better forecast performance in general. Consequently, the VaR models based on Student-t innovations have difficulties in modeling high frequency returns and consistently overestimate the return (risk) of all stocks in different time frequencies.

Table 2.5: The Kupiec test result of IVaR at 5% quantile for the three stocks (p-value)

Stocks	**Model**	**1 minute**	**5 minutes**	**10 minutes**
NR				
	HS	0.8444	0.7686	0.7822
	Normal GARCH	0.6740	0.6449	0.8446
	Student-t GARCH	**0.0000**	**0.0019**	**0.0078**
	Normal EGARCH	0.4178	0.0405	0.8119
	Student-t EGARCH	**0.0000**	**0.0000**	**0.0078**
	MRS-GARCH	0.9115	0.9521	0.9737
RBS				
	HS	0.7917	0.9590	0.9710
	Normal GARCH	0.0783	0.1211	0.4845
	Student-t GARCH	**0.0000**	**0.0000**	**0.0029**
	Normal EGARCH	0.0547	0.0537	0.0824
	Student-t EGARCH	**0.0000**	**0.0000**	**0.0029**
	MRS-GARCH	0.6183	0.4142	0.9613
HSBC				
	HS	0.9663	0.5010	0.3320
	Normal GARCH	0.5646	0.6449	0.8119
	Student-t GARCH	**0.0000**	**0.0002**	**0.0078**
	Normal EGARCH	0.4336	0.2383	0.2383
	Student-t EGARCH	**0.0000**	**0.0000**	**0.0029**
	MRS-GARCH	0.7315	0.7752	0.8446

Kupiec test results for HS, the normal GARCH, Student-t GARCH, normal EGARCH, Student-t EGARCH and MRS-GARCH measures for intraday VaR. A bold number (p-value) denotes that the IVaR model is rejected at a 95% confidence level.

Figures 2.4, 2.5 and 2.6 present the return of forecast sample and the 95% IVaR from the Normal GARCH and MRS-GARCH models with three different time horizons. We can observe that the MRS-GARCH IVaR is less fluctuative than Normal GARCH and the Normal GARCH model is more sensitive when capturing the shocks in volatility. In a certain sense, this could result from the averaging effect of two regimes shifting.

It can be concluded that the HS IVaR model, Normal GARCH model and MRS-GARCH model uniformly perform better compared to the other three models. Nevertheless, in general cases the MRS-GARCH model performs the best under the $p - value$ of the Kupiec test is never smaller than 0.4142.

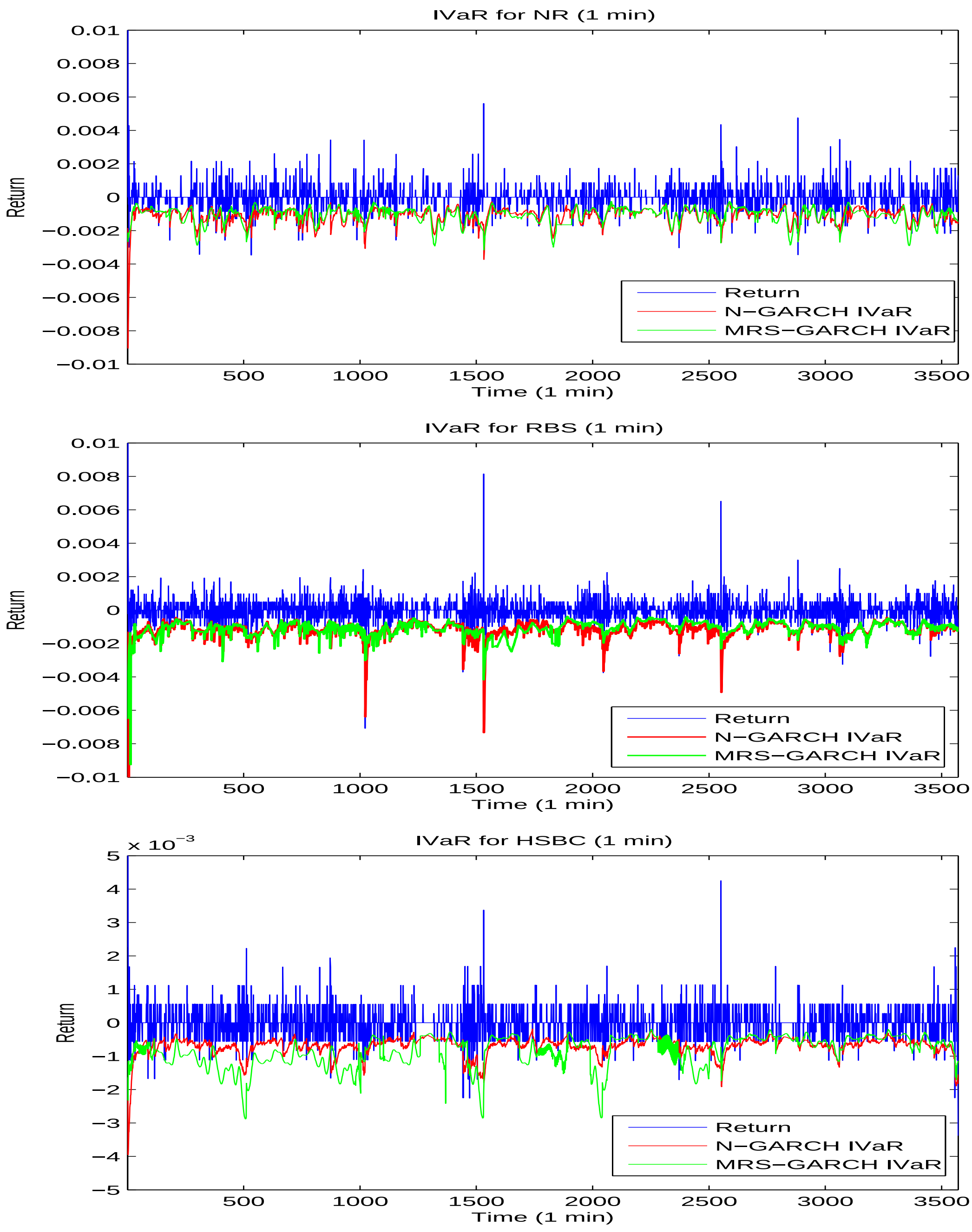


Figure 2.4: The one minute (95%) IVaR estimates from Normal GARCH and MRS-GARCH models for three shocks. The blue line displays the one minute return, the red line shows the IVaR of a normal GARCH model and the green line displays the MRS-GARCH IVaR.

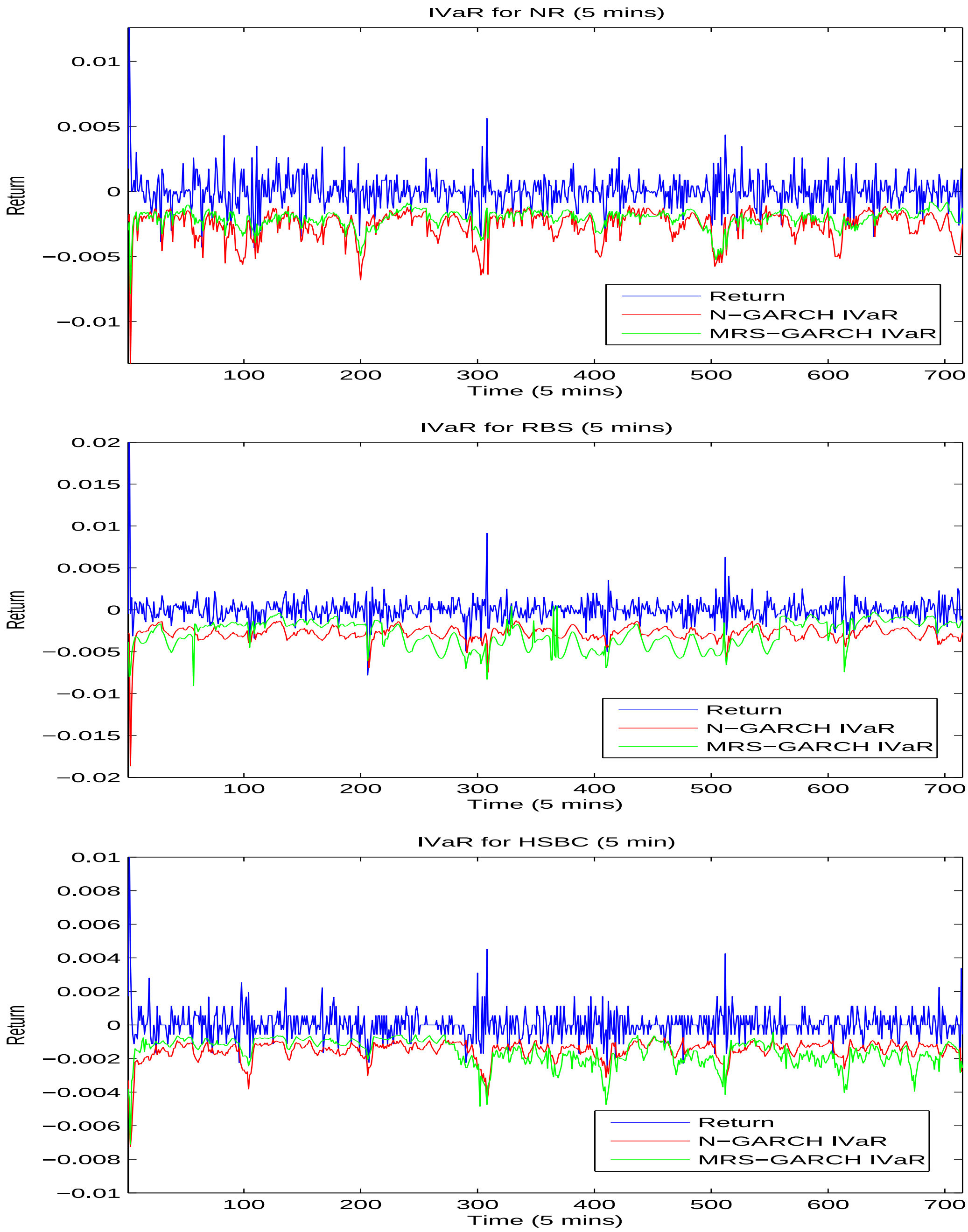


Figure 2.5: The five minute (95%) IVaR estimates from Normal GARCH and MRS-GARCH models for three shocks. The blue line displays the five minute return, the red line shows the IVaR of a normal GARCH model and the green line displays the MRS-GARCH IVaR.

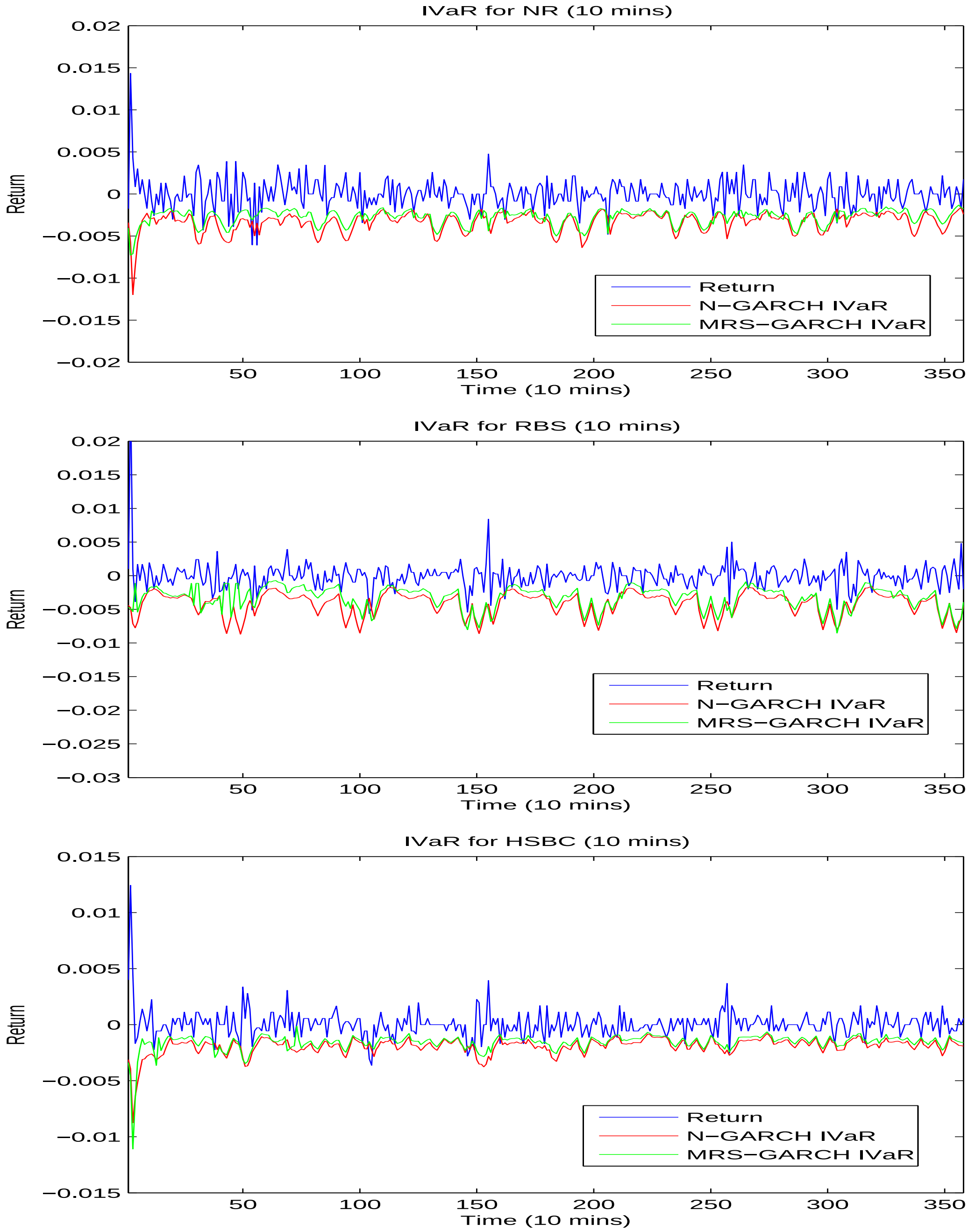


Figure 2.6: The ten minute (95%) IVaR estimates from Normal GARCH and MRS-GARCH models for three shocks. The blue line displays the ten minute return, the red line shows the IVaR of a normal GARCH model and the green line displays the MRS-GARCH IVaR.

2.6 Conclusion

This chapter compared the forecasting abilities of the Intraday VaR (IVaR) models based on high-frequency measures of volatility. Six different models were applied for quantifying intraday market risk, namely the Historical Simulation method, Normal GARCH, Student-t GARCH, Normal EGARCH, Student-t EGARCH and MRS-GARCH model. Three stocks (Northern Rock (NR), Royal Bank of Scotland (RBS) and Hong Kong and Shanghai Banking Corporation (HSBC)) from the FTSE 100 were selected for application. The time horizon of the empirical analysis is much shorter than the conventional VaR method which is computed on a daily basis.

The main task is to measure the intraday VaR using six different models and evaluate their performance in terms of the ability to forecast the IVaR for three different short time frequencies. According to the backtesting results, we can draw the following conclusions.

As a non-parametric model, the simple Historical Simulation method significantly outperforms at all different time horizons. For parametric models, the GARCH type models based on normal innovation are superior to other models in forecasting IVaR. The empirical failure rate of the Student-t distribution based IVaR model is too low and rejected. This finding is different with many existing literatures in the reference which recognised the importance of the fat tails of daily or intraday returns. One cause for risk overestimation for the models under the Student-t distribution innovations might be that the tested period does not have as many significant losses as the estimation sample period. Furthermore, previous studies recognise the importance of capturing the volatility persistence by using regime switching models on daily VaR calculation. We applied a MRS-GARCH model (Klaassen (2002)) into intraday VaR context. The empirical backtesting results confirm that the MRS-GARCH model with normal innovations is a good candidate for forecasting IVaR.

In the evolving financial markets, algorithmic trading plays a very important role in today's trading world. Today's trading system forces firms to continuously build their own strategies to beat the market (Aldridge (2009)). As a final remark, it would be useful to consider implementing an IVaR estimation model into trading or regulation software as daily VaR. Daily risk reporting does not give an accurate picture of the actual market risk for short-term traders. The IVaR models in this chapter, can provide a real-time market risk measurement which is beyond the conventional VaR. For instance, the IVaR is a very important tool for high frequency hedge fund managers to adjust their portfolios in a safer position before their trades get executed.

Chapter 3

Liquidity Adjusted Intraday Value at Risk

The traditional Value at Risk (VaR) is a very popular tool for measuring market risk, but it does not incorporate liquidity risk. This chapter proposes an extended intraday VaR model to integrate liquidity risk introduced in Section 3.2. From a short-term trader's perspective, we estimate the one-step-ahead liquidity adjusted intraday VaR (LAIVaR) for both bid and ask positions, subject to several operational threshold trading sizes over short time horizons. We also extend existing approaches by investigating both the upside (right tail) VaR and downside (left tail) VaR process. In particular, we are interested in differentiating between both bid and ask sides since different market sides have to face different price movements as well. However, the method in this chapter heavily relies on the data based on limit order books.

The main findings in this chapter are as follows: firstly, our empirical results give a better understanding of the role played by liquidity risk components in computing intraday VaR. Secondly, two methods (GARCH(1,1) and HS) are used to estimate the one-step-ahead LAIVaR of both market sides in order to quantify their real risk position. Last but not least, the results confirm that there is an asymmetry in up and down movements in liquidity adjustments with short time horizons (5 minutes and 10 minutes) of an equity market. Downward movements typically have a higher magnitude than upward movements which implies intraday liquidity risk is negatively skewed. Moreover, in this chapter, we apply a dynamic bivariate GARCH model to analyse the correlations of volatility between bid and ask side.

The outline of this chapter is as follows. Section 3.1 introduces the motivations of our study. Section 3.2 provides a literature review on liquidity risk and liquidity adjusted VaR models. Section 3.3 describes the methodology and Section 3.4 presents the data and the empirical results. Section 3.5 summarises the main findings.

3.1 Introduction

Risk management has gained much attention over the past two decades, especially in the current financial crisis. Liquidity risk has been the leading cause of many serious market crises (Matz and Neu (2007)). The infamous disaster from the Long Term Capital Management (LTCM) in late 1998, the Russian financial crisis in 1998 and the collapse of the credit market in 2008 highlight the dangers of ignoring the effects of liquidity (see Pastor and Stambaugh (2003) and Brunnermeier (2009)). In September 2007, the British retail bank Northern Rock could not refinance itself in the credit market and faced bankruptcy due to lack of liquidity. These big lessons teach us that liquidity plays a very important role in financial markets, in particular when it comes to trading. Therefore, a good risk measurement has to take liquidity risk into account. However, the definition of liquidity is ambiguous and has many different interpretations. "*A liquid market is a market in which a bid-ask price is always quoted, its spread is small enough and small trades can be immediately executed with minimal effect on price*" (Black (1971), page 2). In contrast, Kyle (1985) proposes a more practical approach of defining liquidity that includes the following three dimensions: (a) the amount by which the transaction prices deviate from mid-market prices (tightness), (b) the number of shares that can be traded with the observed transaction price (depth), and (c) the pace with which the asset price recovers to the actual fundamental price (resiliency).

A concept that is even more difficult to predict and measure is *liquidity risk*. In a real "friction market", investors hardly get the mid-price that is used in many risk applications, and a more rigorous approach to risk management is needed. Bangia, Diebold, Schuermann, and Stroughair (1999) argue that the liquidity risk is an important component in order to capture the overall risk. Lawrence and Robinson (1997) stress that the failure to consider liquidity may lead to an underestimation of the VaR by 30%.

More and more market practitioners have recognised that liquidity risk is a very serious concern for firms, plenty of studies have analysed the VaR and liquidity separately. Only a few studies incorporate liquidity into VaR, not to speak of VaR at the intraday level (see, for example, Beltratti and Morana (1999), Dionne, Duchesne, and Pacurar (2009), Colletaz, Hurlin, and Tokpavi (2007), Acerbi (2010)). Incorporating liquidity risk in to intraday VaR measurement is the main focus of this chapter.

The literature includes a few former studies where researchers have incorporated liquidity risk with conventional VaR. In general, there are two different methods: the first one is the stochastic horizon method. Lawrence and Robinson (1997) determine the holding period of VaR according to the size of the position and the characteristics of the liquidity market. The second method models market price changes induced by selling the underlying asset within a fixed time horizon. For example, Glosten, Jagannathan, and Runkle (1997) use this method to derive the optimal strategy of liquidation that maximises the value over a pre-specified period. They estimate the VaR of the value under liquidation of the position, on the basis of the cost of liquidity of the position over the predetermined period of execution. Bertsimas and Lo (1998) use a similar method to determine the dynamic optimal strategy for minimising the cost of execution. Acerbi (2010) introduces a new framework to quantify liquidity risk by distinguishing specific liquidity conditions and liquidity policies.

The motivation for this chapter is as follows: Firstly, liquidity risk is a key factor for the health of the financial system. Conventional VaR models do not take liquidity risk into account but heavily rely on the implied assumption that an asset can be traded at a certain price at any quantity within a fixed period of time. This assumption is not realistic under real market conditions, especially in intraday trading, as execution is not always guaranteed, i.e. the conventional VaR models do not capture the liquidity risk that traders and investors are exposed to.

Secondly, today's trading system is characterised by a combination of high volatility and intensify trading. High frequency traders have to continuously build their own strategies to beat the market (Aldridge (2009)). It is not sufficient for the short-term traders to evaluate their risk based just on downside intraday VaR. The issue of distinguishing the liquidity risk for the long and short positions for the bid and ask sides is not extensively discussed in the literature.[1]

This chapter therefore attempts to measure additional risk due to liquidity in the VaR using intraday data and extends the existing literature in the following way: beside the exogenous risk factor, we consider the endogenous liquidity risk, taking into account the volume effect to model the liquidity

[1]Traders have long positions when they expect the underlying asset will rise in value and is contrasted with going short.

adjusted intraday VaR (LAIVaR), which is measured on the basis of the size of the investors' position. We also differentiate between the upside risk and downside risk on both bid and ask sides. The focus on LAIVaR as a practical risk management tool allows short term traders to value the actual risk level and to allocate long and short trading assets according to realistic market trading conditions. In addition, we characterise the liquidity risk by calculating the liquidity adjustments based on the individual trade size and positions.

3.2 Review on Liquidity Risk and Liquidity Adjusted VaR Models

3.2.1 Liquidity and Liquidity Risk

Liquidity plays a very important role in the financial market. However, the definition of liquidity is ambiguous and has several versions (Berkowitz (2000)). Generally speaking, liquidity is the ability for participants to execute large trades rapidly with a small impact on prices (Committee on the Global Financial System (CGFS), 2000). Market liquidity risk can be summarised as the risk arising from the higher cost and difficulty of executing trade which is caused by an illiquid market. According to Dowd (1998) "*a market can be very liquid most of the time, but lose its liquidity in a major crisis*". In general, we can classify the liquidity risk into normal liquidity risk and crisis liquidity risk. During a crisis period, liquidity risk should be taken more seriously into account because the market loses its liquidity. Dowd (1998) also points out the relationship between the liquidity position of VaR and the holding period (see Figure 3.1).

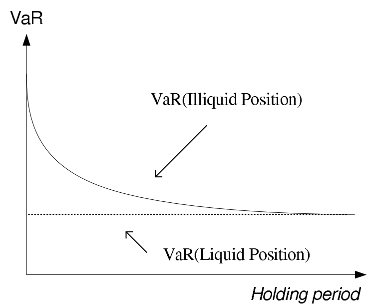


Figure 3.1: The relationship between liquid and illiquid positions of VaR and holding period (adopted from: Dowd (1998), page 198).

In a highly liquid market the investors can settle the position quickly to get the market price without any significant liquidation cost. But in an illiquid market, the extra cost of liquidity for investors to close their position is higher. Generally, the longer the investor waits, the lower the liquidity cost. However, during the waiting period, the asset price can change for the worse.

Bangia, Diebold, Schuermann, and Stroughair (1999) divide the liquidity risk into two components. Exogenous liquidity risk is similar to all market participants, which is influenced by the characteristics of the whole market. Also, exogenous liquidity risk which is connected with bid-ask spread, cannot be affected by any behavior of an individual trader. On the other hand, endogenous liquidity risk is unique for each market participant, which is influenced by one's trading volume (liquidated quantity) and positions. More specifically, the effected of endogenous liquidity risk occurs after the volume exceeds the level of quote depth. Figure 3.2 illustrates the effect of position size on liquidation value. As it can be seen, the endogenous liquidity risk becomes higher when the position size beyond the quote depth point, and the ask-bid spread became larger. In this chapter, we take both endogenous and exogenous components in to account for measuring the market risk.

3.2.2 Models of Liquidity Adjusted VaR

In this section, we provide an brief literature review on models to incorporate liquidity risk to standard VaR frameworks. As shown in Figure 3.3, the conventional VaR model like RiskMetrics (J.P. Morgan (1996)) measures the uncertainty of asset returns and does not include the liquidity risk. So an

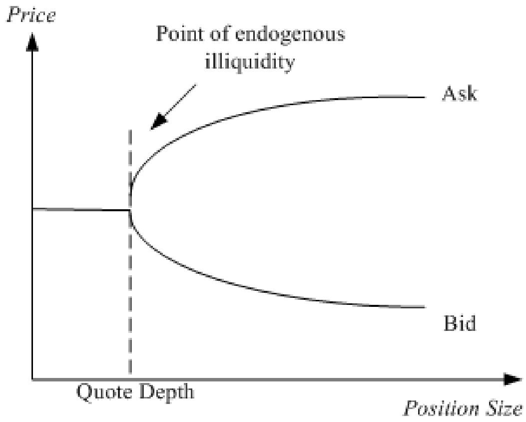


Figure 3.2: The effect of position size on liquidation value (adopted from: Bangia et al.(1999), page 5).

improved approach of modelling VaR should take the whole market risk into account, including both exogenous and endogenous liquidity risk.

The existing models are introduced as three broad categories: (1) optimal execution strategy adjusted models; (2) bid-ask spread adjusted models; (3) intraday liquidity risk models.

Optimal Execution Strategy Adjusted Models

Former studies reported in the literature incorporate liquidity risk with conventional VaR by using the optimal execution strategy. According to Ernst, Stange, and Kaserer (2009), these models are more theoretical than empirically traceable. More specifically, there are two general methods of optimal execution strategy adjusted model: one is the Stochastic Horizon method; another is modelling the changing of market price induced by the selling off with a fixed time horizon.

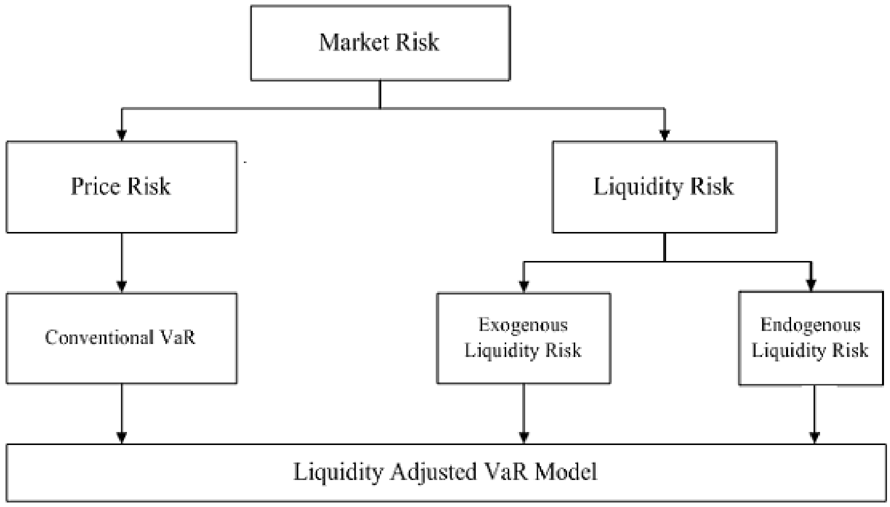


Figure 3.3: Taxonomy of Market Risk and the VaR Models (adopted from: Bangia et al.(1999), page 3).

In the first framework, Lawrence and Robinson (1997) determine the holding period of VaR according to the size of the position and the characteristics of the liquid market. The authors use the second kind of method which derives the optimal strategy of liquidation that will maximise the value over a fixed time horizon. Glosten, Jagannathan, and Runkle (1997) therefore consider the impact of the size of the position and the period of execution on the value of the position under liquidation. Bertsimas and Lo (1998) use a similar method to derive the dynamic optimal strategy with the aim of minimising the expected cost.

Hisata and Yamai (2000) propose a framework for the quantification of the liquidity adjusted VaR (LAVaR) model that considers the market impact induced by the trader's own liquidation. They derive the optimal execution strategy according to the level of market liquidity and the scale of the investor's position. They choose the holding period as an endogenous variable and provide both a discrete and a continuous time model for the LAVaR measurement.

Bid-ask Spread Adjusted Models

Bangia, Diebold, Schuermann, and Stroughair (1999) develop a liquidity risk adjusted VaR model (named the BDSS model after the names of the authors) which is a fundamental framework for integrating liquidity risk into the standard VaR. They split the overall market risk into two components: one is the pure price risk and the other is uncertainty of liquidity costs. Bangia, Diebold, Schuermann, and Stroughair (1999) classify liquidity risk into exogenous and endogenous risk.[2] However, the BDSS model mainly focuses on exogenous liquidity risk which takes the bid-ask spread into account. The LAVaR simply represents the sum of conventional VaR (computed by mid-price) and the liquidity risk adjusted part (computed by bid-ask spread).

Mathematically, the BDSS model is specified as:

$$LAVaR = Mid_t[(1 - e^{\mu - \alpha\sigma}) + \frac{1}{2}(\overline{S} + \alpha'\tilde{\sigma})], \tag{3.1}$$

where Mid_t is the mid-price of the asset at time t; μ is expected return; α is the quantile of the return of mid price; σ is the standard deviation of the return of mid price; $\overline{S} = (Ask - Bid)/Mid$ is the average relative spread; $\tilde{\sigma}$ is the volatility of the relative spread and α' is the quantile of the relative spread distribution.

[2]The details for exogenous and endogenous risk are introduced in Section 3.1.1.

The BDSS approach is simple and easy to implement. However, the BDSS model has several drawbacks: firstly, the model is based on the normal distribution which differs from reality. Secondly, the method ignores the endogenous liquidity risk which is also important.[3] Thirdly, the assumption of perfect correlation between liquidity risk and VaR would lead to an overestimation of LAVaR.

Erwan (2001) extends the BDSS model by using the weighted average spread which incorporates the endogenous risk effect instead of the ask-bid spread. He also points out that for illiquid stocks, the endogenous liquidity risk represents half of the total market risk and must not be neglected.

Agnelidis and Benos (2006) investigated the risk component of bid-ask spread in the Athens Stock Exchange with electronic order book data data. They extended the model from Madhavan, Richardson, and Roomans (1997) by taking both endogenous and exogenous liquidity risk into account. Furthermore, they adjusted the standard VaR with the risk component of high and low capitalisation stocks.

Ernst, Stange, and Kaserer (2008) suggested a liquidity risk adjusted model of VaR with future time variation of prices and spread. This model extends the BDSS model by using non-normal distribution for price and spread instead of normal distribution and historical distribution. However, this model fails to cover endogenous risk.

[3]The endogenous liquidity risk is mainly arisen by the size of trading volume.

Intraday Liquidity Risk Models

In spite of the increasingly important intraday trading in financial market, there is limited published research on how to evaluate market risk accounting for liquidity risk in the intraday context.

Following the BDSS model, Francois-Heude and Van Wynendaele (2001) proposed a new parametric IVaR account for liquidity risk. The advantage of this model is that it incorporates both the exogenous and endogenous liquidity risks. However, the liquidity cost function from the five best limits of the order book of the Paris Stock Exchange and the other hidden quantities listed in the order book are not considered in their research. Furthermore, the intraday data which has strong seasonality is not being filtered or deseasonalised in their analysis.

Giot and Gramming (2006) introduce a GARCH model to derive LAVaR in an automated auction market and quantify the liquidity risk by calculating the weighted average bid price from the real order book data. They incorporate the endogenous risk impact corresponding to the trading volume size and provide evidence that liquidity does affect the intraday VaR estimation. However, the research focuses on downside risk only.

After reviewing several models in the literature, we propose a liquidity risk adjusted IVaR to take account of both endogenous and endogenous impacts. Furthermore, we differentiate between the asymmetric effect of liquidity risk for long and short positions.[4]

[4] The details of our model is introduced in Section 3.3.

3.3 Methodology

3.3.1 Liquidity Adjusted Intraday VaR

Researchers normally use daily time series data to analyse financial problems. Compared with low frequency data, the shorter time horizon of high frequency data can provide more detailed information about the market behaviour.

Ernst, Stange, and Kaserer (2009) test most existing and traceable liquidity risk models based on daily risk estimation. According to their results, models based on limit order book data are outperformed according to the Kupiec (1995) test, including the model from Giot and Gramming (2006).

Previous studies suggested using the top of order book (best bid and ask) information to decide the trading strategies. Recently, researchers found that the activities beyond the top of limit order books contain valuable information (see Kaniel and Liu (2006), Hall and Hautsch (2007), Cao, Hansch, and Wang (2008) and Cao and Wang (2009)). In this chapter, we estimate the liquidity adjusted intraday VaR (LAIVaR) by using the limit order book information instead of the best bid and ask price.

The focus on LAIVaR as the appropriate measure of actual risk faced by short-term traders with fixed limit liquidity horizon and different trade positions according to realistic market trading conditions. In this sense, we assume that different positions face different risks. Based on the limit order book, we investigate the relationship between the midpoint of the best bid and ask and volume weighted average price. Developing the discussed BDSS model further, we estimate the LAIVaR model with different trading volume for the bid side (which is for the investor who wants to buy), as well as for the ask side (which is for the investor who wants to sell).

Let $v_{i,t}$ denote the corresponding volume of orders queuing in the book at time t at positions $i = 1, ..., n$ (i denotes each individual trade at time t). We first define the volume-weighted average prices (VWAP) $B_t(v)$ and $A_t(v)$ for both the bid (B) and ask (A) sides as follows:

$$\begin{aligned} B_t(v) &= \frac{\sum_j B_{i,t} v_{j,t}^{BID}}{v} \\ A_t(v) &= \frac{\sum_j A_{i,t} v_{j,t}^{ASK}}{v} \end{aligned} \tag{3.2}$$

where $B_{i,t}$ and $A_{i,t}$ are the individual bid and ask prices at time t of position i; v denotes a pre-specified threshold volume to be traded against several limit orders when executing at least the first i queuing orders on the bid or ask side, such that $v \leq \sum_{min(n)} v_{i,t}$; $v_{j,t}^{BID}$ and $v_{j,t}^{ASK}$ are the individual limit orders of bid and ask side that adds up to v: $\sum v_{j,t}^{BID} = \sum v_{j,t}^{ASK} = v$. For example, Table 3.1 gives a snapshot of order book data for RBS on 1st March 2007 at 8:01 am. We consider the threshold volume $v = 2000$, in this case, $v_{i,t}^{BID} = (1321, 170, 392, 1042)$, $v_{i,t}^{ASK} = (346, 346, 10242)$, $v_{j,t}^{BID} = (1321, 170, 392, 117)$ and $v_{j,t}^{ASK} = (346, 346, 1308)$.

The VWAP is an ex-ante measure of liquidity which indicates an immediate execution trading cost. With a given volume v (inside the depth), we can compute the impact of price (exogenous liquidity risk component) and volume

Table 3.1: Example of order book data for RBS at 8:01 am (01/03/2007)

	Bid			Ask		
i	Price	Volume	$v_{j,t}^{BID}$	Price	Volume	$v_{j,t}^{ASK}$
1	1128	1321	1321	1131	346	346
2	1128	170	170	1134	346	346
3	1128	392	392	1137	10242	1308
4	1123	10242	1042	1145	117	
5	1122	1503		1147	1339	
6	1120	10000		1150	870	
7	1118	1414		1151	1172	
8	1116	1208		1157	800	
9	1111	1363		1158	870	
...	...	...		...	...	
VWAP						
$B_t(v = 2000) = 1127.7075$						
$A_t(v = 2000) = 1135.4430$						

(endogenous liquidity risk component) by using the information of the full limited order book data.

In order to capture the liquidity risk between bid and ask side, we adopt the model by Giot (2005) and define two log ratio return processes instead of the return based on the weighted spread as

$$r_t^{BID}(v) = \ln \frac{B_t(v)}{B_{t-1}(v)}$$
$$r_t^{ASK}(v) = \ln \frac{A_t(v)}{A_{t-1}(v)} \tag{3.3}$$

representing the VWAP returns.

As mentioned in Chapter 1, the seasonality generally existed in high frequency return. It is reported in former studies (see Goodhart and O'Hara (1997), Andersen and Bollerslev (1999) and Giot (2000)) that financial intraday data have a consistent diurnal pattern of trading activities over the course of a trading day, due to certain institutional characteristics of organised financial markets, such as opening and closing hours or lunch time. Since it is necessary to take the daily *deterministic* seasonality into account (Andersen and Bollerslev 1999), smoothing techniques are required to obtain deseasonalised observations. To remove the seasonality property of high frequency data, Giot and Gramming (2006) assumed a deterministic seasonality in the intraday volatility, and defined the deseasonalised return as

$$
\begin{aligned}
y_t^{BID} &= \frac{r_t^{BID}(v)}{\sqrt{\phi_t^{BID}}} \\
y_t^{ASK} &= \frac{r_t^{ASK}(v)}{\sqrt{\phi_t^{ASK}}}
\end{aligned}
\tag{3.4}
$$

where $r_t^{BID}(v)$ and $r_t^{ASK}(v)$ denote the raw log VWAP-returns for bid and ask side. The ϕ_t^{BID} and ϕ_t^{ASK} are the deterministic seasonality pattern of intraday volatility for bid and ask side returns.

Following the same approach in Chapter 2, we first choose raw return at 30 minute intervals as nodes for the whole trading day and then use cubic splines to smooth the average squared sample returns in order to get the intraday seasonal volatility components ϕ_t^{BID} and ϕ_t^{ASK} for bid and ask side (see also (Giot 2000) and (Giot 2005)).

Having computed the deseasonalized VWAP return process y_t, we apply a

GARCH(1,1) model[5]

$$y_t = \mu_t + \sqrt{h_t}\varepsilon_t \tag{3.5}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \tag{3.6}$$

for both market sides with h_t as the conditional variance for the (deseasonalized) VWAP-returns and ε_t as normally distributed innovations. The LAIVaR at time t for the two return processes given the confidence level α can be modelled as

$$\text{LAIVaR}_t = \mu_t + Z_\alpha \sigma_t \tag{3.7}$$

with σ_t as the volatility component. Based on the estimated conditional variance, the standard deviation of the raw return at time t is $\sigma_t = \sqrt{h_t \phi_t}$. With D_t^{BID} and D_t^{ASK} (Equation 3.4), we can estimate the LAIVaR for both bid and ask sides which can be displayed as LAIVaR_t^{BID} and LAIVaR_t^{ASK}, respectively.

In the "frictionless" market, the frictionless VaR is computed by the mid-price. In order to quantify the liquidity risk adjustment, we also need to compute the intraday VaR based on the mid-price ($IVaR^{MID}$) as a benchmark and compare it with the LAIVaR. We define the log ratio return of

[5] The GARCH(1,1) model based on normal distribution has been proved to have robust results for IVaR estimation. Moreover, the parameters of the GARCH(1,1) model will be used in the DCC model of Section 3.3.3 to obtain the correlations of volatility between bid and ask side.

mid-price $r_{mid,t}$ as

$$r_t^{MID} = \ln \frac{P_t^{MID}}{P_{t-1}^{MID}}, \tag{3.8}$$

where P_t^{MID} is the mid-price at time t and model the mid-price return process using a GARCH(1,1) volatility process. Similarly, the IVaR of mid-price returns at time $t-1$ is given by:

$$IVaR_t^{MID} = \mu_t^{MID} + Z_\alpha \sigma_t^{MID} \quad . \tag{3.9}$$

3.3.2 Liquidity Risk Adjustment

Most studies in the literature ignore upside risk and only focus on the downside risk, however in this chapter the upside risk is a measure for traders who have a short position on their asset. A higher upside risk also means a higher cost. In order to compare the actual liquidity cost of different trading volume and positions intuitively, we first introduce the intraday VaR of price (PIVaR), which is the worst $\alpha\%$ predicted price (P) if one were to trade the asset at time t:

$$PIVaR_t = IVaR_t * P_{t-1} \tag{3.10}$$

Then, we define the liquidity risk adjustment λ_t as the difference between mid-price PIVaR and LAPIVaR. The liquidity risk adjustment λ_t quantified the actual liquidity cost for traders with different trading strategies. In other words, λ_t measures the additional value for liquidity effect not taken in to account by frictionless VaR.

$$\lambda_t = \begin{cases} \frac{1}{T}\sum_{t=1}^{T}(PIVaR_{(t)}^{MID} - LaPIVaR_{(t)}^{BID}) & (DownsideRisk) \\ \frac{1}{T}\sum_{t=1}^{T}(LaPIVaR_{(t)}^{ASK} - PIVaR_{(t)}^{MID}) & (UpsideRisk) \end{cases} \tag{3.11}$$

To summarise, the liquidity risk adjustments provide detailed predictions of liquidity risk for different trading positions. Moreover, the concept of liquidity risk adjustment studied here abstracts an intuitive measure of liquidity risk for upper level managers, who may not be familiar with the statistical analysis. In this work, it is recommended that the intraday risk management method should be considered in asymmetric liquidity risk adjustments and adapt to the specific needs that correspond to different market participants. The asymmetric effect of upside and downside liquidity risk adjustments will be examined in Section 3.4.3.

3.3.3 Dynamic Correlation Analysis

We are also interested in the relative cost of liquidity risk and the difference of the LAIVaR between the bid and ask side. To our knowledge, there is no literature discussing the dynamic correlation of volatility between bid and ask side. To understand the time-varying correlation behind the return of VWAP on both the bid and ask side of the order book *jointly*, we apply the dynamic conditional correlation (DCC) multivariate GARCH model proposed by Engle (2002).

Consider the bivariate filtrated normally distributed return process

$$r_t \mid I_{t-1} \sim N(0, H_t) \tag{3.12}$$

with the covariance matrix

$$H_t = D_t R_t D_t, \tag{3.13}$$

where R_t represents the correlation matrix of the returns on both market sides. Further, Engle (2002) assumes that

$$\begin{aligned} D_t &= diag(\sqrt{h_t}) && (3.14)\\ Q &= (1-a-b)\overline{Q} + a\varepsilon_{t-1}\varepsilon'_{t-1} + bQ_{t-1} && (3.15)\\ R_t &= (diag(Q_t))^{-\frac{1}{2}} Q_t (diag(Q_t))^{-\frac{1}{2}} \quad , && (3.16) \end{aligned}$$

where

$$\overline{Q} = T^{-1} \sum_{t=1}^{T} \varepsilon_t \varepsilon'_t \quad . \tag{3.17}$$

The residuals are assumed to be

$$\varepsilon_{it} = r_{it}/\sqrt{h_{it}} \tag{3.18}$$

with $h_{i,t} = \alpha_0 + \alpha_i \varepsilon^2_{i,t-1} + \beta_i h_{i,t-1}$, where i stand for the i-th asset. Following

Engle (2002), the log-likelihood function can be written as

$$\begin{aligned} L(\theta,\varphi) &= \sum_{t=1}^{T} L_t(\theta,\varphi) \\ &= -\frac{1}{2}\sum_{t=1}^{T}(log|D_tR_tD_t| + r_t'D^{-1}R_t^{-1}D^{-1}r_t) \\ &= -\frac{1}{2}\sum_{t=1}^{T}(\underbrace{2log|D_t| + r_t'D^{-1}r_t}_{L_v(\theta)} - \underbrace{\varepsilon_t'\varepsilon_t + log|R_t| + \varepsilon_t'R_t\varepsilon_t}_{L_c(\theta,\varphi)}) \quad , \end{aligned}$$

allowing a two-step-estimation approach as it can be decomposed into a volatility part

$$\begin{aligned} L_v(\theta) &= -\frac{1}{2}\sum_{t=1}^{T}(2log|D_t| + r_t'D^{-2}r_t) && (3.19) \\ &= \frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{n}(log(h_{i,t} + \frac{r_{i,t}^2}{h_{i,t}})) && (3.20) \end{aligned}$$

and a correlation part

$$L_c(\theta,\varphi) = -\frac{1}{2}\sum_{t=1}^{T}(log|R_t| + \varepsilon_t'R_t\varepsilon_t - \varepsilon_t'\varepsilon_t) \quad . \tag{3.21}$$

Hence, we first estimate the parameters $\hat{\theta} = (\alpha_0, \alpha_i, \beta)$ in (3.20) in the univariate GARCH models, and then substitute $\hat{\theta}$ into (3.21) to estimate the parameter vector $\varphi = (a, b)$.

3.4 Empirical Analysis

3.4.1 Data Description

In this chapter, we use the same three stocks of SETS limit order books as in Chapter 2, which are Northern Rock (NR), Royal Bank of Scotland (RBS) and Hong Kong and Shanghai Banking Corporation (HSBC).[6] Moveover, we use the extra volume information in order book for the empirical analysis in this chapter. The different volume sizes executed have different liquidity risk effects. For illustration purposes, we present three thresholds of liquidity executions in this chapter which are based on small, medium and large sizes of volume. Executing big volume orders has a bigger liquidity risk than executing small volume. We measure the investor's risk on both the downside and upside risk which depends on the investors' trading strategy (short or long position). Table 3.2 provides a list of several average volumes which reflect the liquidity activity for the three selected stocks and shows that HSBC have the largest trade size in every category. If we compare the average cumulated volume of total ask and bid, NR has the smallest size. According to these facts we choose several different representative threshold volume sizes to reflect different liquidity positions for each stock indicated in the last three rows of Table 3.2.

The data sample covers the period from 1st March 2007 to 30th March 2007, excluding the weekends. We use five minute equally sampling data (as in Chapter 2) and split the selected data set into two parts, the first three

[6]Details of the data are introduced in Chapter 2, Section 2.5.1.

Table 3.2: Data description

Average volume of ...	NR	RBS	HSBC
Best ask	2979	2038	28386
Best bid	2802	2039	18450
Best three ask orders	7504	8762	57074
Best three bid orders	6654	9015	41743
Total ask side	345420	1077160	3939042
Total bid side	346030	1116740	4348526
Threshold Size (small)	2000	10000	50000
Threshold Size (medium)	10000	50000	100000
Threshold Size (large)	20000	100000	200000

weeks are used for estimation (estimation sample) and the others (forecasting sample) for backtesting purposes. Then the procedure is also repeated for ten minute frequency data.

In Figure 3.4 the mid-price time series of NR with the whole sample period is showed in panel (a). In order to illustrate the difference between VWAP and mid-price more clearly, we take a close look at the first 100 observations as plotted in panel (b). As illustrated, the difference of VWAP and mid-price between ask and bid sides exhibits asymmetric behaviour. In addition, the bid-ask spread of different threshold volumes is presented in panel (c). We can observe that the spread is bigger when the threshold volume size is larger.

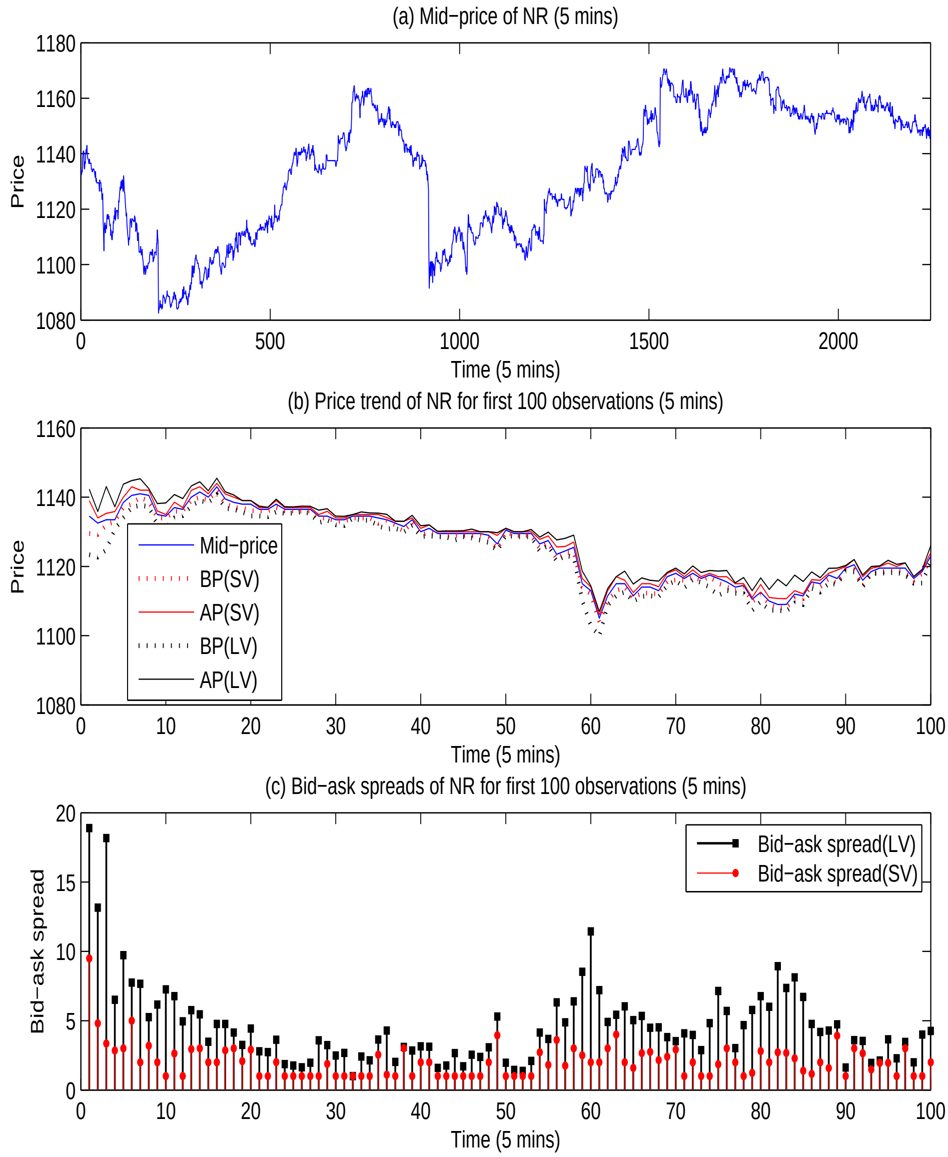


Figure 3.4: (a) The blue line denotes the mid-price (5 minutes) of NR for the whole sample period. (b) The red dotted line represents the VWAP of bid side (BP) with small volume (SV); the red line represents the VWAP of ask side (AP) with small volume (SV); the black dots represents VWAP of the bid side (BP) with large volume (LV) and the black line represents VWAP of the ask side (BP) with large volume (LV). (c) The red circles indicate the bid-ask spread of NR with small volume and the black squares indicate the bid-ask spread for large volume.

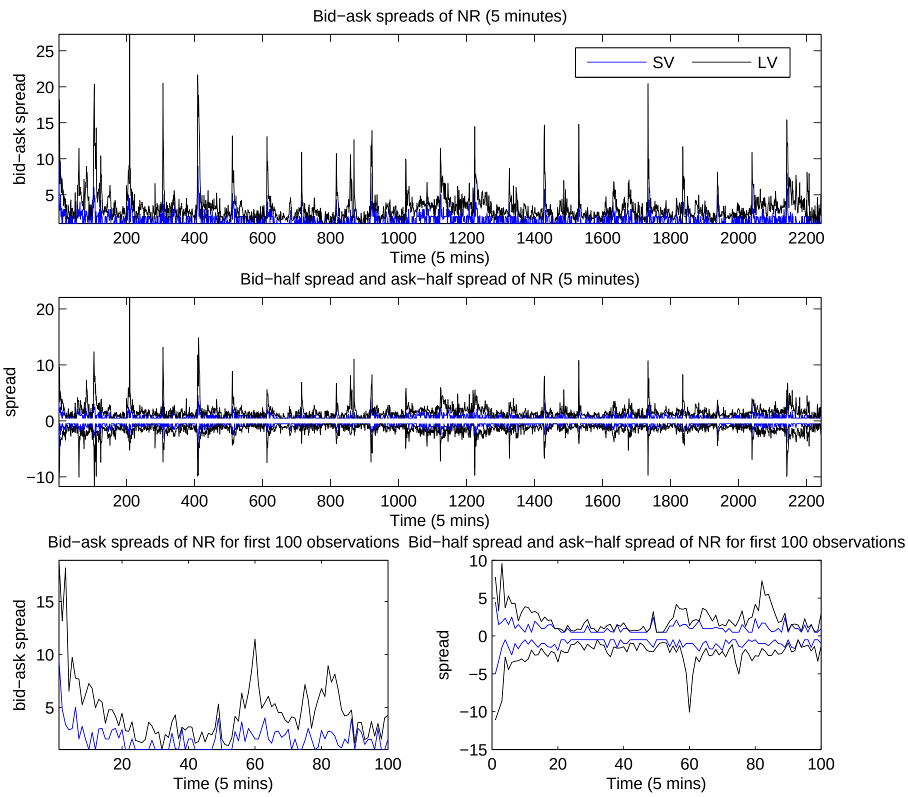


Figure 3.5: The bid-ask spread and spread between bid and ask for NR (SV=small volume; LV=large volume). Figure (a) and (b) show spreads for the whole sample period. The zoom in sub-plots (c) and (d) show spreads for the first 100 observations.

Figure 3.5 illustrates the bid-ask spread based on different trading volume (small and large) for NR. The bid-ask spreads are bigger for larger volume sizes. Furthermore, we are more interested in the asymmetric information between the bid and the ask side[7]. Figure 3.5 also shows the spread between bid and ask side. As it can be seen from the figure, the higher volume size has a positive impact on the asymmetric behaviour between the bid and the ask

[7]Bid-ask spread= $VWAP^{ASK} - VWAP^{BID}$; Bid-half-spread= $VWAP^{BID} - P^{mid}$ and Ask-half-spread= $VWAP^{ASK} - P^{mid}$.

side. For clearer scope, we show two sub-plots with the first 100 observations. The figures of bid-ask spread for the other two stocks are shown in Figure B.1 and B.2 in Appendix B.

3.4.2 Estimation of GARCH Parameters

We filter every five minute and ten minute snapshot of the order book to get equally spaced time series data. Table B.1 presents the GARCH model parameter estimates (with the standard errors in brackets) based on the VWAP returns for the three stocks with different threshold volume values. For the stock of NR and HSBC, all α parameters are, as expected, smaller than β which means that the updated variance is mainly based on the past variance and less affected by "news". However for the stock of the RBS on the bid side, the past variance is mainly dependent on the "innovation" part. Based on the estimated parameters, we compute the frictionless IVaR and LAIVaR for both upside and downside at $\alpha = 5\%$ with a different sampling period.

3.4.3 LAPIVaR and Liquidity Risk Adjustment

For the purpose of studying the measurement of intraday liquidity risk, we estimate the frictionless PIVaR and LAPIVaR through Formula 3.10. In addition, Historical Simulation (HS) [8] is also used to quantify LAPIVaR. As we already pointed out, there is an asymmetric intraday price behaviour

[8] Historical Simulation method has been proved to have robust results for estimating IVaR in Chapter 2.

between bid and ask side. Therefore, it is necessary to estimate the upside risk and downside risk respectively.

In this Section, we first estimate the LAPIVaR by using GARCH(1,1) and HS methods. Figure 3.6 and Figure 3.7 display both the upside and downside LAPIVaR (with α=5%) for NR and compares this with the frictionless PIVaR, based on a five minute and ten minute sampling frequency. In order to illustrate the price impact, three representative threshold volume sizes (large, medium and small volume) to reflect different liquidity positions are shown in Table 3.2. From the figures, we can observe that when executing a huge volume size, the LAPIVaR is always above the frictionless PIVaR for upside risk and lower for downside risk, and the difference is obvious. The LAPIVaR also displays asymmetric behaviour between the upside and downside position. The results for other two stocks are shown in Appendix B.

For the algorithmic trader who always adjusts their position over a short time period, it is important to take liquidity risk into account. For instance, a trader who needs to close his position in very short time has to pay extra for liquidity cost. We assumed that the liquidity cost depends not only on the volume size, but also on the positions. The upside and downside LAIVaR allow traders to know exactly how large the risk of a long and short position is. As shown in Figure 3.6 and Figure 3.7, huge volume entails more liquidity risk and higher cost. Hence, the conventional method which uses mid-price to measure IVaR underestimates this risk.

Next, we examine the effect of our liquidity risk by the liquidity risk adjustment λ (Equation 3.11 in Section 3.3.2). Figure 3.8 displays the forecasted

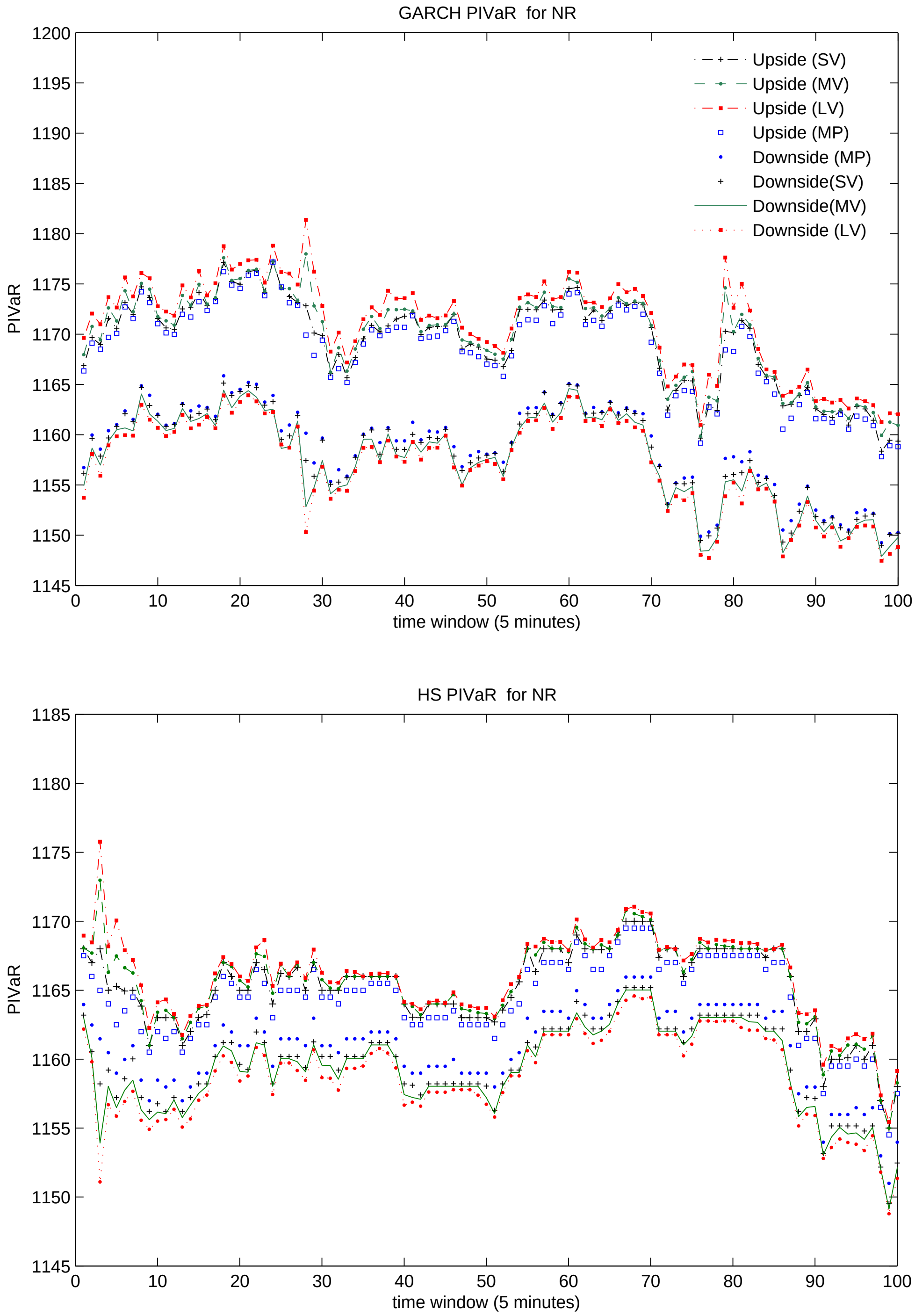
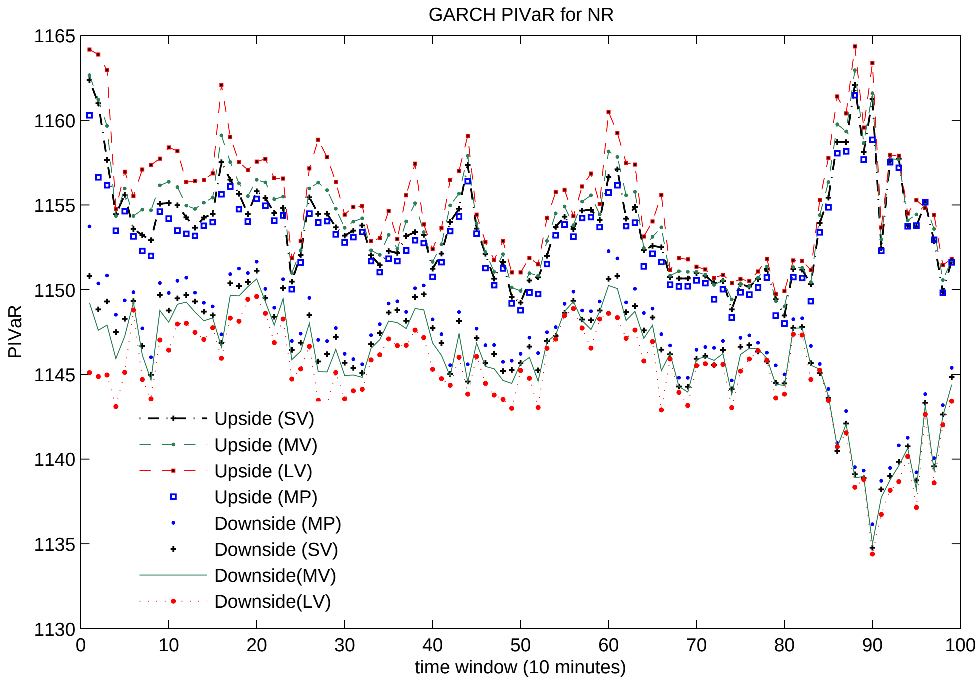


Figure 3.6: Sample subset of GARCH PIVaR and HS PIVaR (α=5%) for the three companies with 5 minutes sampling frequency. In the figures, upside denotes upside risk and downside denotes downside risk; SV=small volume; MV=medium volume; LV=large volume and MP= mid-price.



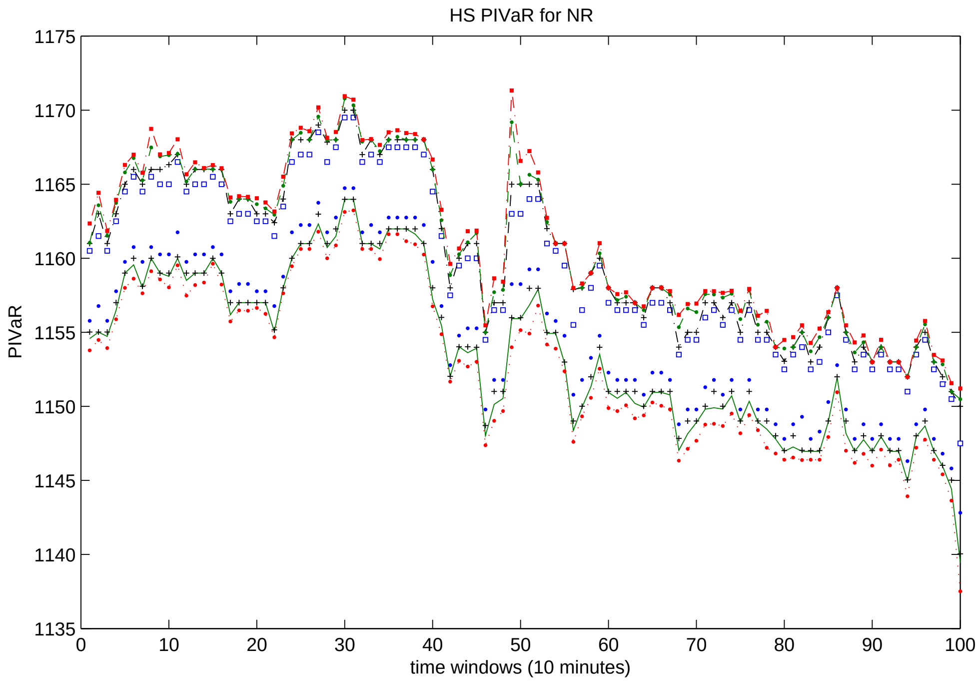


Figure 3.7: Sample subset with PIVaR (α=5%) for the three companies with 10 minutes sampling frequency. In the figures, upside denotes upside risk and downside denotes downside risk; SV=small volume; MV=medium volume; LV=large volume and MP= mid-price.

risk adjustment λ of large and small trading volumes for both the upside and downside risk. Liquidity risk is higher when the volume size is bigger for both upside and downside risk which is exactly the same as the theory predicts. For a larger volume size there are more big jumps in the risk adjustment which can affect traders who plan to execute large volumes within a short time period. The risk adjustments with the same trading volume but different trading positions exhibit different behaviours. This means traders face different risk with long or short trading positions. These considerations are especially relevant for individual investors who may apply active trading strategies within a short time horizon. A full set of figures of risk adjustments for other stocks are included in Appendix B.

The BDSS model based on the bid-ask spread only considers the price impact. For improvement, we propose using the LAIVaR model to adjust the conventional VaR by incorporating simultaneously the exogenous liquidity risk and the endogenous liquidity risk. In order to see whether the conventional VaR methods heavily underestimate the risk, we propose to compute the average liquidity risk adjustment: $\bar{\lambda} = \sum_{t=1}^{T} \lambda_t / T$. The results for three stocks with two different frequencies (5 minutes and 10 minutes) are presented in Table 3.3. The liquidity risk adjustment indicates a significant impact on the entire risk profile, especially in the case of large volume size. In general, we can observed the similar results of $\bar{\lambda}$ which are estimated by GARCH and HS. The liquidity risk are higher for a larger size of threshold volume, since $\bar{\lambda}$ is bigger. In other words, volume is positively related to the intraday liquidity risk. The results also indicate that the liquidity risk should not be ignored,

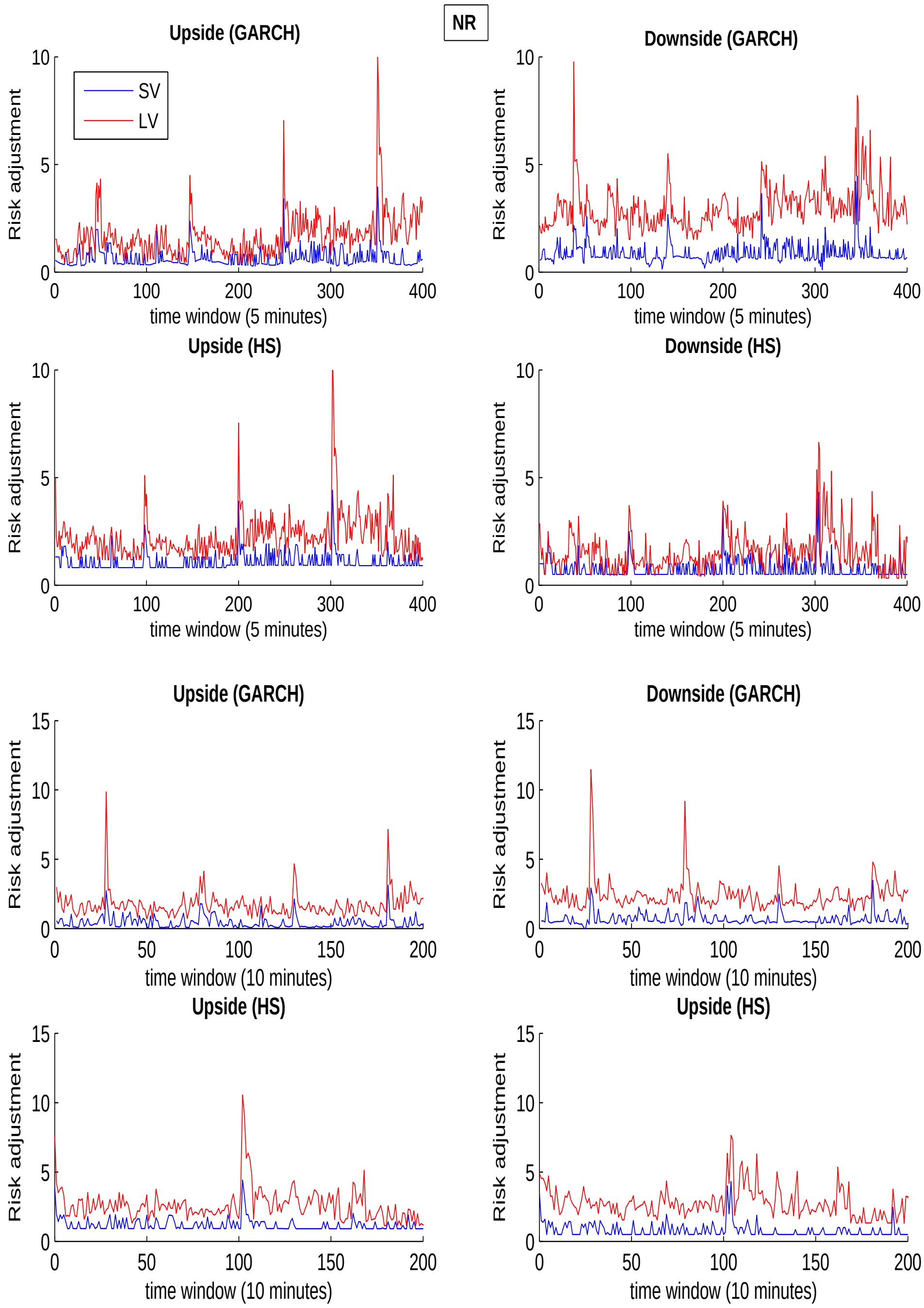


Figure 3.8: Risk adjustment for NR with 5 and 10 minutes sampling frequency. The blue line displays risk adjustment for small volume and the red line is for large volume (SV=small volume; LV=large volume). The four subplots on the left are for upside risk and the other four on the right are for downside risk.

even for the smallest $\bar{\lambda} = 0.2229$.[9]

For the sake of comparison of the risk adjustments with different relative price P, the mean of percentage liquidity risk adjustment is proposed as: $\bar{\lambda}(\%) = \sum_{t=1}^{T} \frac{\lambda_t}{P_t}/T$. And the values of $\bar{\lambda}(\%)$ are provided as figures in bracket in Table 3.3.

Furthermore, in contrast to Giot and Gramming (2006), who investigate the downside liquidity risk adjustment, we are interested in the asymmetric effect of liquidity risk on both the ask and bid side, because the asymmetric information of two sides and also the ask side risk is important, especially for investors who are in the short position. For example in Table 3.3, the average liquidity risk adjustment of different volumes for the NR and RBS stocks are larger on the bid side in both the five minute and ten minute cases. This implies that the downside liquidity risk is bigger than for the upside. For HSBC, the liquidity risk adjustment is roughly the same on both sides. The results also show that liquidity risk adjustment computed by GARCH and HS for all stocks are similar.

In additional, we also provide the backtesting results in Table B.2 of Appendix B. The failure rates of conventional VaR (computed by mid-price) are compared with our two LAIVaR measures (HS and GARCH). The backtesting results give further evidence of that how large is the conventional VaR measure underestimated the market risk, especially for large volume execution.

[9] The liquidity impact of the smallest $\bar{\lambda}$ equals to $\bar{\lambda} * V = 0.2229 * 50000 = 11145$ for HSBC with ten minute frequency.

Table 3.3: The Average Liquidity Adjustment ($\bar{\lambda}$)

Data sample	5 minutes					
volume	SV		MV		LV	
NR	GARCH	HS	GARCH	HS	GARCH	HS
Ask	0.3370	0.5749	1.0938	1.1711	1.6755	1.6778
(%)	(0.0005)	(0.0006)	(0.0010)	(0.0010)	(0.0014)	(0.0015)
Bid	0.8781	0.9702	1.3210	1.4625	2.9364	2.2125
(%)	(0.0008)	(0.0008)	(0.0012)	(0.0013)	(0.0025)	(0.0019)
RBS	GARCH	HS	GARCH	HS	GARCH	HS
Ask	1.3305	0.9309	3.9213	2.3007	9.4878	5.4600
(%)	(0.0007)	(0.0005)	(0.0029)	(0.0018)	(0.0048)	(0.0027)
Bid	1.9241	1.2605	6.7812	4.5538	11.9636	7.9802
(%)	(0.00011)	(0.0006)	(0.0032)	(0.0024)	(0.0054)	(0.0040)
HSBC	GARCH	HS	GARCH	HS	GARCH	HS
Ask	0.9624	0.3234	1.0362	0.4708	1.4567	0.6970
(%)	(0.0010)	(0.0004)	(0.0013)	(0.0005)	(0.0017)	(0.0008)
Bid	0.7133	0.4708	0.07982	0.5038	1.4827	0.8956
(%)	(0.0008)	(0.0005)	(0.0009)	(0.0006)	(0.0017)	(0.0010)

Data sample	10 minutes					
volume	SV		MV		LV	
NR	GARCH	HS	GARCH	HS	GARCH	HS
Ask	0.4063	0.7481	1.2894	1.1715	1.8695	1.6648
(%)	(0.0004)	(0.0007)	(0.0011)	(0.0010)	(0.0016)	0.0015
Bid	0.6982	0.8846	1.1995	1.3158	2.4876	2.3458
(%)	(0.0007)	(0.0008)	(0.0010)	(0.0012)	(0.0021)	(0.0021)
RBS	GARCH	HS	GARCH	HS	GARCH	HS
Ask	0.8963	0.9431	1.6382	1.0192	7.0120	5.3423
(%)	(0.0006)	(0.0005)	(0.0009)	(0.0007)	(0.0048)	(0.0027)
Bid	1.1095	1.4929	3.7678	3.8439	9.5969	7.8016
(%)	(0.0008)	(0.0007)	(0.0019)	(0.0019)	(0.0048)	(0.0039)
HSBC	GARCH	HS	GARCH	HS	GARCH	HS
Ask	0.2229	0.3360	0.4780	0.4610	0.6150	0.6965
(%)	(0.0003)	(0.0004)	(0.0005)	(0.0005)	(0.0007)	(0.0007)
Bid	0.3769	0.3401	0.5975	0.4896	0.6471	0.7262
(%)	(0.0004)	(0.0004)	(0.0007)	(0.0005)	(0.0007)	(0.0008)

The Average Liquidity Adjustment ($\bar{\lambda}$) with corresponding percentage liquidity risk adjustment (in the bracket) are computed by two models (GARCH and HS). Three representative threshold volume sizes are small volume (SV), medium volume (MV) and large volume (LV).

In general, by examining the liquidity risk adjustment, one can reveal the liquidity risk component when measuring the VaR model. An increase in trade size produces a positive impact on liquidity risk adjustment. An investor, especially one who has to execute a large volume of assets, must take into account the effect of liquidity in order to trade more rationally.

3.4.4 The Study of Dynamic Correlation

The empirical study of the dynamic correlation behind bid and ask side can help to better understand the bid-ask spread structure and liquidity risk. In the dynamic correlation model the correlation parameter varies over time. The DCC model is normally used to calculate the volatility of a portfolio, but we apply it into capture the dynamic correlation of volatility between bid and ask side in this chapter. According to the DCC parameters, we can get the correlations between two GARCH(1,1) volatility processes. Figures 3.9 show the dynamic conditional correlation and the conditional variance for bid and ask positions of NR with five and ten minute sampling frequencies . The dynamic correlation between bid and ask volatility is linked to each other. For each asset, there are results for two different volumes and two different sampling frequencies. The correlation is more fluctuant for five minutes volatility. In the five minutes case, for example, the correlation of volatility ranges from -0.7 to 1 for the small volume size of sample of NR. The correlation results for the other two stocks are shown in Appendix B.

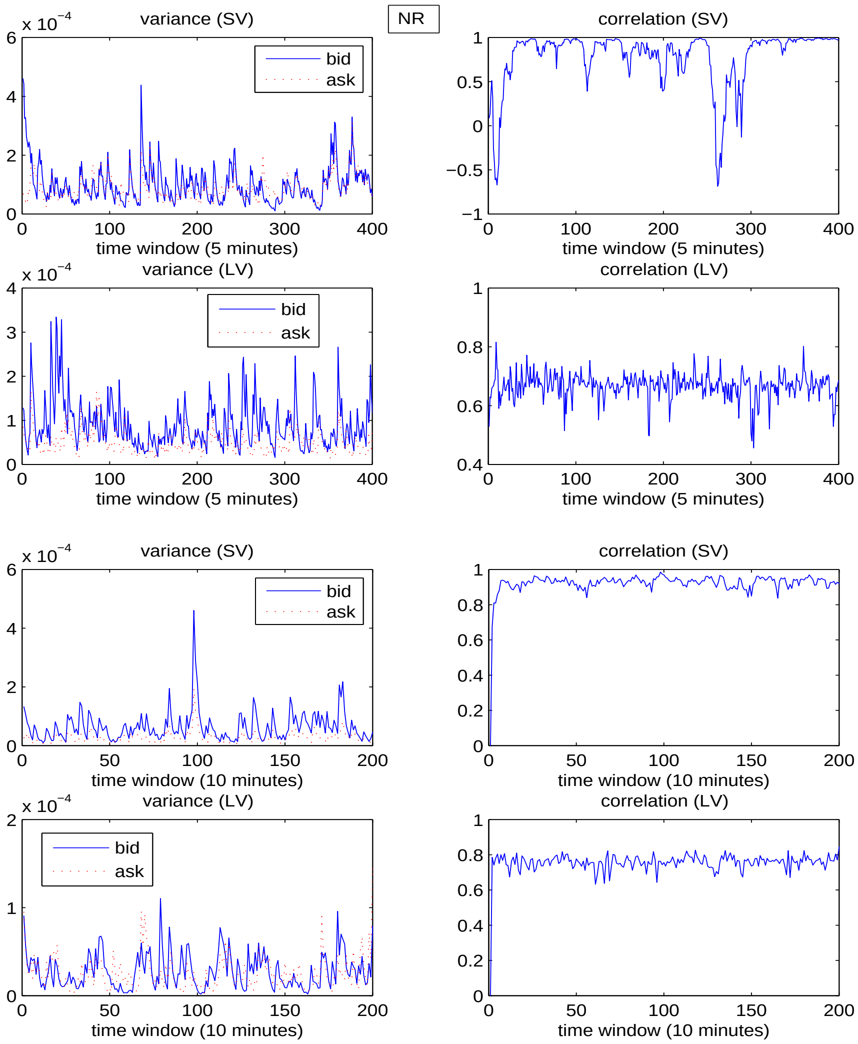


Figure 3.9: Variance and correlation for NR with different volume sizes (SV=small trading volume, LV=large trading volume)

3.5 Conclusion

This chapter extends the conventional VaR measurement methodology by introducing the notion of liquidity adjusted intraday VaR (LAIVaR) which incorporates the liquidity risk of asset trading and the trading positions of the market participators. The limited order book data is used to quantify the liquidity risk and the asymmetric risk effect for short time intervals.

In this chapter, we proposed a new practical empirical technique which can help the algorithmic trader to quantify their risk depending on their market position. This approach extends the famous BDSS model (Bangia, Diebold, Schuermann, and Stroughair (1999)) by integrating the endogenous liquidity risk effect instead of the ask-bid spread. We establish the liquidity risk adjustment to quantify the liquidity risk between different volume sizes which provides a specified structure of liquidity risk. Compared with Giot and Gramming (2006), we use both bid and ask side real-return processes which can reflect the real market information to measure. Furthermore, we emphasise the role of potential price impact and position impact on the value incurred by the liquidation.

The model presented in this chapter focuses on three main aspects. Firstly, we estimate the LAIVaR more precisely by taking into account the price impact of hidden quantities in limited order book data. Secondly, the liquidity risk is quantified by liquidity risk adjustment λ. The empirical results show that an increase in trade size produces a positive impact on liquidity risk adjustment λ. This indicates that the endogenous liquidity risk is a crucial factor in estimating VaR. Negligence of liquidity costs as in a conventional VaR model will lead to an underestimation of risk. Thirdly, we further study the patterns of LAPIVaR and liquidity risk adjustment between the bid and ask side of an order driven stock market. The asymmetric behaviours are observed and highlighted in our analysis. The method of using VWAP data of the bid and ask side for deriving the LAIVaR gives some understanding of intraday liquidity risks in the stock market.

Finally, our model is flexible and practical so that the investors can quantify the LAIVaR by setting the parameters of volume for a long or short position according to their unique need. Therefore, the modelling of the LAIVaR could be valuable for short-term traders in setting trading limits, hedging decisions and overall risk evaluation. However, this model heavily relies on the data based on limit order books.

Chapter 4

Forecasting Market Risk in the High Frequency FX Market with Scaling Laws

Chapters 2 and 3 investigated the issue of risk measurement on a single time scale method basis. In this chapter, a new multiple time scale based empirical framework for the measurement of financial risk is proposed. Ultra-high frequency data is used in the empirical analysis to estimate the parameters of empirical scaling laws which gives a better understanding of the dynamic nature of the FX market. We introduce three new scaling laws based on the original mean maximal price change (MPC) scaling law by Glattfelder, Dupuisy, and Olsen (2011) in Section 4.3. Two experiments are run to check for robustness for the results from the MPC scaling law method. Moreover, a comparison of risk forecasting performance between the traditional risk measurements and scaling law methods during the crisis period of 2008 are also presented.

The advantage of the new scaling law method in this chapter is its robustness, flexibility and the multiple time scales incorporating all information available in the market. The traditional method to measure daily VaR by extracting only one asset price per day has the disadvantages that (a) it needs an adequately long period data set to collect enough in-sample data points and (b) it neglects the dynamic nature of financial markets during the day. Instead, real-time ultra-high frequency data is used to estimate the parameters of the maximal price change (MPC) scaling law that take full information of the market into account, not just one arbitrarily chosen number per trading day.[1] It is a challenging computation task to analyse this huge amount of data. The proposed scaling laws would enable both traders and investors with different investment horizons to better understand and predict market price movements.

The main finding is that the new scaling law method is more accurate and flexible compared to traditional VaR method. Among the scaling law methods, the new exponential moving average maximal price change (EMAMPC) scaling law performs the best for all five currencies. Furthermore, the forecasting errors are smaller when shorter in-sample data is used. The scaling law methods with one month data provide good forecasting on the maximum loss within 10 days.

[1] In this chapter, we will analyse all the tick data with different time scales, instead of equally spaced sampling high frequency data.

The outline of this chapter is as follows. Section 4.1 motivates the application of scaling law method in market risk measuring and forecasting. Section 4.2 provides a literature review of power law applications in economics and finance. Section 4.3 describes the scaling law methodology and Section 4.4 presents the data and the empirical results. The last section discusses the main results.

4.1 Introduction

On 19th October, 1987, the S&P 500 index fell about twice as much as any day recorded throughout all of its history. About \$500 billion were lost in one day while major stock markets around the world crashed as well. "*The crash of October 1987 and its Black Monday on October 19 remains one of the most striking drops ever seen in stock markets, both by its overwhelming amplitude and its encompassing sweep over most markets worldwide*" (Sornette, Malevergne, and Muzy (2003)). In 1989, the price bubble of Japanese stock started to deflate and three years later, the Nikkei index dropped to 17,000 from 39,000, leading to an financial crisis in Japan. In December 1994, Orange County declared bankruptcy by announcing that the investment pool of interest rates had suffered losses around \$1.6 billion. This was the largest financial failure ever recorded by a local government in US history. For banks and financial institutions, risk management plays a very important role, because if there is an error in estimating the risk level, then it may affect a bank's whole investment strategy.

In 2008, economic storms were rife all over the world. Merrill Lynch and AIG which were supposed to be "too big to fail" finally went wrong under the heavy weight of leverage, liquidity problems and poor risk management. Since the 1960s the flexible exchange rate has led to more volatility in the FX market. The reality is challenging the whole financial theoretic system because of the increasing volatility. To manage the risks, a robust risk measurement model is needed to prevent the major crises and send advanced warning signals.

There are several existing standard risk measurement models discussed in the literature. Value-at-Risk (VaR) is one of the most widely used tools for measuring the market risk of financial assets. According to the Basel Committee, banks have to report the 10-day VaR of portfolios. The common way to forecast the 10-day VaR is to use the square-root-of-time method which scales the short-term horizon VaR to a longer-term horizon VaR, implicitly assuming that the quantile of the distribution scales like the variance (Fama (1965)). However, this method is doubted by many researchers as it is not valid when the underlying asset return distribution is not Gaussian. The stylised facts of empirical financial asset returns violating the normality assumption have been documented by many authors (see, e.g., Cont (2001)). In particular, Danielsson and Zigrand (2004) analyse the behaviour of the square root rule when the underlying stochastic process has a jump diffusion component, with the conclusion that the resulting VaR measure underestimates the risk (see also Menkens (2008)).

The key motivation of this chapter is to suggest a more flexible and robust approach to forecast the market risk, based on the power law framework that recently has gained much attention for its flexibility and scale free mathematical properties. The traditional methodology is to build a model first based on some assumptions, then to draw the conjecture. The scaling law methods in this chapter analyse all tick-by-tick market data, then develop models to explain the patten based on the observed events and the statistical properties of the data. Modelling financial time series using the scaling law method can overcome the limits of return distribution assumptions. For example, the stylised facts of financial returns have been first documented to violate the log-normality assumption (Fama (1965)). Also, fat-tails and asymmetry are commonly found in financial return distributions. Glattfelder, Dupuisy, and Olsen (2011), recently found statistical relationships of different features of tick data in the FX market, namely scaling laws. Among the 12 different scaling laws, we link the maximal price change scaling laws to predict the market risk. We further extend their method and propose innovative scaling law models for risk management. In the rest of this chapter, we will present three new scaling laws based on the positive or negative maximal price change within a given period of time. The new scaling laws can give a better forecast of the market risk (10 day losses), even in crisis periods.

4.2 Literature Review

4.2.1 Concept of Power Laws

Power laws are found in many fields, such as physics, computer science, bionomics, population statistics, economics and finance. Over a half century, power law theory has gained much attention due to its special mathematical properties, which sometimes brings on marvelous consequences.

Power laws are scaling functions[2], which propose a special mathematic relationship between two scalar quantities. Mathematically, a power law function can be written as:

$$f(x) = cx^k$$

with $c > 0$ and $k \in \mathbb{R}$,

where k is a scaling parameter or power law index. On logarithmic scales, power laws are straight lines, and the relationships become linear,

$$lnf(x) = lnc + klnx$$

with $c > 0$ and $k \in \mathbb{R}$,

where k is the slope of the straight line.

A so-called power-law distribution implies that small occurrences are ex-

[2]Power laws have the same meaning as scaling laws. We use different names in this chapter with respect to the original authors and works.

tremely common, whereas large instances are extremely rare. " *Power law distributions are scale invariant in the sense that the relative probability to observe an event of a given size and an event ten times larger is independent of the reference scale* " (Bauhaus (2001), page 2).

The most important property of power laws is scale invariance which means a power law does not change for any scale. Several famous power laws observed in many science areas give us a preliminary understanding of scale invariance (see details in Mitzenmacher (2003)). The most fundamental and famous two power law distributions are the Pareto distribution Pareto (1896) and the Zipf distribution (Zipf (1949)).

In the nineteenth century, Pareto (1896) discovered the "80-20 rule" in income distribution which means 80% of the wealth is owned by 20% of the population as a whole. He created a mathematical formula of a power law distribution as $P(W) \cong \frac{(W_0^{\mu})}{(W^{(1+\mu)})}$, where $P(W)$ is the individual wealth distribution density; W_0 is the minimum possible value of W and μ represent the large W's decay factor of distribution: a smaller value of μ denotes a slower decay and a larger poverty gap.

The well known Zipf's law named after the Harvard linguistic professor George Kingsley Zipf (1949) for the probability of occurrence of words or income and other items. The general form of the Zipf law is

$$P(r) = Kr^{-q} \quad with \quad P(r) > P(r+1), \tag{4.1}$$

where $P(r)$ is probability of an event (such as: the frequency of word will occur in English language) which is related to its's rank; K is a scaling constant; r is its rank in descending order and q is some parameter of the distribution. Shiode and Batty (2000) gives a normal way to fit Zipf's distributions to data which is to perform a linear regression of $log(P(r))$ on $log(r)$ where the parameters $log(K)$ and q are the intercept and slope of the relationship

$$log(P(r)) = log(K) - qlog(r), \tag{4.2}$$

respectively.

Vandewalle, Briscois, and Lefebvre (2000) apply a new Zipf-like method to find patterns and correlations in the world trading systems. They emphasise that the n-Zipf analysis is very useful for abnormal correlations in financial data or cross-correlations between different markets.

Newman (2005) gives plenty of examples of different power laws, such as the distributions of earthquakes, the sizes of cities, solar flares, people's investments and people's personal fortunes all appear to follow power laws. Power laws as an important modelling clue are also useful in the financial data analysis. Gabaix (2009) presents a detailed survey on empirical power laws in economics and finance with many empirical examples.

The discussion surrounding the concept of power laws, thus far has both defined power laws and subsequently identified two valuable laws. However has only done so from a theoretical viewpoint. Therefore, the next section will now introduce and discuss the real-world applications of power laws in relation to financial risk management.

4.2.2 Power Law Applications in Financial Risk Management

Academic research on power law scaling in financial time series is fairly recent. Müller, Dacorogna, Olsen, Pictet, and Morgenegg (1990) find an empirical scaling law of the mean absolute changes of logarithmic prices against the time interval. Bauhaus (2001) claims that the effort of research in power law in financial areas is insufficient. Furthermore, he presents several models of power-law distributions and power-law correlations in a financial time series. His active study gives many motivations and ideas to people who intend to dedicate themselves to this area.

The power law concept has also been applied to risk management and volatility modelling. Generally speaking, there are two method categories which are volatility scaling and scaling of quantiles. Barndorff-Nielsen (1998) analyses the empirical scaling law on volatility at different time scales. He defines the standard deviation in high frequency data as the average of absolute logarithmic price changes which is the same as Müller, Dacorogna, Olsen, Pictet, and Morgenegg (1990). The authors also find a power law relationship between volatility and time interval. Alentorn and Markose (2008) find two empirical scaling laws for Economic VaR (E-VaR) and implied volatility. By using the scaling law, they can remove the maturity dependence to compute E-VaR values at any time horizon.

The regulators commonly use the square-root-of-time (SQRT) rule to get 10-day VaR. The idea of SQRT is using the daily VaR to scale up to get d-day

VaR (like 10-day VaR) and the formula can be written as (see Danielsson and Zigrand (2004)):

$$VaR_a P^d = \sqrt{d} VaR_a(P^1).$$

However, this method is not valid when the underlying asset return is non-normal distribution. Danielsson and Zigrand (2004) analyse the behaviour of the SQRT rule when the underlying stochastic process is a jump diffusion process. They point out that the VaR estimated by using the SQRT rule tend to underestimate the market risk. Menkens (2008) applies the Hurst coefficient to extend the common method to compute d days VaR from one day VaR for several quantiles. Menkens transforms the formula as

$$VaR_a P^d = d^H VaR_a(P^1)$$

where H is the Hurst coefficient. From his results, we can calculate how much the risk will be over-or-underestimated.

In risk management areas, most of the literature related to power law applications deal with the issue of converting 1-day volatility (or VaR) to d-days volatility (or VaR). However, those works are all done within single time scale analysis. In this chapter, we contribute new knowledge to the current body of literature. The new scaling law method proposed is based on multiple time scale analysis and can be a promising way to gain new insights into the risk measurement. The details of the scaling law methodology will be introduced in the next section.

4.3 Scaling (or Power) Law Methodology

The most common sampling method in traditional time series analysis is only to record the last observation at equally spaced time intervals. The equal time sampling method ignores excessive dynamic price movements during the sampling period if the window chosen is too wide. In contrast, the empirical scaling law method applied in this chapter is based on a multiple time scale analysis. The advantage of multiple time scale study is that all data points observed in different sampling frequencies are accounted for analysing purposes.

Glattfelder, Dupuisy, and Olsen (2011) discover 12 new scaling law relationships, which capture different stylise facts in FX data. Among the 12 scaling law relationships in Glattfelder, Dupuisy, and Olsen (2011), the maximal price change (MPC) scaling law is the one we might expect to contribute to risk management. In this section, we will introduce the MPC scaling law first, and then present three new scaling laws. The core message of this study is that the scaling laws can be used to capture the volatility of the fundamental asset and its dynamic nature across time.

4.3.1 The MPC Scaling Law

Consider an electronic trading platform recording all market activities in real-time and let X denote the mid-price of the quote with $X_t = (bid_t + ask_t)/2$. Define the maximal price change (MPC) as the difference between the highest and the lowest observed price X within a certain time interval Δt, i.e.

$$\Delta_X^{max}(\Delta t) = \max(X(t-\Delta t,t)) - \min(X(t-\Delta t,t)) \quad . \qquad (4.3)$$

where $\Delta_X^{max}(\Delta t)$ denotes the MPC ; $\max(X(t-\Delta t,t))$ and $\min(X(t-\Delta t,t))$ are the highest and lowest observed price during a time horizon.

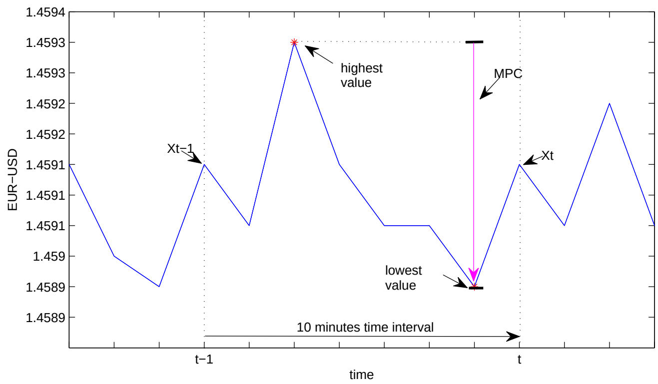


Figure 4.1: An illustration for calculation of the maximal price change (MPC) in a 10 minute interval. In the figure, the MPC is the difference between the highest value and the lowest value of the FX within 10 minute time intervals. The 10 minutes price difference with equally spaced time is calculated by $X_t - X_{t-1}$.

Figure 4.1 illustrates that the price change event is captured by a total price move between the highest and lowest price in a 10 minute time interval. The common price change $\Delta X(\Delta t)$ with $\Delta t = 10$ (minute), is computed by conventional equal-time sampling method $\Delta X(\Delta t) = X_t - X_{t-1}$, which is very close to 0 in this case. However, the magnitude of MPC in 10 minutes is: $\Delta_X^{max}(\Delta t)$= 0.04%. This indicates that the conventional method ignores the dynamic change of events during the time interval.

We apply the original MPC scaling law (L) by Glattfelder, Dupuisy, and Olsen (2011) that describes

$$L : \langle \Delta_X^{max}(\Delta t) \rangle_p = c(\Delta t)^k \tag{4.4}$$

where $\Delta_X^{max}(\Delta t)$ denotes average maximal price change within a certain time interval Δt; c is a constant and k is called the scaling exponent. $\langle x \rangle_p$ denotes the average operator which is defined as: $\langle x \rangle_p = (1/n \sum_{j=1}^{n} x_j^p)^{1/p}$ where n the number of the observations and $p = 1, 2$, which is the arithmetic mean when $p = 1$ and the standard variance when $p = 2$.

This scaling law L investigates the relationship between the average maximal price movement within a time interval (which is a random variable) and the size of that time interval (which can be pre-specified). If the scaling-law L exists, the quantity Δt should satisfy

$$\langle \Delta_X^{max}(\Delta t) \rangle_p \propto \Delta t^k \tag{4.5}$$

which means that $\langle \Delta_X^{max}(\Delta t) \rangle_p$ is directly proportional to Δt^k (see, e.g., Clauset, Shalizi, and Newman (2008)). If we simply take the logarithm of both sides of equation (4.4), then the power law relationship changes to

$$L^* : log(\langle \Delta_X^{max}(\Delta t) \rangle_p) = \log(c) + k \cdot \log(\Delta t) \quad . \tag{4.6}$$

The transformed scaling-law L^* now describes a linear relationship controlled by the slope k and the intercept $log(c)$. We chose many different threshold time intervals $\Delta t = \{0.5, 1, 2, 4, 8, 16, 32, 63, 128, 256, 512\}$ (hours) to obtain the respective average MPC for the individual sampling window, $E(\Delta_X^{max}(\Delta t))$. Then, we used simple linear OLS regression to estimate the MPC scaling law parameters in equation 4.6 (see also Arnold (1983)). Figure 4.2 shows an example of the obtained regression line for the FX pair EUR-USD using data from 2007.

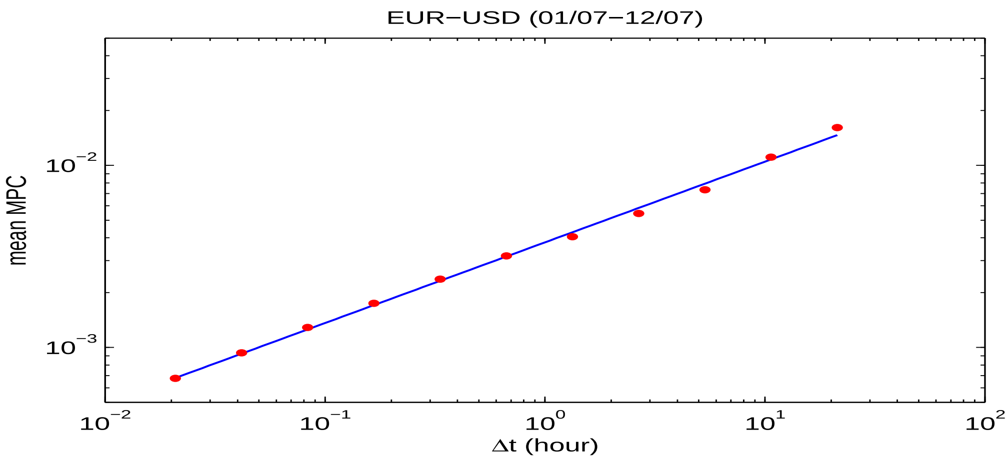


Figure 4.2: An example of the estimated scaling law regression line L^* for the FX pair EUR-USD using data observed in 2007
Note: The x-axis shows the time interval Δt and the y-axis is the average MPC for the chosen time interval.

In practice, comparing dips and peaks are important information for traders, which is hard to visualize in real time. The biggest challenge is that you never know when the dips and peaks started and ended. To solve this problem, the MPC in L catches every peaks and low point due to the properties of multi-timescale analysis. Furthermore, we also can calculate how large of the price depth is based on the relationship from L. In the next section, we will propose three new extended scaling laws which can be used to capture the volatility of the fundamental asset and its dynamic nature across time.

4.3.2 New Extended Scaling Laws

Extending the maximal price change event paradigm by Glattfelder, Dupuisy, and Olsen (2011), we further observe new, stable patterns of scaling laws by replacing $E(\Delta_X^{max}(\Delta t))$ in equation 4.4 with three new modified measures, which are the positive (or negative) MPC scaling law ($L1$), the exponential moving average MPC scaling law ($L2$) and the expected tail loss MPC scaling law ($L3$).

The Positive or Negative MPC Scaling Law

In the first model extension $L1$, we differentiate between the positive and negative maximal price change (PMPC and NMPC) within a certain time horizon. The aim is to separate the different dynamic volatility components generating a profit or a loss:

$$L1 : E(\Delta_X^{max\pm}(\Delta t)) = c(\Delta t)^k \tag{4.7}$$

where

$$\Delta_X^{max+}(\Delta t) = \max(X_s - X_r) \tag{4.8}$$

$$\Delta_X^{max-}(\Delta t) = \min(X_s - X_r) \tag{4.9}$$

X_s (or X_t) is the observed price at time s (or t) and $\forall s, r \in \{t - \Delta t, t\}$ with $s > r$.

Figure 4.3 illustrates how to obtain the positive and the negative MPC during a 10 minute time interval. In the previous example (see Figure4.1), the MPC $\Delta_X^{max}(\Delta t)$= 0.04% and $\Delta X(\Delta t) \cong 0.00\%$. For the new scaling law L1, we observed two extremal price change events (with $\Delta t = 10$ (minute)): $\Delta_X^{max+}(\Delta t) = 0.02\%$(PMPC) and $\Delta_X^{max-}(\Delta t) = 0.04\%$ (NMPC). The new scaling law ($L1$) also reflects the the asymmetric price impact between gain and loss. As shown in Figure 4.3, the new law L1 captures two observed extreme events: Positive MPC and negative MPC. Within the the selected 10 minutes, the magnitude of price change is larger in loss than gain.

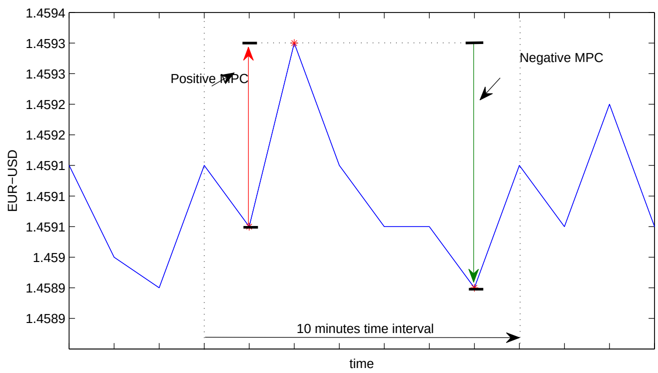


Figure 4.3: An illustration for calculation of the positive and negative maximal price change in a 10 minute interval.

The Exponential Moving Average MPC Scaling Law

In the second new scaling law ($L2$), the exponential moving average maximal price change (EMAMPC) is used instead of the MPC to examine the exponential decay impact in consecutive time intervals. The idea behind the EMA method is to treat recent data as more relevant and more important compared to the historical data. Similar to the other models, the entire in-sample time span T is again divided into n equally-spaced sub-intervals. However, the EMAMPC approach employs the exponentially decreasing weights which give the least weight to the oldest MPC and increase weights for more recent ones, i.e.

$$EMAMPC(\Delta t) = \sum_{i=1}^{n} w_i(\Delta_{X_{n-i}}^{max\pm}(\Delta t)) \tag{4.10}$$

where w_i is the weight factors of PMPC (or NMPC) which decrease exponentially as: $w_i = \frac{\delta \times (1-\delta)^{(i-1)}}{\sum_{i=1}^{n} \delta \times (1-\delta)^{(i-1)}}$ with property $\Sigma_{i=1}^{n} w_i = 1$ and $\delta = \frac{2}{n+1}$. This yields

$$L2 : EMAMPC(\Delta t) = c(\Delta t)^k \quad . \tag{4.11}$$

Compared with $L1$, the EMAMPC scaling law $L2$ applies weighting factors which decrease exponentially and moving forward. In other words, we treat the recent MPC events more valuable. In investing, traders are alwasys waiting for the signal to buy or sell. The EMAMPC scaling law places more emphasis on recent events which can give the trader more clear signals faster.

Expected Tail Loss MPC Saling Law

The demand for extreme risk analysis is growing in the wake of financial crisis. The last new scaling law ($L3$) focuses on the extreme tail loss distribution of $E(\Delta_X^{max}(\Delta t))$ for a given Δt (defined as Formula 4.12). The biggest problem of extreme risk analysis is inadequate data for robust quantification. The new $L3$ is based on a multiple time scale analysis and filtered the valuable information from every price change event. This method which focus on the worst losses, has sufficient data for estimation to tackle this problem.

The expected tail loss scaling law (ETLMPC) $L3$ evolves from the standard expected tail loss (ETL) definition. First, we defined the ETLMPC as the mean of the q-tail distribution of MPC and depends on both q and Δt as

$$ETLMPC(q, \Delta t) = E(\Delta_X^{max\pm}(\Delta t) | \Delta_X^{max\pm}(\Delta t) < \Delta_X^{max\pm}(\Delta t)_q)$$

where q is the probability of the quantile $\Delta_X^{max\pm}(\Delta t)_q$.

Accordingly, the ETLMPC scaling law $L3$ is depend on the size of the potential mean PMPC or NMPC (defined in equation 4.7) beyond a given quantile $\Delta_X^{max\pm}(\Delta t)_q$. Then we can derive

$$L3 : ETLMPC(q, \Delta t) = c(\Delta t)^k \tag{4.12}$$

The $L3$ measures the expected MPC in the worst q% of the cases, which is more sensitive to the shape of the tail of the MPC distribution. The study of ETLMPC scaling law has led to important research on extreme risk management. In the worst scenario cases, we can use $L3$ to measure and monitor the extreme market risk.

In this section, we introduced four scaling laws which are suggested for potential application in risk management. In the next section, we focus on the empirical application.

4.4 Empirical Analysis

The scaling law analysis in this chapter heavily relies on the price change event-driven process of empirical time series. The first main experiment in Section 4.3.2 shows the estimation of parameters for the standard MPC scaling law model (L) and the robustness check (Section 4.3.2). The second main results exhibit the risk forecasting performance of different proposed models (Section 4.3.3).

4.4.1 Data Description

The empirical experiment uses tick-by-tick data for five currency pairs which are EUR-USD (Euros - United States Dollars), AUD-EUR (Australian Dollars - Euros), SGD-USD (Singapore Dollars - United States Dollars), HKD-USD (Hong Kong Dollars - United States Dollars) and AUD-USD (Australian Dollars - United States Dollars). The data set is provided by Olsen Financial Technology and the sampling period is from 1st January, 2005 to 31st December, 2008. It covers the financial crisis in this period.

4.4.2 Estimation of MPC Scaling Law Parameters and Robustness Check

Experiments are run to check for the stability of the original MPC scaling law parameters. We assume a linear relationship between the mean of log MPC and log (Δt) as defined in equation 4.6. Estimation of the scaling laws

Table 4.1: The window sizes of the two experiments

	In sample data	Out of sample data	Rolling window size
Experiment 1	6 months	6 months	6 months
Experiment 2	1 year	6 months	6 months

is done by simple OLS regression with different in-sample window sizes.

In this section, the first step is to estimate the MPC scaling law parameters with different window sizes to forecast the MPC value. The second step is the performance test with different out-of-sample data sets. Details of two empirical experiments are shown in Table 4.1. The only difference between the two experiments is the size of the in-sample data. For example, when the out-of-sample period is from 01/07 to 06/07, the in-sample period for experiment 1 (E1) is from 07/06 to 12/06 and for experiment 2 (E2) from 01/06 to 12/06. We set the same period for the out-of-sample size as 6 months for reasons of better comparison.

Parameters Estimation

The power law distribution is fit by the given data set, with the assumption that the law parameters depend on the time horizon thresholds and the mean of the MPC value. First, 11 threshold time periods are chosen with regular steps $\Delta t = [0.5; 1; 2; 4; 8; 16; 32; 64; 128; 256; 512](hours)$. Because of the scale invariance property of scaling law, the law should hold for any chosen set of thresholds. Then we calculate the mean of MPC in equation (4.3) with different threshold periods.[3]

Table 4.2: Estimated MPC scaling law parameter values

Experiment1 (E1)			Experiment2 (E2)		
AUD-EUR					
In-sample	k	$log(c)$	In-sample	k	$log(c)$
07/06-12/06	0.4409	-5.6643	01/06-12/06	0.4428	-5.5692
01/07-06/07	0.4506	-5.6117	07/06-06/07	0.4459	-5.6375
07/07-12/07	0.4805	-4.9972	01/07-12/07	0.4698	-5.2575
01/08-06/08	0.4565	-5.2179	07/07-06/08	0.4698	-5.1011
AUD-USD					
In-sample	k	$log(c)$	In-sample	k	$log(c)$
07/06-12/06	0.4202	-5.1686	01/06-12/06	0.4346	-5.0881
01/07-06/07	0.4386	-5.0755	07/06-06/07	0.4301	-5.1204
07/07-12/07	0.4656	-4.4874	01/07-12/07	0.4558	-4.7384
01/08-06/08	0.4539	-4.5896	07/07-06/08	0.4600	-4.5371
EUR-USD					
In-sample	k	$log(c)$	In-sample	k	$log(c)$
07/06-12/06	0.4522	-5.6665	01/06-12/06	0.4451	-5.5847
01/07-06/07	0.4351	-5.4984	07/06-06/07	0.4431	-5.5783
07/07-12/07	0.4595	-4.9188	01/07-12/07	0.4506	-5.1666
01/08-06/08	0.4778	-4.3757	07/07-06/08	0.4713	-4.6098
HKD-USD					
In-sample	k	$log(c)$	In-sample	k	$log(c)$
07/06-12/06	0.4170	-9.7988	01/06-12/06	0.4469	-9.6871
01/07-06/07	0.4877	-9.4021	07/06-06/07	0.4585	-9.5776
07/07-12/07	0.4423	-9.3153	01/07-12/07	0.4623	-9.3548
01/08-06/08	0.4739	-9.6828	07/07-06/08	0.4541	-9.4812
SGD-USD					
In-sample	k	$log(c)$	In-sample	k	$log(c)$
07/06-12/06	0.4611	-6.3379	01/06-12/06	0.4662	-6.3104
01/07-06/07	0.5060	-6.0699	07/06-06/07	0.4862	-6.1939
07/07-12/07	0.4899	-5.9666	01/07-12/07	0.4968	-6.0157
01/08-06/08	0.5192	-5.8162	07/07-06/08	0.5052	-5.8878

Note: $log(c)$ denotes the slope and k denotes the intercept of the linear relationship of MPC scaling law (see equation (4.6)).

The MPC parameters are estimated with a rolling time window with a fixed size of in-sample data. Table 4.2 presents the estimated MPC scaling law

[3] We only imply the mean MPC scaling law with $p = 1$ for better comparison with other scaling laws.

parameters for five currency pairs of data which are EUR-USD, AUD-EUR, SGD-USD, HKD-USD and AUD-USD. The MPC scaling law parameters $log(c)$ and k are the estimated slope and intercept, respectively. For example, the estimated parameters $k = 0.4409$ and $log(c) = -5.6643$ are shown on the first row left-hand side of Table 4.2 and the in-sample data period covers from June 2006 to December 2006 for AUD-EUR. Then we can get a linear relationship as : $log(\langle \Delta_X^{max}(\Delta t)\rangle_1) = -5.6643 + 0.4409 * log(\triangle t)$.

Generally speaking, the exponents k for all pairs of currencies are around 0.45, which indicates the effect of a one-unit change in variable $log(\Delta t)$ on variable $log(\langle \triangle(Xmax)\rangle_{p=1}$. In other words, if time increases by a logarithm of one hour, the MPC value would increase around half a unit and the estimated parameters would change in different time periods. However, the differences are relatively small except for the parameters in 2008 (see Table 4.1). Basically, the increase in value of the parameters means a higher volatility. The main reason for this is the crash of the financial market in 2008 and the higher extreme price movements led to the increase of the MPC scaling law parameters. According to the estimated results we can distinguish the stylised facts of the normal and abnormal market. For example, we can set a benchmark of risky signals with the MPC scaling law parameters in 2008. In the next section, modified scaling laws with different time lengths of in-sample data size are used to forecast the risk in 2008.

Robustness Check for MPC scaling Law

Due to the scale invariance property of scaling law method, we can use the estimated parameters to predict the expected maximal price change within any time interval. The MPC scaling law performance test starts with several different threshold time intervals namely $\Delta t =$[3; 6; 12; 24; 48; 96; 192](hours). We choose these time thresholds that are different to those time thresholds for estimating the scaling law on purpose. Then we calculate the forecasted MPC value using the estimated MPC scaling law parameters and compare this with the real mean MPC movement. The objective of the "backtesting" is to test the forecasting ability of the MPC scaling law. We use the prediction error terms $MPC^p_{SL} - MPC^p_{ob}$ to check the forecasting performance of MPC, where MPC^p_{SL} is the estimated value and MPC^p_{ob} is the MPC observed in the out-of-sample data.

Figure 4.4 plot the estimated MPC scaling laws (in-sample) and corresponding forecasting errors (out-of-sample) for AUD-EUR, and the z-axis shows the different in-sample time horizon. For the scaling law 3D-plots (a1 and a2 in Figure 4.4), the x-axis indicates the threshold ticks; y-axis shows the mean MPC thresholds of the observations. The error terms are plotted, where the x-axis shows the selected time interval thresholds of the observations and y-axis shows the prediction errors. The bias tends to increase with the threshold time horizon. The 3D plots show the scaling relationship of in-sample log mean MPC within different threshold time intervals. The prediction errors are presented in four sub figures which express the different out-of sample period. The results for the other four currency pairs are provided in Figure C.1 to C.4 in Appendix C.

The forecasting errors for all five currency pairs are quite small and exhibit similar patten, except in the out-of-sample period of year 2008. For sub-figures b1 to b4 in Figure 4.4, the error terms are negative when the MPC scaling law of the experiment underestimated the risk, especially in the period of 07/2008 to 12/2008 (sub-figure b4). During the crash period, more big jumps of the price influence the value of mean MPC. The estimated parameters are not satisfied anymore when the whole market risk level change too much[4]. Furthermore, we also can observe that both experiment 1 and experiment 2 have similar forecasting results for the period of 2007. However, for 2008, the errors of experiment 1 (red dots) are smaller which means that the MPC scaling law computed with half year in-sample data performed better than using one year in-sample data. This lead us to shorten the in-sample period to get better results on the empirical experiment in the next subsection.

In general, we find the self-similarity of the relationship between the MPC and certain time intervals of the underlying process. In other words, the MPC scaling law reveals the self-similarity property which is an important characteristic of fractals. The scaling relationships will repeat themselves in time. By examining the MPC scaling law, one can get better forecasts of the market risk. Moreover, the flexibility of the MPC scaling law model allows users to choose any period according to their requirement to track market risk sentiment.

[4]To solve this problem, the rolling window size is chosen as 1 day instead of 6 month in the next section.

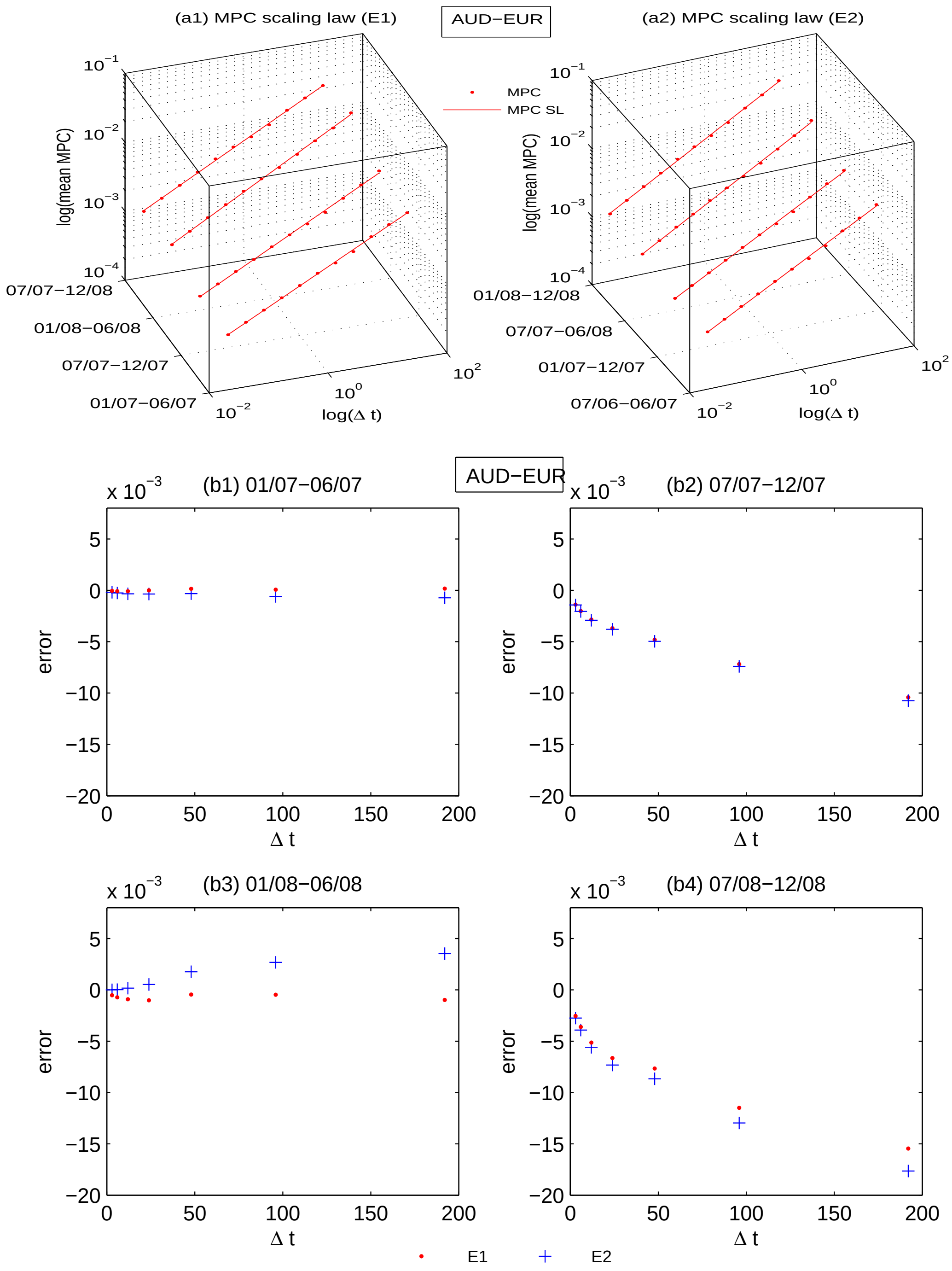


Figure 4.4: The scaling law plot and prediction error plot of out-of-sample data for AUD-EUR. Note: E1: experiment 1 (6 month in-sample data); E2: experiment 2 (1 year in-sample data).

4.4.3 Application of Scaling Laws to Financial Risk Assessment

In the following study, we compare the performance of the risk forecasts of the four proposed scaling laws $L1$-$L3$ with conventional VaR measures and orignal MPC scaling law L. The conventional VaR measures include the 5%-VaR ($V1$), the 5%-ETL, (i.e. the expected tail loss of the 5% tail loss distribution ($V2$)), the 1%-VaR ($V3$) and the 1%-ETL ($V4$). A backtesting procedure is introduced to compare the quality of different models.

Design of the Experiment

We conduct an empirical experiment to analyse the performance of proposed market risk forecasting models. The data used for estimation and forecasting are the daily prices and tick-by-tick data of five currency pairs which are EUR-USD, AUD-EUR , SGD-USD, HKD-USD and AUD-USD. Figure 4.5 plots the daily prices for the five different currency pairs from January 2007 to December 2008. Interestingly, the trend of the price curves is similar, except for HKD-USD.

According to the regulation of Basel III, banks are required to report their maximum loss over 10 trading days with a 99% quantile. We fit the scaling laws with tick data with different lengths of in-sample windows to get the parameters of scaling laws. We compute the daily VaR and ETL using the previous three years' daily data (from January 2005 to December 2007). The criterion to assess the model performance is the daily prediction error of

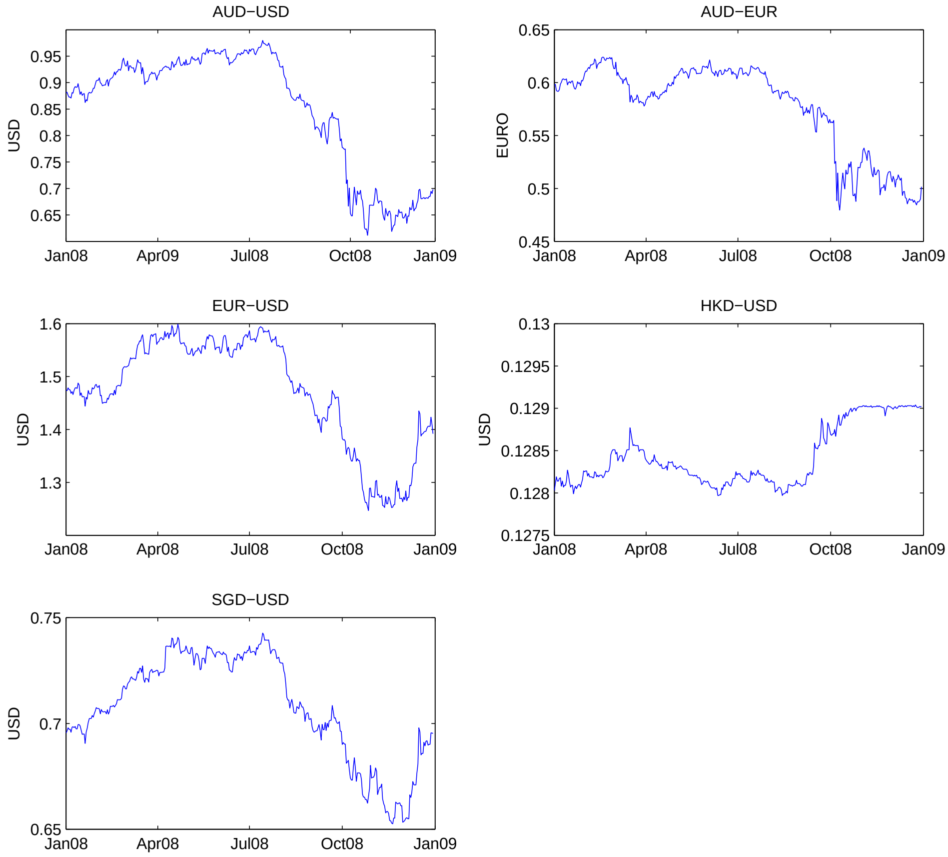


Figure 4.5: The daily price of five currency pairs in 2008.

the 10 day actual occurring maximum loss (Basel III), that is the biggest negative change of daily prices occurred within a 10 day time window. The whole out-of-sample period is from 1st January 2008 to 31st December 2008. We take every 10 days maximum loss to be the out-of-sample data from 1st January 2008 and roll forward by one day.

The first predicted maximum loss will be compared against the actual "maximum loss in the *next* 10 days", which can be observed at the earliest on 10th

January 2008, that is the first "realised" maximum loss within the (*last*) 10 days. The first prediction error is then simply defined as:

first realised maximum loss - first predicted maximum loss ,

expressed in the respective monetary currency units of the FX rates. And the detailed procedure is as follows:

First, we forecast the daily VaR and ETL (Expected Tail Loss) by using historical simulation method with daily prices. The square-root-of-time (SQRT) rule is applied to obtain the 10-days VaR and ETL, i.e. $VaR_aP^{10} = \sqrt{d}VaR_a(P^1)$. As the risk forecast for the next 10 days usually has to be provided on a daily basis, the in-sample windows keep moving forward day by day, more specifically, dropping one of the oldest data out and taking one of the newest data in.

Then we forecast 10-days' market risk with scaling law methods by using tick data. The first prediction for the "maximum loss in the *next* 10 days" is made on 1st January 2008. The obtained number is the first predicted maximum loss. The scaling law parameters are estimated with five different in-sample data windows which are 1 year, 6 months 3 months, 2 months and 1 month, and the time window moves forward with a step size of one day. In other words, the parameters are re-estimated day by day with updated in-sample tick-by-tick data sets. The scaling laws state that there is a fixed relationship between some kinds of average price changes and the time interval . With estimated parameters, it is straightforward to obtain a linear relationship between two variables of scaling law. Due to the scale invariance property of scaling law method, one can choose any length of the time variable to estimate the other variable. In this study, the time variable is set as 10-days for model comparison purposes.

Finally, the whole crash year of 2008 is selected to be the out-of-sample data period when most risk management tools were claimed failed. We compare the forecast errors for the five currency pairs as it is more important for a bank's risk manager to see whether the necessary capital requirement is actually achieved or violated. We compare the standard sample moments of both prediction errors and relative prediction errors for all forecast models. The forecast errors are the difference between the 10 day actual maximum loss in the out-of-sample and the predicted maximum loss by the models.

Forecasting Performance Results

As mentioned in the last subsection, the parameters of scaling laws are estimated by simple linear OLS regression with five different in-sample data windows and move forward by one day.[5]

Figure 4.6 shows the estimated scaling law parameters of EUR-USD with 1 month in-sample data. For comparison purpose, the plot of MPC (L) scaling law parameters (black line) in the figure are replicated in different sub-figures. Basically, the different shapes of PMPC and NMPC scaling law parameters show the asymmetric effect. We can observe that the parameters of NMPC (red line) for new proposed scaling laws tend to more volatile patterns. The slope parameters of different scaling laws are shown in sub-figures a2, b2, c2

[5] For our approach, one day rolling window means large volumes of tick-by-tick data.

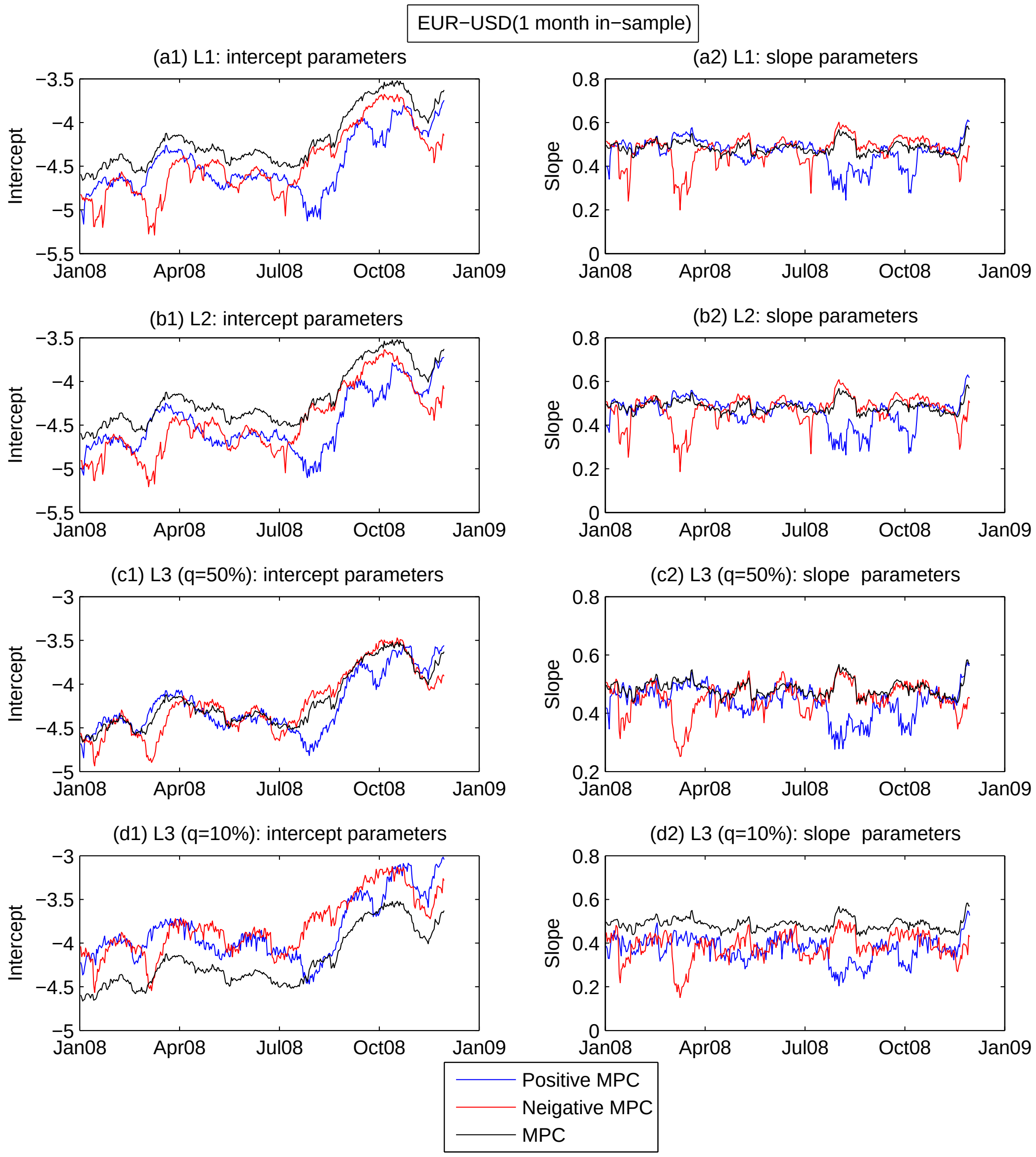


Figure 4.6: The estimated parameters of scaling laws for EUR-USD with 1 month in-sample data (Out-of-sample period is from 01/01/2008 to 31/12/2008). The black lines in the figure display the parameters from the original MPC scaling law (L). The blue and red lines display the parameters of positive MPC and negative MPC, respectively, which are used in the new scaling laws (L1-L3).

and d2. We can observe that the slope of new scaling laws are more sensitive than the original scaling law (L). In the sub-figure d1, the intercept of L is smaller than the intercept of $L3$ with q=10% in most of cases. Graphs of the other results are shown in full in Appendix B.

The performance criteria are calculated by prediction errors for all models. We compared the results of the four scaling law models with different in-sample data windows and compare them with the benchmarks $V1$-$V4$. The full table of the sample moments of the forecast errors for all five currencies are showed in Tables B1-B4 in the Appendix B. Table 4.3 shows one currency example, which describe the descriptive statistics of forecast errors (for the entire out-of-sample) for EUR-USD. Generally speaking, the forecast errors of new $L1$ and $L2$ have a smaller absolute mean than standard methods. Furthermore, the absolute mean of the forecast errors for same scaling law method decrease when in-sample period is shorten. The standard deviation changes with a smaller range. The smaller the kurtosis, the less likely we are to overestimate or underestimate the risk. Significant negative or positive skewness implies asymmetry of the error's distribution, which here corresponds to under-or-overestimation of risk. Generally, kurtosis value are smaller with the decreasing size of in-sample window.

Table 4.3: Sample moments of the forecast errors for the VaR and scaling laws models for EUR-USD.

	EUR-USD			
Standard methods	Mean	Std	Skewness	Kurtosis
V1: VaR (5%)	-0.0207	0.0227	1.2733	4.7533
V2: ETL (5%)	-0.0542	0.0080	0.4187	3.2199
V3: VaR (1%)	-0.0443	0.0235	1.2884	4.7530
V4: ETL (1%)	-0.0699	0.0099	-0.5934	4.4132
1 year	Mean	Std	Skewness	Kurtosis
L: MPC	-0.0142	0.0228	1.1147	4.4674
L1: NMPC	-0.0048	0.0227	1.1469	4.4771
L2: EMAMPC	-0.0037	0.0227	1.3613	4.9643
L3a: ETLMPC (50%)	-0.0169	0.0234	0.7347	3.8628
L3b: ETLMPC (10%)	-0.0381	0.0264	-0.1120	3.1835
6 month	Mean	Std	Skewness	Kurtosis
L: MPC	-0.0298	0.0274	-0.2766	3.2112
L1: NMPC	-0.0213	0.0284	-0.4627	3.0587
L2: EMAMPC	-0.0251	0.0308	-0.6727	2.9690
L3a: ETLMPC (50%)	-0.0399	0.0358	-0.9024	2.9502
L3b: ETLMPC (10%)	-0.0749	0.0558	-0.9183	2.4541
3 month	Mean	Std	Sskewness	Kurtosis
L: MPC	-0.0304	0.0261	0.4314	3.8657
L1: NMPC	-0.0217	0.0260	0.3041	3.6495
L2: EMAMPC	-0.0239	0.0266	0.1317	3.6062
L3a: ETLMPC (50%)	-0.0375	0.0292	-0.1707	3.4069
L3b: ETLMPC (10%)	-0.0640	0.0380	-0.7254	3.0580
2 month	Mean	Std	Skewness	Kurtosis
L: MPC	-0.0277	0.0263	0.3043	3.2957
L1: NMPC	-0.0183	0.0253	0.2118	3.3467
L2: EMAMPC	-0.0186	0.0241	0.2632	3.3783
L3a: ETLMPC (50%)	-0.0355	0.0278	-0.1263	3.1494
L3b: ETLMPC (10%)	-0.0581	0.0302	-0.7138	3.2662
1 month	Mean	Std	Skewness	Kurtosis
L: MPC	-0.0293	0.0255	0.2449	3.3726
L1:NMPC	-0.0187	0.0244	0.3385	3.3058
L2:EMAMPC	-0.0185	0.0242	0.2618	3.4404
L3a:ETLMPC (50%)	-0.0262	0.0247	0.1092	3.1189
L3b:ETLMPC (10%)	-0.0395	0.0279	-0.1792	3.008

For conventional data sampling, one month data means around 30 daily data sets for the FX market and 22 daily data sets for the stock market which is too short to analyse. We found a remarkable feature of our scaling law method was that only one month tick-by-tick data is required to achieve better forecast. To highlight this feature, Figure 4.7 summarises the four standard moments of forecast errors between the benchmarks $V1$-$V4$ (5%-VaR ($V1$), the 5%-ETL, ($V2$)), the 1%-VaR ($V3$) and the 1%-ETL ($V4$)) and the scaling law method with 1 month in-sample data. From the five graphs in the first left-hand column of Figure 4.7, we can observe that the mean of forecast error is smaller for $L2$ with one month in-sample data. For example, the absolute value of mean forecast error of EUR-USD for $V1$ to $V4$ is 0.0207, 0.0542, 0.0443, 0.0699 respectively, but the mean error for $L2$ is 0.0185. The standard deviation of forecast errors for the common VaR with quantile ($\alpha = 1\%$) is higher than other methods. Significant negative or positive skewness implies asymmetry of the error's distribution, which here corresponds to under-or-overestimation of risk. From the bar chart (Figure 4.7), one can observe that the new scaling law methods perform better even with a short data sample. The EMAMPC (L2) is found to be robust, as it produces more accurate forecasts that exhibit stable results for all five currencies.

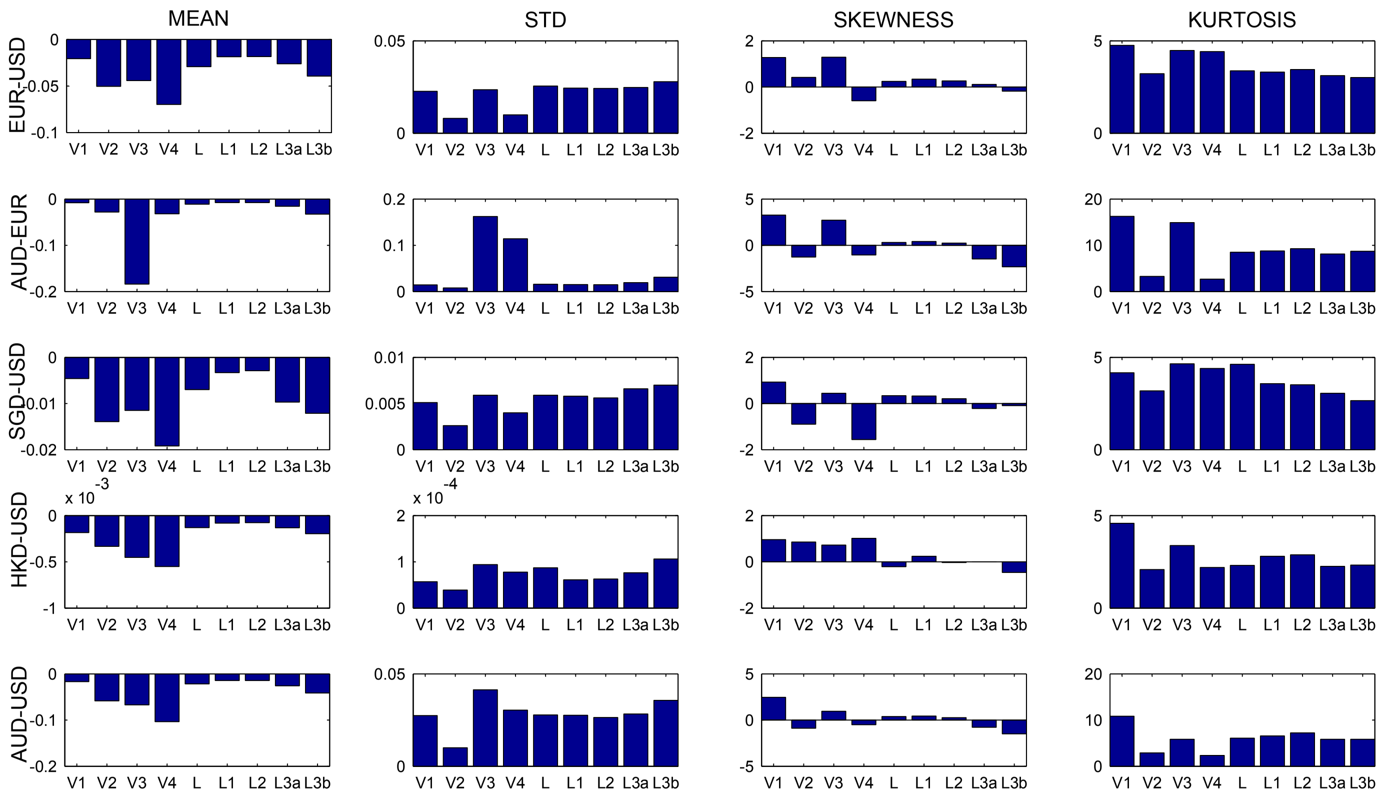


Figure 4.7: The comparison of forecast errors between conventional VaR method and the scaling law methods with 1 month in-sample data.
Note: V1(VaR($\alpha = 5\%$)); V1 (ETL ($\alpha = 5\%$)); V2 (VaR($\alpha = 1\%$)); V2 (ETL ($\alpha = 1\%$)); L (MPC); L1 (NMPC); L2 (EMAMPC); L3a (ETLMPC ($q = 0.5$));L3b(ETLMPC ($q = 0.1$)).

Our basic criterion for choosing the best performance model is minimising the forecast errors subject to the 10-days maximum loss under the Basel III Accord. Additionally, we consider the closest movement of maximum risk which is more important for investors to have a clear picture of future market risk.

Figures 4.8 to 4.12 present the graphic results of 10-days' risk forecasts using different models (VaR, ETL and scaling laws), where the vertical axis represents returns, and the horizontal axis represents the period from 1st January 2008 to 31st December 2008. In all figures, the blue line reports for a particular day the observed maximum loss within the last 10 days. The forecasting performance in the conventional VaR method is shown in the top left panels in Figures 4.8 to 4.12, which is neither stable nor acceptable. For 10-days VaR method, the backtesting numerical results for EUR-USD has violation rates of magnitude 10.90 % and 5.07 % for 5% VaR and 1% VaR, respectively, which are underestimating the market risk. For HKD-USD, the VaR method totally overestimates the risk. The other panels in the figures also show the forecasted mean MPC estimated by the MPC scaling law (L: light blue line), the NMPC scaling law (L1: green line), the EMAMPC scaling law (L2: black line) and the ETLMPC scaling law (L3: red line and purple line) with five different sizes of in-sample data.

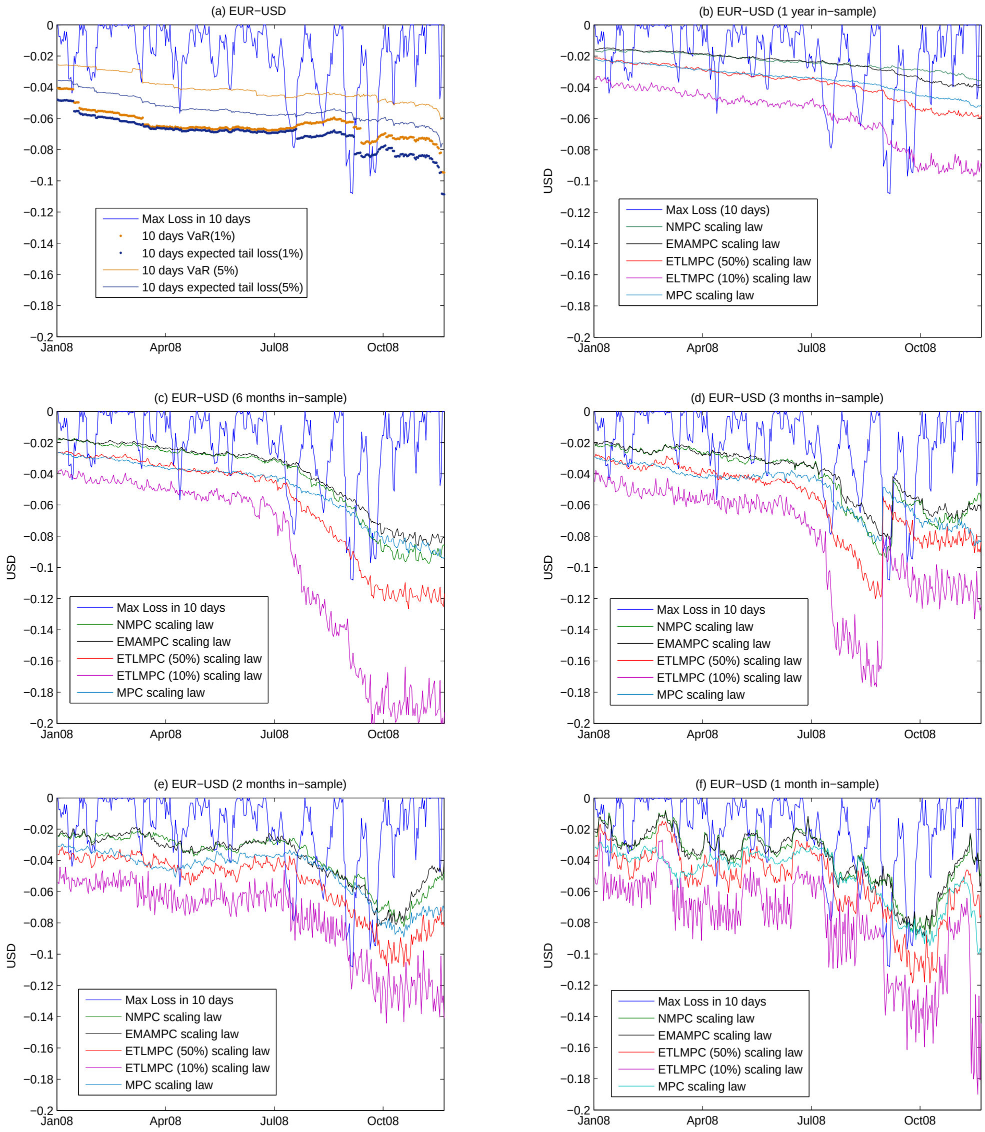


Figure 4.8: The 10-day VaR, scaling laws forecasting results and expected tail loss for EUR-USD (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008). The forecasting performance of the conventional VaR methods are shown in Figure (a), which are VaR (orange line) and ETL (dark blue line). The other sub-figures (b) to (f) show the forecasting results for five scaling law methods using different in-sample length, which are MPC scaling law L (light blue line), the NMPC scaling law $L1$(green line), the EMAMPC scaling law $L2$ (black line) and the ETLMPC scaling law $L3$ with $q = 50\%$ and $q = 10\%$ (red line and purple line).

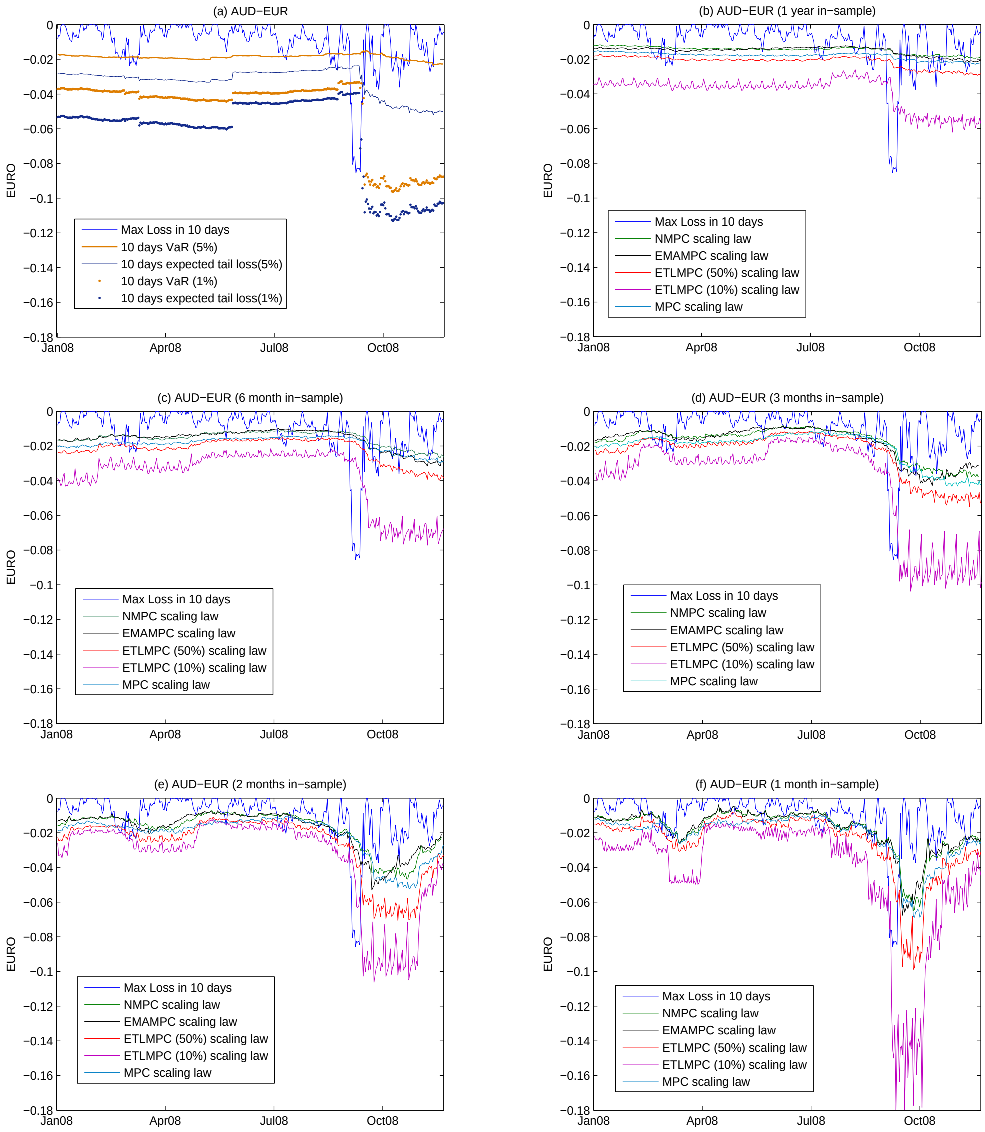


Figure 4.9: The 10-day VaR, scaling laws forecasting results and expected tail loss for AUD-EUR (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008). The forecasting performance of the conventional VaR methods are shown in Figure (a), which are VaR (orange line) and ETL (dark blue line). The other sub-figures (b) to (f) show the forecasting results for five scaling law methods using different in-sample length, which are MPC scaling law L (light blue line), the NMPC scaling law $L1$(green line), the EMAMPC scaling law $L2$ (black line) and the ETLMPC scaling law $L3$ with $q = 50\%$ and $q = 10\%$ (red line and purple line).

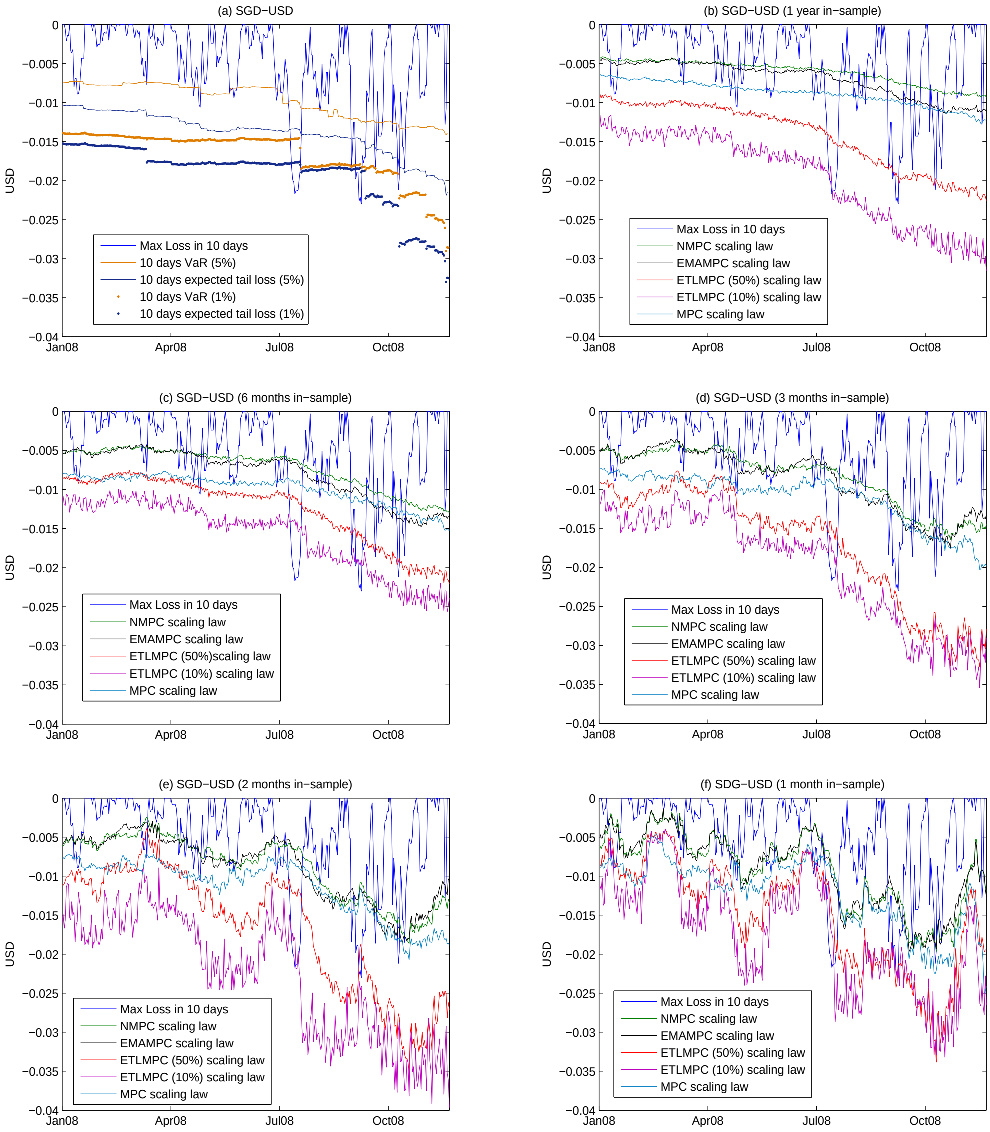


Figure 4.10: The 10-day VaR, scaling laws forecasting results and expected tail loss for SGD-USD (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008). The forecasting performance of the conventional VaR methods are shown in Figure (a), which are VaR (orange line) and ETL (dark blue line). The other sub-figures (b) to (f) show the forecasting results for five scaling law methods using different in-sample length, which are MPC scaling law L (light blue line), the NMPC scaling law $L1$(green line), the EMAMPC scaling law $L2$ (black line) and the ETLMPC scaling law $L3$ with $q = 50\%$ and $q = 10\%$ (red line and purple line).

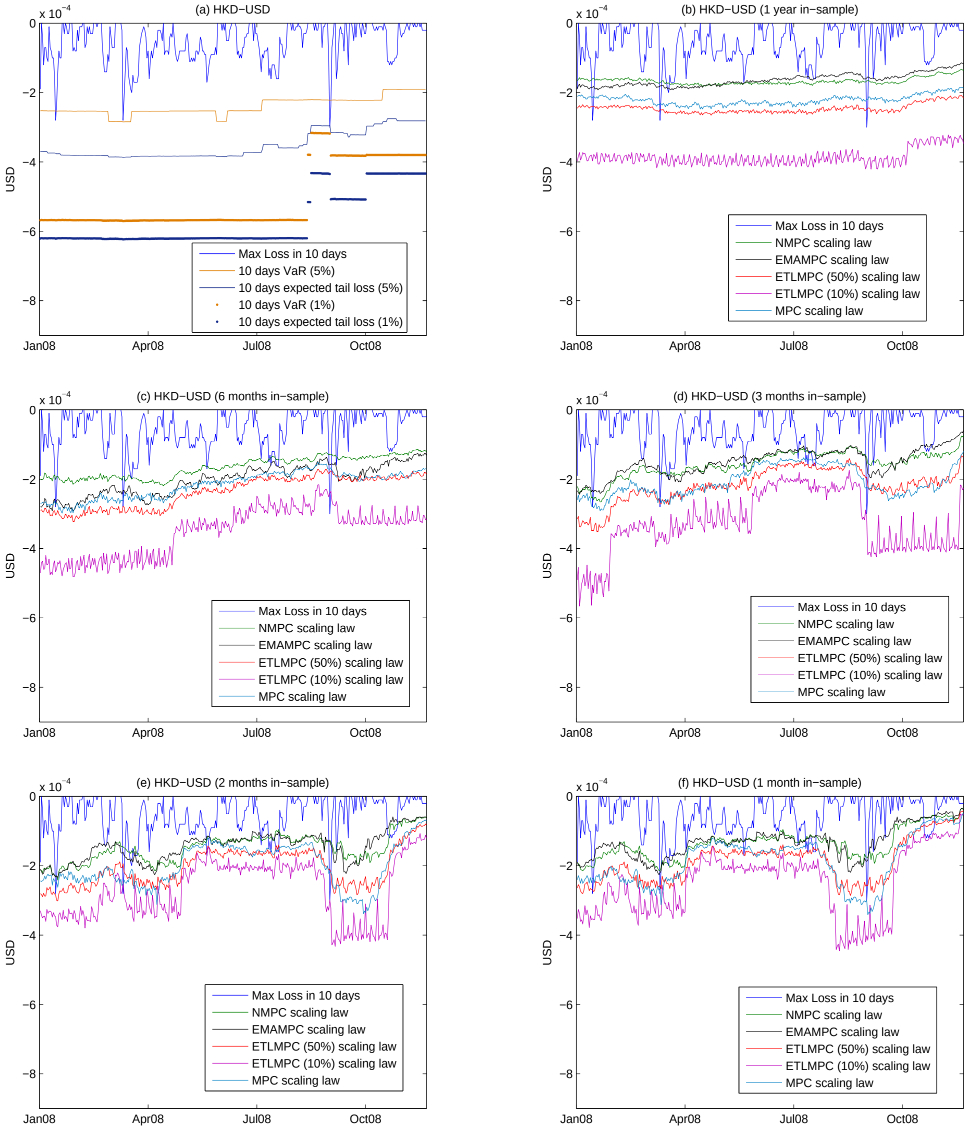


Figure 4.11: The 10-day VaR, scaling laws forecasting results and expected tail loss for SGD-USD (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008). The forecasting performance of the conventional VaR methods are shown in Figure (a), which are VaR (orange line) and ETL (dark blue line). The other sub-figures (b) to (f) show the forecasting results for five scaling law methods using different in-sample length,which are MPC scaling law L (light blue line), the NMPC scaling law $L1$(green line), the EMAMPC scaling law $L2$ (black line) and the ETLMPC scaling law $L3$ with $q = 50\%$ and $q = 10\%$ (red line and purple line).

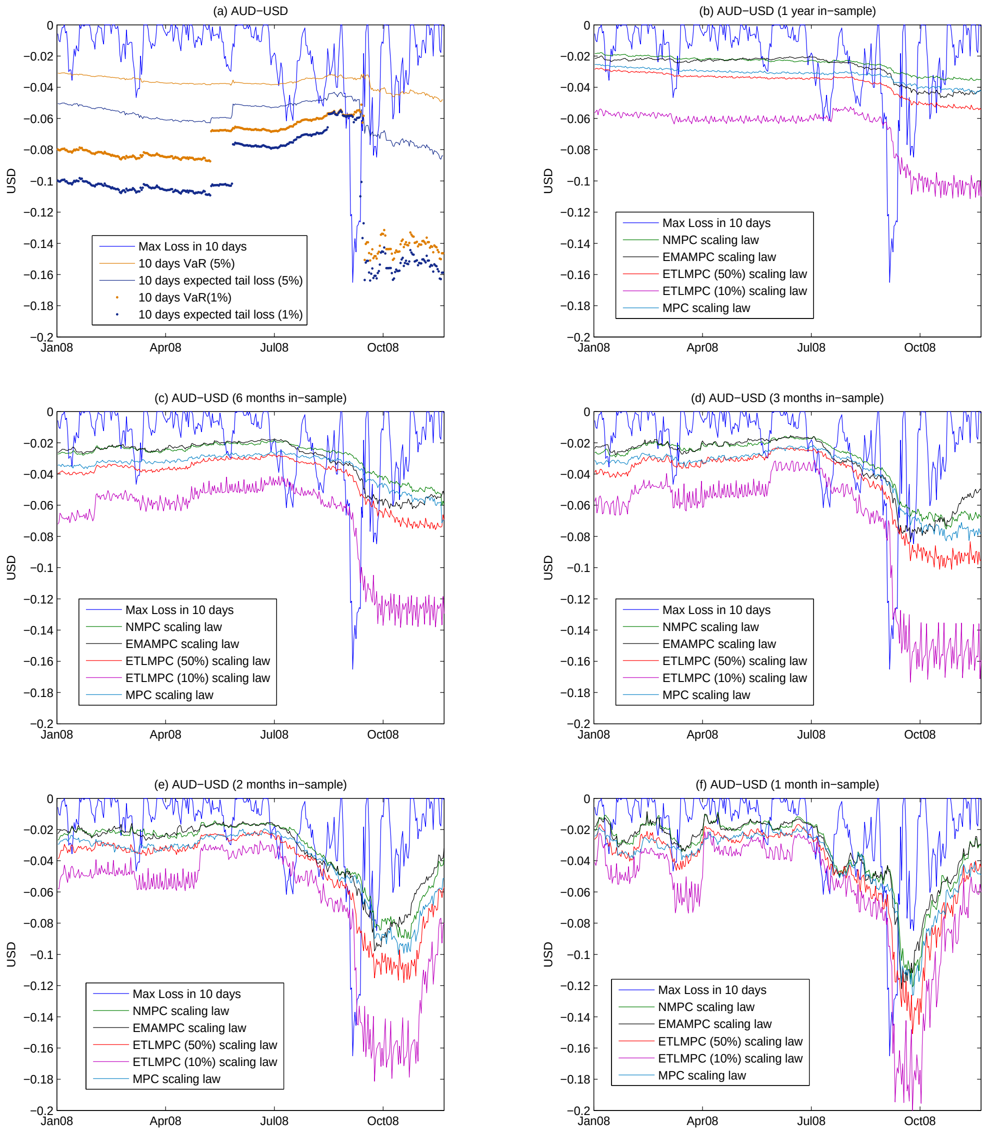


Figure 4.12: The 10-day VaR, scaling laws forecasting results and expected tail loss for AUD-USD (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008). The forecasting performance of the conventional VaR methods are shown in Figure (a), which are VaR (orange line) and ETL (dark blue line). The other sub-figures (b) to (f) show the forecasting results for five scaling law methods using different in-sample length, which are MPC scaling law L (light blue line), the NMPC scaling law $L1$(green line), the EMAMPC scaling law $L2$ (black line) and the ETLMPC scaling law $L3$ with $q = 50\%$ and $q = 10\%$ (red line and purple line).

In general, the VaR and ETL models tend to respond weakly to new data. Surprisingly, we can observe that the scaling law methods have more “sensitive” responses to the new data, especially for the smallest in-sample (1 month) window case (sub-figure (f) in Figures 4.8 to 4.12). Furthermore, we find that sometimes shortening the length of the in-sample window gives better results. The forecast results with 1 month in-sample show the most volatile movements but are also closest to the dynamic trend of the maximum loss. From the graphical results we can also observe that our model is better able to predict the dynamic movement of risk and volatility than the conventional VaR model and expected tail loss model. For all series, the maximum loss has some big jumps from June 2008 to October 2008, but after that period, the maximum loss shrinks. This trend is coincidental with the forecasting result of scaling law models with 3 months, 2 months and 1 month in-sample data (sub-figure (d), (e) and (f) in Figures 4.8 to 4.12). However, the conventional risk measures all fail to forecast the extreme losses. Overall, the shape of the curve for 1 month in-sample data is closest to the real loss movement.

The ETLMPC scaling laws $L3$ turned out to overestimate the risk in most cases, especially under the quantile α=10%. However, $L3$ can be used to forecast the extreme event risk for some future time interval. As shown in the graphs, the risk curves (red line) forecasted by $L3$ with q=50% are closer to every big jump of maximum loss in 10 days. The users can obtain the optimal model by changing different quantile constraint.

The empirical results obtained by the MPC scaling law methods show that the new method with multiple time scales can forecast the market risk better and use a data set of a much shorter time period. The most striking feature of our methods is that ‘intraday’ proportionality is the same as that of longer-term data of days, weeks, months and even years. This self-similarity of scaling law allows financial analysts to check the maximum loss in any time interval, not only for traders with very short investment horizons (of probably less than 1 day) but also investors with long holding periods.

4.5 Conclusion

The objective of this chapter is to provide alternative robust risk management tools based on multi-time scale analysis. The principal advantage of embedding the multi-time scale method into risk management. Following an approach by Glattfelder, Dupuisy, and Olsen (2011), we propose a new empirical framework for measurement of risk to obtain a robust risk management strategy. According to Basel III, all banks are required to report the 10-day VaR, and the conventional method is to scale the 1-day VaR to the 10-day VaR which is not valid as the return of underlying assets are usually not normally distributed. However, scaling law methods are distribution free.

The widely applied method of sampling daily return in the common VaR framework ignores dynamic price changes during the trading day. The empirical scaling law method in this chapter is based on the multiple time scale analysis. Ultra-high frequency data sets contain all time scales like the footprint of traders which comprises all information of the market. The principle advantage of such models is the ability to capture a full picture of possible loss scenarios of different time scales which gives a better understanding of the dynamic nature of the FX market.

Our first contribution is to propose three new scaling law methods in risk measuring and forecasting. Three modified MPC scaling law methods were compared with the general MPC scaling law and normal VaR method for the period from January 2008 to December 2008. The backtesting results provide evidence that the modified MPC scaling law methods are robust, in that it minimises the forecast errors and is closer to the real loss movement, including the tail event in a crisis.

Furthermore, the attraction for researchers in using the scaling law method is that they do not need to worry about the shortage of the data. Our second contribution is that we only use one month of in-sample high frequency data which can already provide good predictions, whereas for the conventional VaR at least two years of data is needed to obtain a reasonable result as the conventional risk measurement method requires regularly spaced time series which samples data with particular time horizons (usually daily). In contrast, the scaling law method incorporates all the information for different time scales.

Furthermore, larger window length may also contain the old data which has become irrelevant, especially in a financial crisis period. A financial crisis is like an earthquake; it is very dangerous and costs a lot if the likelihood or the size of a major crash is underestimated. Hence, we need to think more carefully when we measure the risk during a crisis. One reason for conventional risk measurement failing is that it is based on a single time scale data set. Longer time horizon data sets which include before, during or after the crisis period is needed to forecast risk. Extreme events may occur by extremely small changes during the long time horizon. In other words, when in a crisis, we measure VaR with a confidence level equal to 95% or 99%. It is equivalent to the framework for the short term data set but does not hold for the long term data set. For example, over a ten year data term, the 2008 crisis event is the 99.9999% tail event, not the 99% event. When the model is estimated using a long term data set, we need to incorporate the correct factor for that. Due to the scale invariance and short term data properties needed, this chapter suggests using the scaling law method to measure and forecast risk during a crisis period. When we use short term data within a crisis to analyse extreme event risk, it is much like the chance of being hit by lightning during a storm compared to a clear sunny day (long term data).

Therefore, the approach based on the MPC scaling law should be considered as a new standard measurement of risk. Also, this study is just a starting point, more efforts need to be put into the scaling laws research in financial risk management in order to improve the understanding of the market behaviour. For example, incorporating the relationship between volume and price impact could evaluate the liquidity risk of the asset.

Chapter 5

Summary

This thesis has investigated the risk measurement applications using high-frequency data. The aim was to address three important issues: the intraday Value at Risk (IVaR) forecasting performance, the asymmetric effects of liquidity risk adjustment in long and short position VaR, and using scaling laws to measure market risk. The first part of the study, comprising of chapters 2 and 3, investigated the issue of risk measurement on a single time scale method basis. The second part of the study, the analysis in Chapter 4, is extended to a multiple time scale framework by using an empirical scaling law method.

Today's trading system forces firms to continuously build their own strategies to beat the market. In Chapter 2, we explored the market risk measurement based on high-frequency measures of volatility with selected stocks in three different sectors. The conventional VaR is a rudimentary measure of risk and needs to be improved. IVaR can provide a real-time market risk measurement which is beyond the common daily calculated conventional VaR model. Parametric as well as non-parametric procedures are discussed. Beyond the results by Giot (2005), we quantify intraday market risk with much shorter time intervals. So far, popular approaches of computing IVaR mainly concentrate on single regime analysis. This chapter further contributes towards a more accurate IVaR based on a Markov Regime Switch (MRS) GARCH model. Nevertheless, the MRS-GARCH model can capture the structure changes in high-frequency return.

Chapter 2 has also provided the backtesting results for comparing the risk forecasting performance of different risk measurements. The proposed historical simulation method provided a really good performance considering the observed failure rates and results of the Kupiec test. According to results of the failure rate and Kupiec's test, the GARCH type models based on normal innovation are superior to the models with Student-t innovations. The IVaR results are too conservative for the models that rely on the Student-t distribution, especially in the 1 minute frequency case. The empirical results confirm that the MRS-GARCH model with normal innovations lead to considerable improvement in forecasting IVaR for all three different time frequencies. The empirical findings for MRS-GARCH IVaR, in the high frequency context, are new to the literature. The chapter therefore provided a practical application of intraday VaR measures for high-frequency traders to quantify the risk before the end of the business day.

Chapter 3 proceeds into the analysis of liquidity risk in high-frequency trading. The assumption of conventional VaR measures is that the asset can be executed at mid-price. However, this assumption does not hold in reality, in particular when investors execute large trades. The motivation of this chapter was to provide a new practical empirical technique which can help users to quantify their risk level as a function of the trading size and positions. We proposed an extended VaR measurement by incorporating the liquidity risk in intraday trading strategies when analysing limit order book data. The focus is on the integration of asymmetric information, upward or downward risks, into the input factors of forecast variables.

Chapter 3 also suggests to analyse bid spread and ask spread instead of the common bid-ask spread. We find that there exists an asymmetric behaviour of bid spread and ask spread between different trading volume. By taking account of the actual liquidity risk faced by investors with different trading size and positions, we proposed a liquidity adjusted intraday VaR (LAIVaR). Our approach improves the BDSS model by incorporating the endogenous liquidity risk effect instead of the bid-ask spread. In contrast to Giot and Gramming (2006), the study in Chapter 3 focuses on the asymmetric behaviour of both upside and downside LAIVaR and provides liquidity risk adjustments to specify the proportion of liquidity risk. The liquidity adjustments provides significant and specific information for short-term investors who want to go long or short. Therefore, the modelling of the LAIVaR allows traders to adjust positions with a benchmark for the optimal order scheduling. Furthermore, we apply the bivariate analysis to investigate the asymmetric effect of the bid and ask side. The results of liquidity adjustment show that the liquidity risk is a crucial factor in estimating VaR. Neglecting liquidity cost will lead to underestimation of risk in a conventional VaR model.

In the second part of this thesis, we extended the research by introducing a multiple time scale method based on scaling law for risk measurement and forecasting in the FX market. The multiple time scale analysis has the advantage that it incorporates information about the footprint of traders using different time horizons and gives a better understanding of the dynamic nature of the financial market. In Chapter 4, the 10-day VaR is calculated as a benchmark of performance comparison which is required by regulatory authorities. The conventional measurement is to scale the 1-day VaR to the 10-day VaR which is not appropriate, given that the return of financial assets are usually not normally distributed. This chapter proposed three new empirical scaling laws based on the original maximal price change (MPC) scaling law by Glattfelder, Dupuisy, and Olsen (2011). The most valuable property is the scale invariance which allows financial analysts to assess the maximum loss for any time interval. The strong evidence of the effectiveness of the new scaling law methods are provided in the model performance section. The out-of-sample data covers the special crisis period in 2008 when most risk measurement tools failed. The forecasting performance of three modified MPC scaling law methods with the general MPC scaling law and normal VaR method are provided. The empirical results show that the 10-day maximum loss calculated by using a modified MPC scaling law exponent performs better and is closer to the real loss, including the tail event in the crisis.

Furthermore, using a high frequency data set provides the fundamental statistical properties and full information of the market. The scaling law method with only one month in-sample data can already provide a good prediction of risk, whereas for the conventional VaR at least ten years worth of data is needed to obtain a reasonable result.

Overall, this thesis has presented a very practical and comprehensive study of the risk measurement based on high-frequency data. Subject to today's trading environment, a detailed empirical analysis is proposed to overcome the limitations of current popular risk assessment models. In terms of future research, some suggestions are highlighted. As a most widely used risk measurement, the VaR model has to be more sophisticated. The main volatility forecasted model used in this thesis is based on the GARCH family. The extended VaR model should go beyond market risks to take other risk factors into account. Secondly, the multiple time scale analysis in this thesis is based on the different empirical scaling laws. The empirical results of the scaling laws method show that modified model specifications for forecasting the market risk perform better and use a much shorter time period data set. It would be interesting to develop new scaling laws. Additional research could also attempt to incorporate the relationship between volume and price impact which can capture liquidity risk more accurately.

Bibliography

ABRAMSON, A., AND I. COHEN (2007): “On the stationarity of Markov switching GARCH processes,” *Econometric Theory*, 23(3), 485–500.

ACERBI, C., AND G. SCANDOLO (2008): “Liquidity Risk Theory and Coherent Measures of Risk,” *Quantitative Finance*, 8(7), 681–692.

ACERBI, C.AND FINGER, C. (2010): “The value of liquidity,” RiskMetrics Technical paper.

AGNELIDIS, T., AND A. BENOS (2006): “Liquidity Adjusted Value at Risk based on the components of bid-ask spread,” *Applied Financial Economics*, 16(11), 835–851.

AKGIRAY, V. (1989): “Conditional Heteroskedasticity in Time Series of Stock Returns: Evidence and Forecasts,” *Applied Financial Economics*, 62(1), 55–88.

ALBERG, D., H. SHALIT, AND R. YOSEF (2008): “Estimating stock market volatility using asymmetric GARCH models,” *Applied Financial Economics*, 18(15), 1201–1208.

ALDRIDGE, I. (2009): *High-Frequency Trading: A Practical Guide to Algorithmic Strategies and Trading Systems.* John Wiley and Sons, NY.

——— (2010): *High-Frequency Trading: A Practical Guide to Algorithmic Strategies and Trading Systems.* John Wiley and Sons.

ALENTORN, A., AND S. MARKOSE (2008): *Generalized Extreme Value Distribution and Extreme Economic Value at Risk (EE-VaR).* Erricos John Kontoghiorghes, Berc Rustem, Peter Winker(ed), Book: Computational Methods in Financial Engineering, pp 47-72, Springer.

ANDERSEN, T. G., AND T. BOLLERSLEV (1998): "Answering the skeptics: yes, standard volatility models do provide accurate forecasts," *International Economic Review*, 49(4), 885–905.

——— (1999): "Intraday periodicity and volatility persistence in financial markets," *Journal of Empirical Finance*, 4(3), 115–158.

ANDERSEN, T. G., T. BOLLERSLEV, F. X. DIEBOLD, AND H. EBENS (2001): "The distribution of realized stock return volatility," *Journal of Financial Economics*, 61(1), 43–76.

ANDREA, B., AND M. CLAUDIO (2001): "Computing Value at Risk with High Frequency Data," *Journal of Emperical Finance*, 6(1), 431–455.

ARNOLD, B. C. (1983): *Pareto Distributions.* International Cooperative Publishing House, Fairland, Maryland.

ARTZNER, P.AND FREDDY, D., J. EBER, AND D. HEATH (1999): "Coherent Measures of Risk," *Mathematical Finance*, 9(3), 203–228.

Aussenegg, W., and T. Miazhynskaia (2006): "Uncertainty in Value-at-risk Estimates under Parametric and Non-parametric Modeling," *Financial Markets and Portfolio Management*, 20(3), 243–264.

Bangia, A., F. Diebold, T. Schuermann, and J. Stroughair (1999): "Modeling Liquidity Risk With Implications for Traditional Market Risk Measurement and Management," The Wharton Financial Institutions Center, Working Paper No. 99-06.

Barndorff-Nielsen, O. E. (1998): "Processes of Normal Inverse Gaussian Type," *Finance and Stochastics*, 2(1), 41–68.

Bauhaus, J. (2001): "Power-Laws in Economy and Finance: Some Ideas from Physics," *Quantitative Finance*, 1(1), 105–112.

Beltratti, A., and c. Morana (1999): "Computing value at risk with high frequency data," *Journal of Empirical Finance*, 6(5), 431–455.

Berkowitz, J. (2000): "Computing value at risk with high-frequency data," *Journal of Derivatives*, 5(2), 32–44.

Bertsimas, D., and A. W. Lo (1998): "Optimal control of execution costs," *Journal of Financial Markets*, 1(1), 1–50.

Black, F. (1971): "Towards a fully automated exchange : Part 1," *Financial Analyst Journal*, 27(1), 29–34.

Black, F. (1976): "Studies of Stock Market Volatility Changes," Business and Economic Statistics Section (177-181).

BOLLERSLEV, T. (1986): "Generalized autoregressive conditional heteroskedasticity," *Journal of Econometrics*, 31(3), 307–327.

BOLLERSLEV, T., R. ENGLE, AND J. WOOLDRIDGE (1998): "A capital asset pricing model with time varying covariances," *Journal of Political Economy*, 96(1), 116–131.

BROOKS, C., AND G. PERSAND (2000): "Value at Risk and Market Crashes," *Journal of Risk*, 2(4), 5–26.

BRUNNERMEIER, M. K. (2009): "Deciphering the Liquidity and Credit Crunch 2007ÍC2008," *Journal of Economic Perspectives*, 23(1), 77ÍC100.

CAO, C., O. HANSCH, AND X. WANG (2008): "Order placement strategies in a pure limit order book market," *Journal of Financial Research*, 31(2), 113–140.

CAO, C.AND HANSCH, O., AND X. WANG (2009): "The information content of an open limit-order book," *The Journal of Futures Markets*, 29(1), 16–41.

CLAUSET, A., C. SHALIZI, AND M. NEWMAN (2008): "Power-law distributions in empirical data," SIAM Review 51, 661-703 (2009).

COLLETAZ, G., C. HURLIN, AND S. TOKPAVI (2007): "Irregularly Spaced Intraday Value at Risk (ISIVaR) Models - Forecasting and Predictive Abilities," Working Paper No. 00162440, University of Orléans.

CONT, R. (2001): "Empirical properies of asset returns: stylized facts and statistical issues," *Quantitative Finance*, 1(2), 223–236.

Dacorogna, Gençay, Muller, Olsen, and Pictet (2001): *An Introduction to High-Frequency Finance*. Academic Press., New York.

Danielsson, J., and J. Zigrand (2004): "On time-scaling of risk and the square-root-of time rule," FMG Discussion Papers. Working Paper No. dp439.

Dionne, G., P. Duchesne, and M. Pacurar (2009): "Intraday Value at Risk (IVaR) Using Tick-by-tick Data with Application to the Toronto Stock Exchange," *Journal of Emprical Finance*, 16(5), 777–792.

Dowd, K. (1998): *Beyond Value at Risk : the New Science of Risk Management*. John Wiley and Sons., Chichester and New York.

——— (2002): *An Introduction to Market Risk Measurement*. John Wiley and Sons., Chichester and New York.

Engle, R., and K. Kroner (1995): "Multivariate simultaneous generalized ARCH," *Journal of Political Economy*, 11(6), 122–150.

Engle, R., and J. Russell (2006): *Analysis of high frequency data*. In: Ait Sahalia, Y., Hansen, L.P.(Eds.), Handbook of Financial Econometrics. Springer.

Engle, R., and J. R. Russell (1998): "Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data," *Econometrica*, 66(5), 1127–1162.

Engle, R. F. (1982): "Autoregressive conditional heteroskedasticity with estimates of the variance of the United Kingdom inflation," *Econometrica*, 50(4), 987–1007.

——— (2002): "Dynamic Conditional Correlation: A Simple Class of Generalized Autoregressive Conditional Heteroskedasticity Models," *Journal of Business and Economic Statistics*, 20(3), 339–350.

Engle, R. F., and V. Ng (1993): "Measuring and Testing the Impact of News on Volatility," *Journal of Finance*, 48(5), 1749–1778.

Ernst, C., S. Stange, and C. Kaserer (2008): "Accounting for Nonnormality in Liquidity Risk.," CEFS working paper 2008 No. 14.

——— (2009): "Measuring Market Liquidity Risk - Which Model Works Best?," CEFS Working Paper Series 2009 No. 1.

Erwan, L. S. (2001): "Incorporating Liquidity Risk in VaR Models," Working Paper, University of Rene.

Fama, E. (1965): "The behaviour of stock market prices," *Journal of Business*, 38(1), 34–105.

Francois-Heude, A., and P. Van Wynendaele (2001): "Integrating Liquidity Risk in a Parametric Intraday VaR Framework," Working Paper, University of Perpignan.

Gabaix, X. (2009): "Power Laws in Economics and Finance," *Annual Review of Economics*, 1, 255–294.

Giot, P. (2000): "Time transformations, intraday data, and volatility models," *Journal of Computational Finance*, 4(2), 31–62.

——— (2005): "Market risk models for intraday data," *The European of Journal of Finance*, 11(4), 309–324.

Giot, P., and J. Gramming (2006): "How Large is Liquidity Risk in an Automated Auction Market?," *Empirical Economics*, 30(9), 867–887.

Giot, P., and S. Laurent (2004): "Modeling daily Value-at-Risk using realized volatility and ARCH type models," *Journal of Empirical Finance*, 11(3), 379–398.

Glattfelder, J., A. Dupuisy, and R. B. Olsen (2011): "Patterns in high-frequency FX data: discovery of 12 empirical scaling laws," *Quantitative Finance*, 11(4), 599–614.

Glosten, L. R., R. Jagannathan, and D. E. Runkle (1997): "Mopping up liquidity," *Risk*, 10(12), 170–173.

Goodhart, C., and M. O'Hara (1997): "High frequency data in financial markets: Issues and applications," *Journal of Empirical Finance*, 4(2), 73–114.

Gray, S. (1996): "Time transformations, intraday data, and volatility models," *Journal of Financial Economics*, 42(1), 27–62.

Group of Thirty (1993): *Derivatives: Practices and Principles*. Group of Thirty, Washington, DC.

Hall, A. D., and N. Hautsch (2007): "Modelling the buy and sell intensity in a limit order book market," *Journal of Financial Markets*, 10(3), 249–286.

HAMILTON, J. D., AND R. SUSMEL (1994): "Autoregressive Conditional Heteroskedasticity and Changes in Regime," *Journal of Econometrics*, 64(2), 307–333.

HISATA, Y., AND Y. YAMAI (2000): "Research toward the practical application of liquidity risk evaluation methods," *Monetary and Economic Studies*, 18(2), 83–127.

HORCHER, K. (2005): *Essentials of financial risk management.* John Wiley & Sons, Hoboken, NJ.

HULL, J., AND A. WHITE (1997): "Value at Risk When Daily Changes in Market Variables Are Not Normally Distributed," *Journal of Derivatives*, 5(3), 9–19.

JORION, P. (2006): *Value at Risk: The New Benchmark for Managing Financial Risk (3rd Ed).* McGraw-Hill, NY.

J.P. MORGAN (1996): "Morgan Technical Document," pubulished on line: avilable at: http://www.riskmetrics.com/publications/techdocs/rmcovv.html.

KANIEL, R., AND H. LIU (2006): "So What Orders Do Informed Traders Use?," *Journal of Business*, 79(4), 1867–1914.

KISSELL, R., AND R. MALAMUT (2006): "Algorithmic Decision-Making Framework," *The Journal of Trading*, 1(1), 12–21.

KIYMAZ, H., AND H. BERUMENT (2003): "The day of the week effect on stock market volatility and volume: International evidence," *Review of*

Financial Economics, 12(4), 363–380.

Klaassen, F. (2002): "Improving GARCH Volatility Forecasts," *Empirical Economics*, 27(2), 363–394.

Koopman, S. J., B. Jungbacker, and E. Hol (2005): "Forecasting daily variability of the S&P100 stock index using historical, realized and implied volatility measurements," *Journal of Empirical Finance*, 12(4), 445–475.

Kupiec, P. H. (1995): "Techniques for verifying the accuracy of risk measurement models," *Journal of Derivatives*, 2(9), 73–84.

Kyle, A. S. (1985): "Continuous Auctions and Insider Trading," *Econometrica*, 53(6), 1315–1335.

Lawrence, C., and G. Robinson (1997): "Liquidity, Dynamic Hedging and Value at Risk," *Risk Management for Financial Institutions*, 3(2), 63–72.

Linsmeier, T., and N. Pearson (1996): "Risk Measurement: An Introduction to Value at Risk," Working Paper No. 9609004, University of Illinois at Urbana-Champaign.

Ljung, G., and E. Box (1978): "On a Measure of a Lack of Fit in Time Series Models," *Biometrika*, 65(2), 297–303.

Madhavan, A., M. Richardson, and M. Roomans (1997): "Nonstationarities in financial time series, the long- range dependence, and the IGARCH effcts," *Review of Economics and Statistics,*, 86(1), 378–390.

Marcucci, J. (2005): "Forecasting stock market volatility with regime-switching GARCH models," *Studies in Nonlinear Dynamics & Econometrics*, 9(4), 1–53.

Markowitz, H. M. (1952): "Portfolio Selection," *Journal of Finance*, 7(1), 77–91.

——— (1959): *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley and Sons.

Matz, L. M., and P. Neu (2007): *Liquidity risk Measurement and Management*. New York: John Wiley and Sons.

McMillan, D., A. Speight, and O. Gwilym (2000): "Forecasting UK Stock Market Volatility," *Applied Financial Economics*, 10(4), 435–448.

Menkens, O. (2008): *Value at risk and self-similarity*. Miller, John and Edelman, David and Appleby, John A.D (ed), Numerical Methods for Finance, pp 226-252, Chapman and Hall/CRC.

Mikosch, T., and C. Starica (2004): "Why do security prices change? A transaction-level analysis of NYSE stocks," *Review of Financial Studies*, 10(4), 1035–1064.

Mitzenmacher, M. (2003): "A History of and New Directions for Power Law Research," *Harvard University, Internet Mathematics*, 1(2), 226–251.

Müller, U. A., M. M. Dacorogna, R. B. Olsen, M. Pictet, O.V. Schwarz, and C. Morgenegg (1990): "Statistical study of foreign exchange rates, empirical evidence of a price change scaling law and intraday analysis," *Journal of Banking and Financecs*, 14(6), 1189–1208.

Nelson, D. B. (1991): "Conditional heteroskedasticity in asset returns: A new approach," *Econometrica*, 59(5), 347–370.

Newman, M. E. J. (2005): "Power laws, Pareto distributions and Zipf's law," *Contemporary Physics*, 46(5), 323–351.

Nocera, J. (2009): "Risk Mismanagement," available at: http://www.nytimes.com/2009/01/04/magazine/04risk-t.html, New York Times (January 2, 2009).

Pareto, V. (1896): *Cours d'Economie Politque.* Geneva, Droz.

Pastor, L., and R. F. Stambaugh (2003): "Liquidity Risk and Expected Stock Returns," *The Journal of Political Economy*, 111(3), 642–685.

Poon, S., and C. Granger (2003): "Forecasting Volatility in Financial Markets: A Review," *Journal of Economic Literature*, 41(2), 478–539.

Sajjad, R., J. Coakley, and J. Nankervis (2008): "Markov-Switching GARCH Modelling of Value-at-Risk," *Studies in Nonlinear Dynamics & Econometrics*, 12(3), Article 7.

Shiode, N., and M. Batty (2000): "Power Law Distribution in Real and Virtual World," Centre for Advanced Spatial Analysis (UCL),Working Papers (19).

Sornette, D., Y. Malevergne, and J. Muzy (2003): "What causes crashes?," *Journal of Risk*, 16(2), 61–71.

Taylor, S. J. (1986): *Modeling Financial Time Series*. John Wiley and Sons, Chichester.

Taylor, S. J., and X. Xu (1997): "The incremental volatility information in one million foreign exchange quotations," *Journal of Empirical Finance*, 4(4), 317–340.

Ulrich, A. (2000): "Volatility Computed by Time Series Operators at High Frequency," Working Paper No. 323, Olsen & Associates.

Van Den Goorbergh, R., and P. Vlaar (1999): "Value-at-risk analysis of stock returns: historical simulation, variance techniques or tail index estimation?," Dnb staff reports no.40, Netherlands Central Bank, Amsterdam.

Vandewalle, N., F. Briscois, and P. H. Lefebvre (2000): "Managing both sign and size of fluctuations within the n-Zipf framework," *International Journal of Theoretical and Applied Finance*, 3(3), 409–414.

Vasanta, M. C. (2004): "Regulation and risk Management: Value-at Risk as a Unifier," avalable online: http://www.gtnews.com/article/5642.cfm.

Wilmott, P. (2007): *Quantitive Finance*. Wiley; 2 edition.

Appendices

Appendix A

A.1 Estimated Parameters for AR-GARCH Type Models

Table A.1: Estimated parameters for NR (In-sample)

		NR		
		1 minute	5 minutes	10 minutes
AR-GARCH	μ	0.0233 (0.0124)	0.0131 (0.0258)	0.0540 (0.0363)
	δ	0.0125 (0.0155)	0.0551 (0.0381)	-0.1361 (0.0471)
	a_0	0.0440 (0.0017)	0.5048 (0.0502)	0.4234 (0.0498)
	a_1	0.1173 (0.0031)	0.3960 (0.0332)	0.3211 (0.0773)
	β_1	0.8749 (0.0038)	0.2200 (0.0237)	0.4536 (0.0393)
AR-GARCH-T	μ	$2.9E-6$ ($6.2E-4$)	0.0066 (0.0314)	0.0217 (0.0370)
	δ	-0.0083 (0.0073)	-0.0882 (0.0257)	-0.0542 (0.0442)
	a_0	$2.0E-5$ ($1.7E-8$)	0.3547 (0.1068)	0.3152 (0.1202)
	a_1	0.8485 (0.00858)	0.4237 (0.0427)	0.3152 (0.1066)
	β_1	0.1015 (0.0015)	0.5734 (0.0636)	0.5858 (0.0978)
	ν	2.1326 (0.0116)	2.8388 (0.2740)	3.2088 (0.5310)
AR-EGARCH	μ	0.0027 (0.0073)	0.0499 (0.0285)	0.0748 (0.0446)
	δ	0.0102 (0.0130)	-0.0370 (0.0328)	-0.1718 (0.0561)
	a_0	0.0636 (0.0020)	0.2335 (0.0350)	0.1354 (0.0269)
	a_1	0.1849 (0.0041)	0.5399 (0.0152)	0.5771 (0.0421)
	β_1	0.9692 (0.0010)	0.6634 (0.0311)	0.7306 (0.0477)
	γ	0.0477 (0.0026)	0.1984 (0.0290)	0.0855 (0.0289)
AR-EGARCH-T	μ	0.0013 (0.0058)	$-6.0E-9$ ($1.0E-8$)	0.0139 (0.0364)
	δ	-0.0068 (0.0086)	-0.0840 (0.0259)	-0.0556 (0.0447)
	a_0	0.0101 (0.0036)	0.2140 (0.0873)	0.0924 (0.0606)
	a_1	0.9527 (0.0055)	0.8031 (0.0027)	0.5245 (0.1038)
	β_1	0.7269 (0.0032)	0.8307 0.0320)	0.7895 (0.0678)
	γ	0.0612 (0.0542)	0.0743 (0.0630)	0.0580 (0.0664)
	ν	2.0165 (0.0058)	2.6326 (0.3171)	3.2943 (0.5541)

Table A.2: Estimated parameters for RBS (In-sample)

		RBS		
		1 minute	5 minutes	10 minutes
AR-GARCH	μ	-0.0327	-0.0102	-0.0720
		(0.0312)	(0.0371)	(0.0464)
	δ	0.0099	0.0131	-0.0603
		(0.0340)	(0.0293)	(0.0121)
	a_0	0.3403	0.0453	0.5309
		(0.0449)	(0.0083)	(0.0728)
	a_1	0.3099	0.0534	0.5123
		(0.0449)	(0.0084)	(0.0426)
	β_1	0.5050	0.9208	0.2267
		(0.0572)	(0.0077)	(0.0081)
AR-GARCH-T	μ	-0.0015	-0.0025	-0.0078
		(0.0268)	(0.0270)	(0.0046)
	δ	-0.0018	-0.0022	-0.1030
		(0.0296)	(0.0240)	(0.0081)
	a_0	0.2326	0.2558	0.5457
		(0.0943)	(0.0299)	(0.0814)
	a_1	0.3851	0.3635	0.6501
		(0.0124)	(0.0785)	(0.0590)
	β_1	0.5815	0.6184	0.3183
		(0.0583)	(0.0562)	(0.0137)
	ν	2.8868	3.0431	3.6109
		(0.0939)	(0.3553)	(0.8818)
AR-EGARCH	μ	-0.0461	-0.0242	-0.0335
		(0.0417)	(0.0377)	(0.0051)
	δ	0.0496	0.0134	-0.0670
		(0.0359)	(0.0283)	(0.0091)
	a_0	0.1312	0.0239	0.1503
		(0.0163)	(0.0040)	(0.0568)
	a_1	0.2611	0.1155	0.6762
		(0.0239)	(0.0124)	(0.0067)
	β_1	0.8369	0.9741	0.5659
		(0.0242)	(0.0041)	(0.0069)
	γ	0.0084	0.0211	0.1791
		(0.0150)	(0.0130)	(0.0312)
AR-EGARCH-T	μ	-0.0060	-0.0011	-0.0018
		(0.0001)	(0.0269)	(0.0047)
	δ	-0.0034	-0.0033	-0.1055
		(0.0279)	(0.0289)	(0.0097)
	a_0	0.0807	0.0822	0.1505
		(0.0341)	(0.0365)	(0.0631)
	a_1	0.4185	0.4643	0.7078
		(0.0426)	(0.0498)	(0.0386)
	β_1	0.8983	0.8825	0.7235
		(0.0267)	(0.0292)	(0.0411)
	γ	0.0391	0.0131	0.1517
		(0.0034)	(0.0319)	(0.0518)
	ν	2.8577	3.0504	3.5558
		(0.2931)	(0.3609)	(0.9001)

Table A.3: Estimated parameters for HSBC (In-sample)

		HSBC		
		1 minute	5 minutes	10 minutes
AR-GARCH	μ	-0.0159	0.0014	0.0327
		(0.0140)	(0.0282)	(0.0373)
	δ	-0.0752	-0.1363	0.0540
		(0.0145)	(0.0327)	(0.0470)
	a_0	0.0631	0.0424	0.0645
		(0.0029)	(0.0056)	(0.0137)
	a_1	0.0710	0.1155	0.2174
		(0.0037)	(0.0061)	(0.0326)
	β_1	0.8848	0.8611	0.7550
		(0.0050)	(0.0086)	(0.0345)
AR-GARCH-T	μ	-0.0007	-0.0139	-0.0223
		(0.0003)	(0.0230)	(0.0307)
	δ	-0.0201	-0.1438	-0.0685
		(0.0119)	(0.0257)	(0.0409)
	a_0	0.0002	0.0189	0.0391
		(0.0001)	(0.0077)	(0.0169)
	a_1	0.1540	0.1210	0.2117
		(0.0050)	(0.0257)	(0.0503)
	β_1	0.8239	0.8789	0.7811
		(0.0161)	(0.0207)	(0.0328)
	ν	2.1799	3.8704	4.1301
		0.0728	(0.4502)	(0.0478)
AR-EGARCH	μ	-0.0363	0.0552	0.0478
		(0.0121)	(0.0262)	(0.0301)
	δ	-0.0624	-0.1179	-0.0593
		(0.0137)	(0.0328)	(0.0452)
	a_0	0.0443	0.0407	0.0268
		(0.0023)	(0.0049)	(0.0155)
	a_1	0.1449	0.2722	0.3748
		(0.0056)	(0.0145)	(0.0329)
	β_1	0.9438	0.9377	0.8962
		(0.0027)	(0.0075)	(0.0225)
	γ	0.0155	0.0273	0.1285
		(0.0038)	(0.0128)	(0.0417)
AR-EGARCH-T	μ	0.0018	-0.0092	-0.0282
		(0.0087)	(0.0211)	(0.0287)
	δ	-0.0168	-0.1464	-0.0717
		(0.0127)	(0.0277)	(0.0407)
	a_0	0.0241	0.0077	0.0085
		(0.0080)	(0.0081)	(0.0187)
	a_1	0.5926	0.3717	0.4161
		(0.0358)	(0.0372)	(0.0826)
	β_1	0.9404	0.9795	0.9538
		(0.0024)	0.0080)	(0.0207)
	γ	0.1993	0.0131	0.0526
		(0.0219)	(0.0293)	(0.0564)
	ν	2.0273	3.4752	3.8102
		(0.0917)	(0.3730)	(0.6715)

A.2 Estimated Parameters for MRS-GARCH Model

Table A.4: Estimated parameters of MRS-GARCH model

Stocks	Parameters	1 minute	5 minutes	10 minutes
NR	$\mu^{(1)}$	-0.0236 (0.0133)	-0.0147 (0.0081)	0.0110 (0.0070)
	$\mu^{(2)}$	0.1398 (0.0844)	0.1118 (0.0124)	-0.0273 (0.0130)
	$a_0^{(1)}$	0.2955 (0.0534)	0.0005 (0.0001)	0.0108 (0.0027)
	$a_0^{(2)}$	0.7968 (0.8323)	0.0388 (0.1356)	0.0037 (0.0004)
	$a_1^{(1)}$	0.0023 (0.0285)	0.0618 (0.0019)	0.0034 (0.0007)
	$a_1^{(2)}$	0.3405 (0.1389)	0.7131 (0.0778)	0.5503 (0.2447)
	$\beta_1^{(1)}$	0.5491 (0.0398)	0.3848 (0.0086)	0.4550 (0.0773)
	$\beta_1^{(2)}$	0.6549 (0.0464)	0.2809 (0.0356)	0.4491 (0.0761)
	p	0.9447 (0.0127)	0.7560 (0.2840)	0.7525 (0.0421)
	q	0.1710 (0.0851)	0.1703 (0.0254)	0.1120 (0.0342)
RBS	$\mu^{(1)}$	0.0010 (0.0170)	0.0058 (0.0278)	-0.0205 (0.0170)
	$\mu^{(2)}$	-0.0657 (0.0479)	-0.0274 (0.0126)	0.0384 (0.0313)
	$a_0^{(1)}$	0.0115 (0.0009)	0.0074 (0.0006)	0.0064 (0.0021)
	$a_0^{(2)}$	0.9842 (0.0502)	0.0094 (0.0012)	0.0097 (0.0031)

cont

Stocks	Parameters	1 minute	5 minutes	10 minutes
	$a_1^{(1)}$	0.0375 (0.0099)	0.0987 (0.0266)	0.0135 (0.0422)
	$a_1^{(2)}$	0.0511 (0.0573)	0.0173 (0.046)	0.0550 (0.0677)
	$\beta_1^{(1)}$	0.6447 (0.0150)	0.3406 (0.0774)	0.3541 (0.0749)
	$\beta_1^{(2)}$	0.9120 (0.2750)	0.9644 (0.0546)	0.9448 (0.1552)
	p	0.9846 (0.0007)	0.8215 (0.1627)	0.8459 (0.0171)
	q	0.3940 (0.0356)	0.1044 (0.0147)	0.1180 (0.0482)
HSBC	$\mu^{(1)}$	−0.0255 (0.0081)	−0.0151 (0.0196)	0.0056 (0.0383)
	$\mu^{(2)}$	−0.8377 (0.0287)	0.1595 (0.1289)	−0.0802 (0.0283)
	$a_0^{(1)}$	0.0098 (0.0017)	0.0002 (0.0001)	0.0000 (0.0000)
	$a_0^{(2)}$	0.0166 (0.0042)	0.8826 (0.0320)	2.1519 (0.0182)
	$a_1^{(1)}$	0.0183 (0.0218)	0.0841 (0.0414)	0.0004 (0.0001)
	$a_1^{(2)}$	0.0141 (0.7702)	0.0000 (0.0000)	0.0014 (0.0007)
	$\beta_1^{(1)}$	0.6401 (0.0446)	0.5809 (0.0631)	0.6598 (0.0342)
	$\beta_1^{(2)}$	0.9856 (0.0773)	0.9582 (0.645)	0.9354 (0.0277)
	p	0.1069 (0.0865)	0.9203 (0.0270)	0.7550 (0.0345)
	q	0.9705 (0.0062)	0.0644 (0.0100)	0.7550 (0.0345)

Appendix B

B.1 Multivariate GARCH Models

Multivariate GARCH models were initially developed in the late 1980s. Basically the models study the moving process both of variance and covariances which is different with univariate GARCH models. There are three important classes of multivariate models, namely (a) the VECH model, (b) the diagonal VECH model (Bollerslev, Engle, and Wooldridge (1998)) and (c) the BEKK model (Engle and Kroner (1995)).

The VECH model which is the original version of the multivariate GARCH model, is proposed by Bollerslev, Engle, and Wooldridge (1998):

$$Y_t = \mu_t + \varepsilon_t \tag{B.1}$$

with $\varepsilon_t \mid \Psi_{t-1} \sim N(0, H_t)$, and

$$Vech(H_t) = C + \sum_{i=1}^{q} A_i Vech(\varepsilon_{t-i}\varepsilon'_{t-i}) + \sum_{j=1}^{p} B_j Vech(H_{t-j}) \tag{B.2}$$

where Y_t is an $N \times 1$ vector which denotes the return at time t; μ_t is the conditional mean of Y_t; ε_t is the innovation vector; Ψ_{t-1} is the set of information available at time $t-1$; C is a $N(N+1)/2 \times 1$ vector, A_i and B_j are $N(N+1)/2 \times N(N+1)/2$ matrices $Vech(.)$ denotes the column-stacking operator applied to the lower portion of an $N \times N$ symmetrical matrix.

The number of parameters in the VECH model equals: $(2N(N+1)+N^2(N+1)^2(p+q))/4$. For example, if we assume the simple GARCH(1,1) model and $N = 2$, then there are 21 parameters that need to be estimated; for $N = 3$, there are 78 parameters that need to be estimated. Thus, the estimation of VECH model is very complex. Therefore, Bollerslev, Engle, and Wooldridge (1998) develop the diagonal VECH model in order to reduce the parameters that need to be estimated. The VECH model is written as

$$h_{ij,t} = \omega_{ij} + \alpha_{ij}\varepsilon_{i,t-1}\varepsilon_{j,t-1} + b_{ij}h_{ij,t-1} \quad , \tag{B.3}$$

where ω_{ij}, α_{ij} and b_{ij} are parameters.

Later, Engle and Kroner (1995) present the BEKK model which imposes positive definiteness restrictions to ensure the H matrix being positive. The general format of the conditional covariance matrix can be represented as

$$H_t = CC' + \Sigma_{k=1}^{K}\Sigma_{i=1}^{q} A_{ik}\varepsilon_{t-i}\varepsilon'{}_{t-i}A'_{ik} + \Sigma_{k=1}^{K}\Sigma_{i=1}^{p} B_{ik}H_{t-i}B'_{ik} \quad , \tag{B.4}$$

where C is a lower triangular parameter matrix, A_{ik} and B_{ik} are $N \times N$ matrices. As long as C is definitely positive, the conditional covariance matrix is also definitely positive because the other terms in (4) are expressed in quadratic form. For example, we assume $K = 1$ and apply a GARCH(1,1) model,

$$H_t = CC' + A_{11}\varepsilon_{t-1}\varepsilon'_{t-1}A'_{11} + B_{11}H_{t-1}B'_{11} \quad . \tag{B.5}$$

In the bivariate case, the BEKK becomes

$$\begin{aligned} H_t = CC' &+ \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \varepsilon^2_{1t-1} & \varepsilon_{1t-1}\varepsilon_{2t-1} \\ \varepsilon_{2t-1}\varepsilon_{1t-1} & \varepsilon^2_{2t-1} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}' \\ &+ \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} h_{11t-1} & h_{12t-1} \\ h_{21t-1} & h_{22t-1} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}' \quad . \end{aligned} \tag{B.6}$$

The common method for estimating a multivariate GARCH model is the conditional log likelihood function, which has the form

$$L(\theta) = -\frac{TN}{2}ln2\pi - \frac{1}{2}\sum_{t=1}^{T}(ln|H_t| + \varepsilon_t H_t^{-1}\varepsilon_t) \quad , \tag{B.7}$$

where θ denotes the parameter vector, and $H_t = (\sigma_{ijt})_{N\times N}$. Numerical maximization yields the maximum likelihood estimates with asymptotic standard errors.

B.2 GARCH Model Parameters for LAIVaR

Table B.1: Estimated parameters of GARCH model

Stocks	Volume	Parameters	5 minute		10 minute	
			Ask	Bid	Ask	Bid
NR	v=2000	a_0	$2.8E-7$	$2.6E-7$	$1.8E-6$	$2.2E-6$
			($3.6E-8$)	($3.5E-8$)	($7.8E-5$)	($5.7E-5$)
		a_1	0.2367	0.2013	0.1817	0.1136
			(0.0121)	(0.0100)	(0.0218)	(0.0143)
		β_1	0.7202	0.7570	0.5255	0.5499
			(0.0193)	(0.0169)	(0.0088)	(0.0087)
	v=10000	a_0	$3.6E-7$	$4.7E-7$	$2.1E-6$	$2.3E-6$
			($4.5E-8$)	($7.5E-8$)	($7.3E-5$)	($5.8E-5$)
		a_1	0.2609	0.1585	0.0961	0.1867
			(0.0105)	(0.0131)	(0.0230)	(0.0526)
		β_1	0.6924	0.73557	0.5601	0.5200
			(0.0187)	(0.0298)	(0.0940)	(0.0701)
	v=20000	a_0	$2.6E-7$	$4.3E-7$	$2.2E-6$	$3.9E-6$
			($2.6E-8$)	($5.0E-8$)	($5.5E-5$)	($4.1E-5$)
		a_1	0.2631	0.1564	0.1641	0.1836
			(0.0087)	(0.0103)	(0.0103)	(0.0204)
		β_1	0.7148	0.7630	0.4838	0.4796
			(0.0128)	(0.0189)	(0.0082)	(0.0068)
RBS	v=10000	a_0	$7.6E-7$	$1.2E-6$	$4.7E-6$	$4.4E-6$
			($3.6E-8$)	($4.7E-8$)	($4.8E-6$)	($4.3E-6$)
		a_1	0.2706	0.4580	0.3543	0.5556
			(0.0102)	(0.0264)	(0.0016)	(0.0163)
		β_1	0.5710	0.3041	0.4001	0.0012
			(0.0168)	(0.0204)	(0.0214)	(0.0211)
	v=50000	a_0	$1.9E-7$	$1.4E-6$	$5.1E-6$	$4.4E-6$
			($1.4E-8$)	($4.6E-8$)	($1.8E-7$)	($1.6E-7$)
		a_1	0.3472	0.6126	0.5928	0.77175
			(0.0164)	(0.0226)	(0.0125)	(0.0258)
		β_1	0.5736	0.3041	0.0231	0.064
			(0.0164)	(0.0204)	(0.0168)	(0.0243)
	v=100000	a_0	$1.3E-6$	$1.5E-6$	$4.8E-6$	$5.9E-6$
			($1.9E-5$)	($4.0E-8$)	($4.8E-6$)	($5.0E-6$)
		a_1	0.4953	0.2300	0.5622	0.5763
			(0.0142)	(0.0201)	(0.0142)	(0.0136)

cont

Stocks	Volume	Parameters	5 minute		10 minute	
			Ask	Bid	Ask	Bid
		β_1	0.5046 (0.0106)	0.2318 (0.0132)	0.3017 (0.0213)	0.012 (0.0332)
HSBC	v=50000	a_0	$1.3E-7$ $(7.0E-9)$	$1.5E-7$ $(7.2E-9)$	$2.3E-7$ $(5.0E-7)$	$2.3E-7$ $(5.2E-7)$
		a_1	0.3012 (0.0168)	0.2579 (0.0157)	0.4032 (0.0213)	0.4255 (0.0221)
		β_1	0.6371 (0.0124)	0.6429 (0.0171)	0.5344 (0.0246)	0.5526 (0.0165)
	v=100000	a_0	$1.0E-7$ $(7.0E-9)$	$1.6E-7$ $(7.7E-9)$	$5.1E-7$ $(3.0E-8)$	$3.7E-7$ $(2.0E-8)$
		a_1	0.2383 (0.0146)	0.2953 (0.0152)	0.5839 (0.0323)	0.4229 (0.0234)
		β_1	0.7131 (0.0182)	0.6201 (0.0121)	0.4090 (0.0190)	0.5630 (0.0273)
	v=200000	a_0	$1.3E-7$ $(7.5E-9)$	$1.4E-7$ $(5.9E-9)$	$2.3E-7$ $(5.8E-7)$	$2.4E-7$ $(5.9E-7)$
		a_1	0.2781 (0.0164)	0.2932 (0.0126)	0.4871 (0.0244)	0.4332 (0.0216)
		β_1	0.6429 (0.0161)	0.6381 (0.0126)	0.5002 (0.0192)	0.5582 (0.0298)

B.3 Figures of Spreads

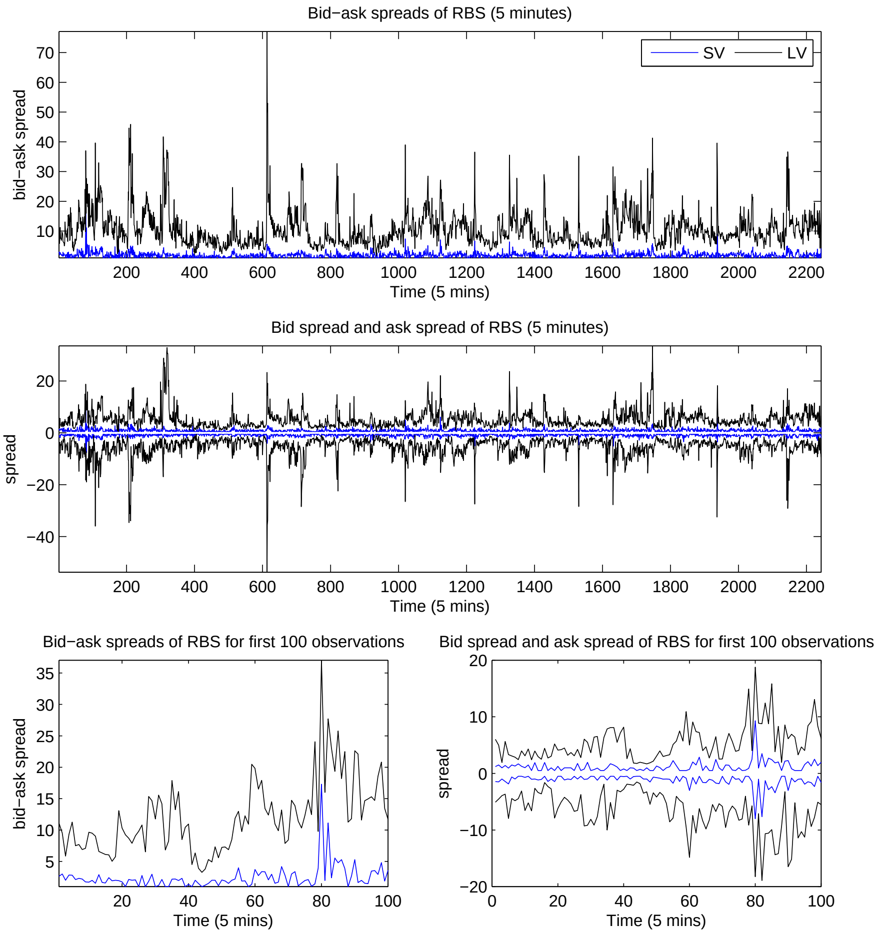


Figure B.1: The bid-ask spread and spread between bid and ask for NR (SV=small volume; LV=large volume).

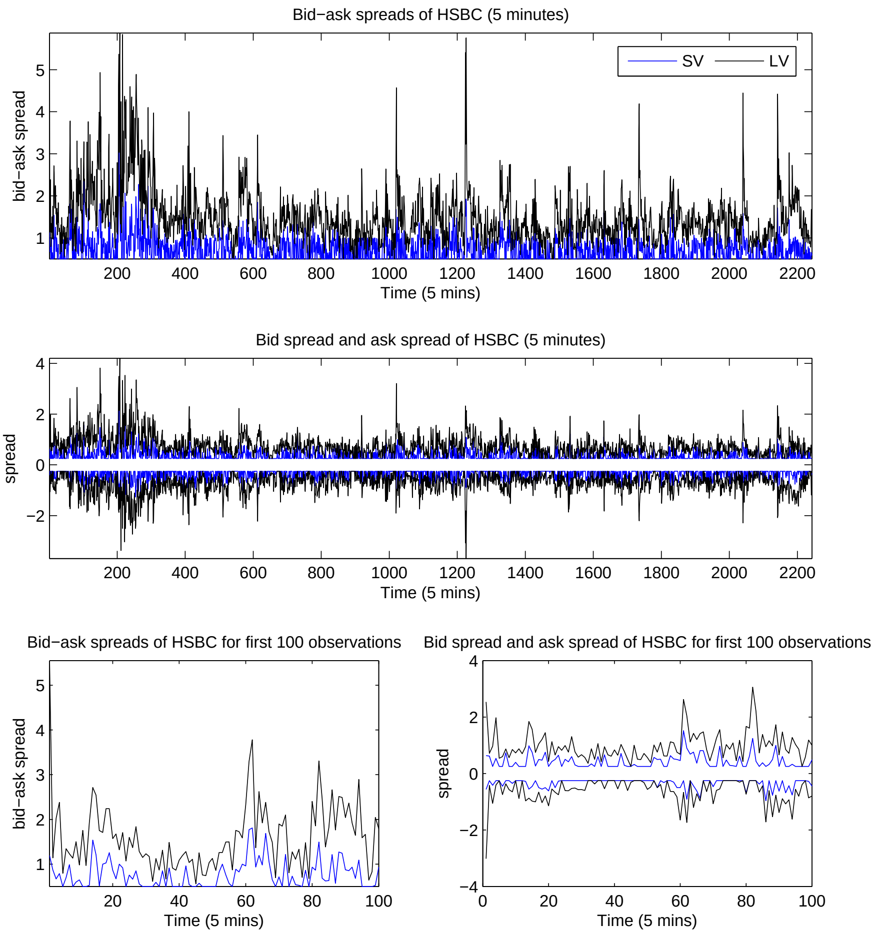
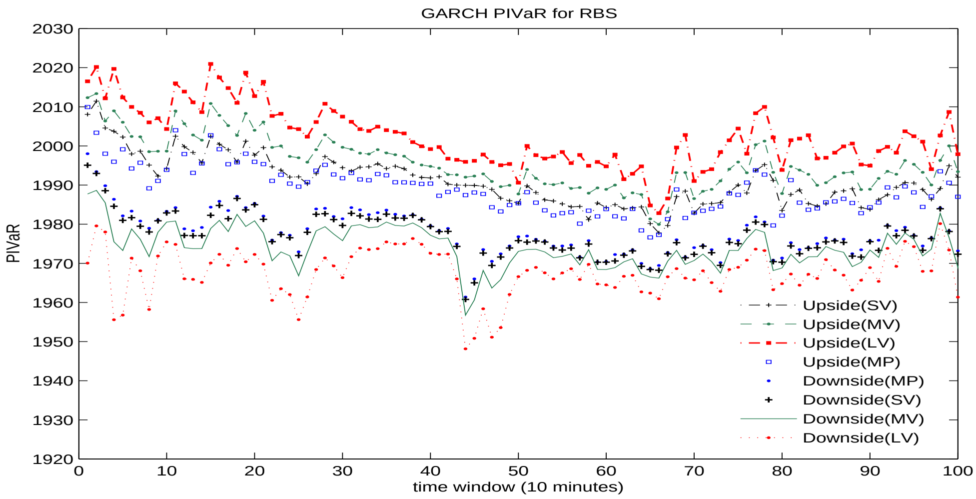


Figure B.2: The bid-ask spread and spread between bid and ask for HSBC (SV=small volume; LV=large volume).

B.4 Figures of PIVaR

Figure B.3: Sample subset of GARCH PIVaR and HS PIVaR (α=5%) for the three companies with 5 minutes sampling frequency. In the figures, upside denotes upside risk and downside denotes downside risk; SV=small volume; MV=medium volume; LV=large volume and MP= mid-price.



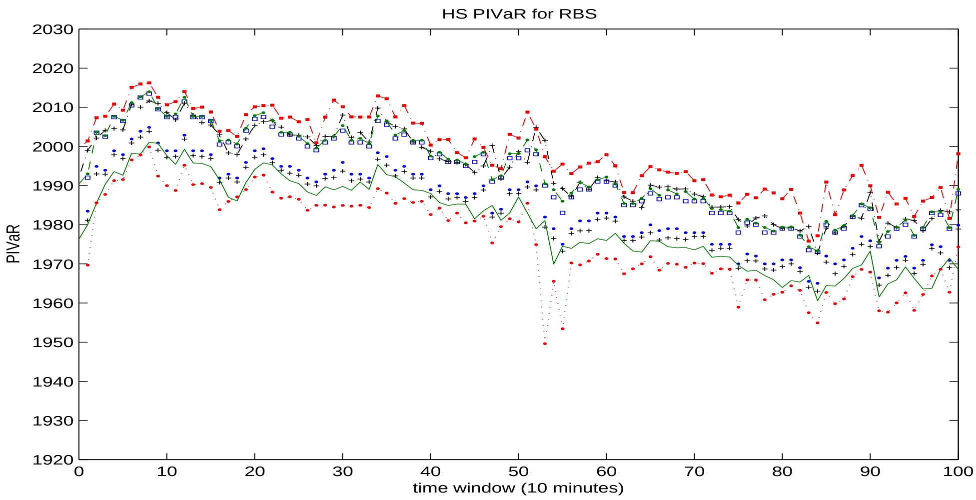


Figure B.4: Sample subset with PIVaR (α=5%) for the three companies with 10 minutes sampling frequency. In the figures, upside denotes upside risk and downside denotes downside risk; SV=small volume; MV=medium volume; LV=large volume and MP= mid-price.

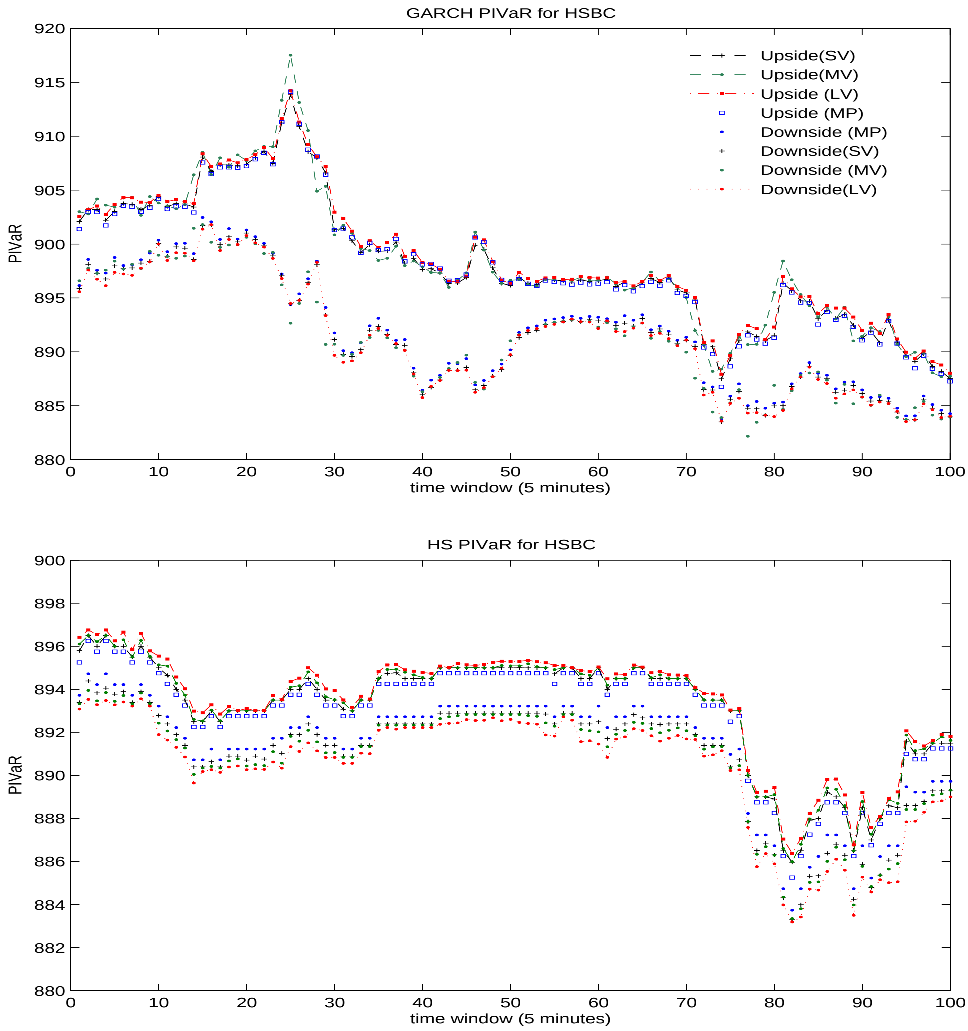
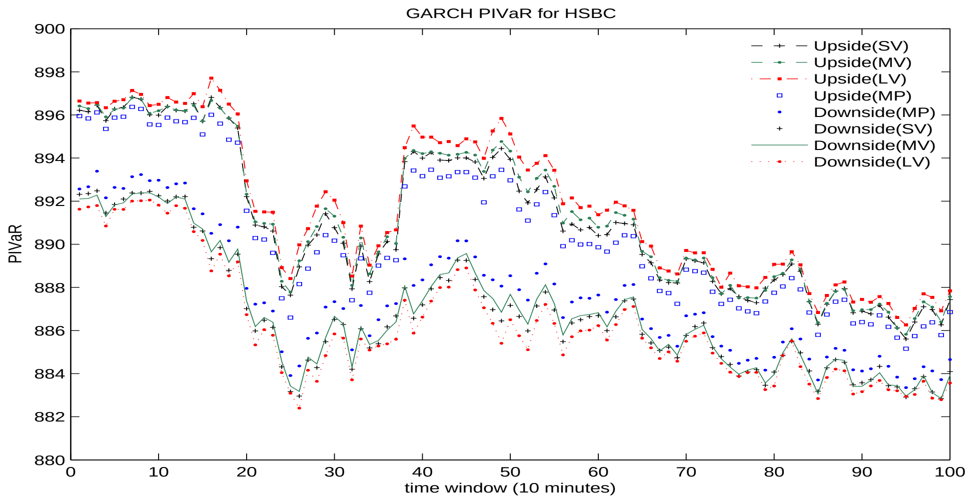


Figure B.5: Sample subset of GARCH PIVaR and HS PIVaR (α=5%) for the three companies with 5 minutes sampling frequency. In the figures, upside denotes upside risk and downside denotes downside risk; SV=small volume; MV=medium volume; LV=large volume and MP= mid-price.



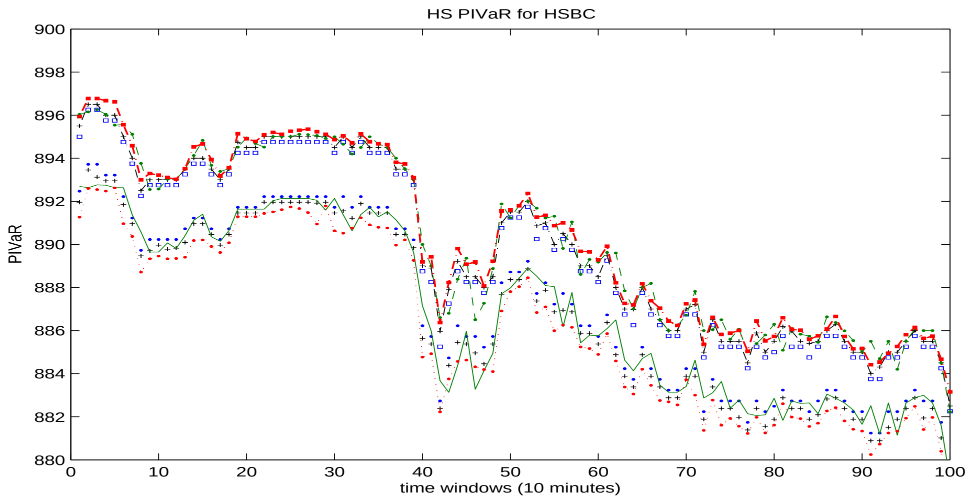


Figure B.6: Sample subset with PIVaR (α=5%) for the three companies with 10 minutes sampling frequency. In the figures, upside denotes upside risk and downside denotes downside risk; SV=small volume; MV=medium volume; LV=large volume and MP= mid-price.

B.5 Figures of Risk Adjustment

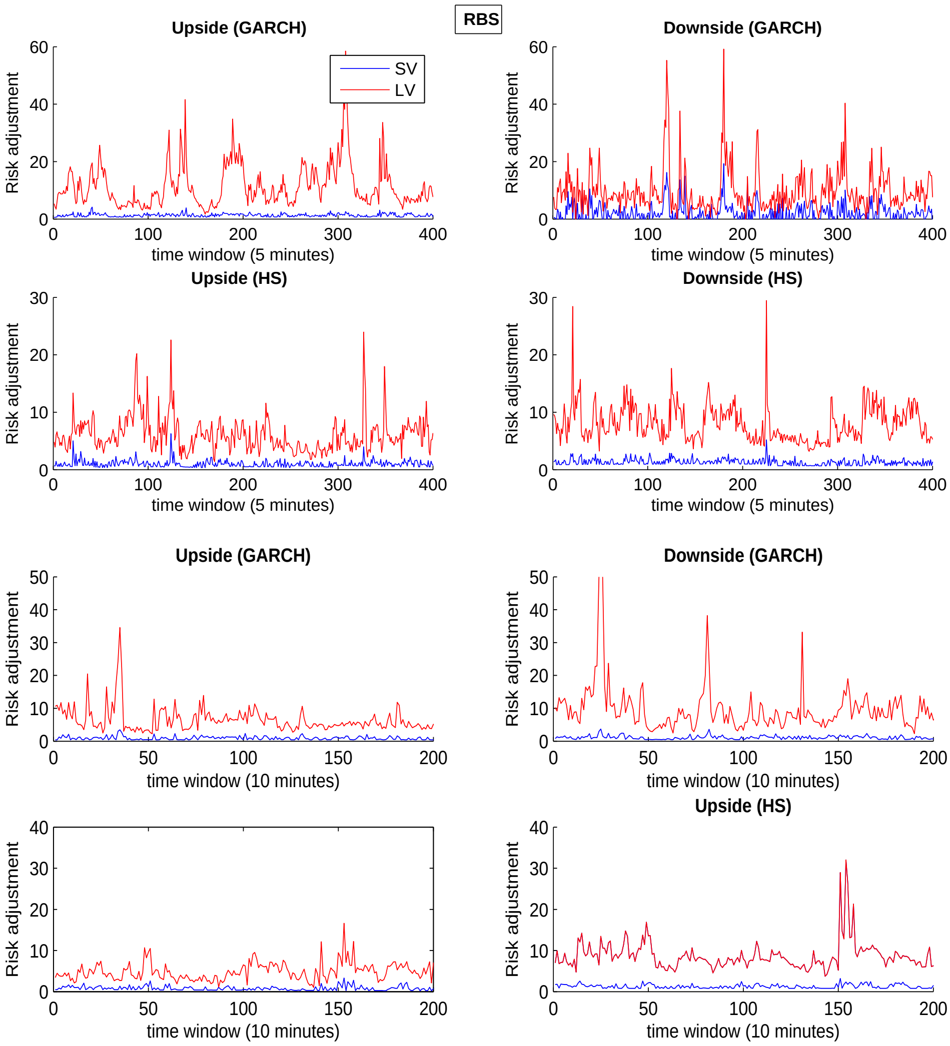


Figure B.7: Risk adjustment for RBS with 5 and 10 minutes sampling frequency. The blue line displays risk adjustment for small volume and the red line is for large volume (SV=small volume; LV=large volume). The four subplots on the left are for upside risk and other four on the right are for downside risk.

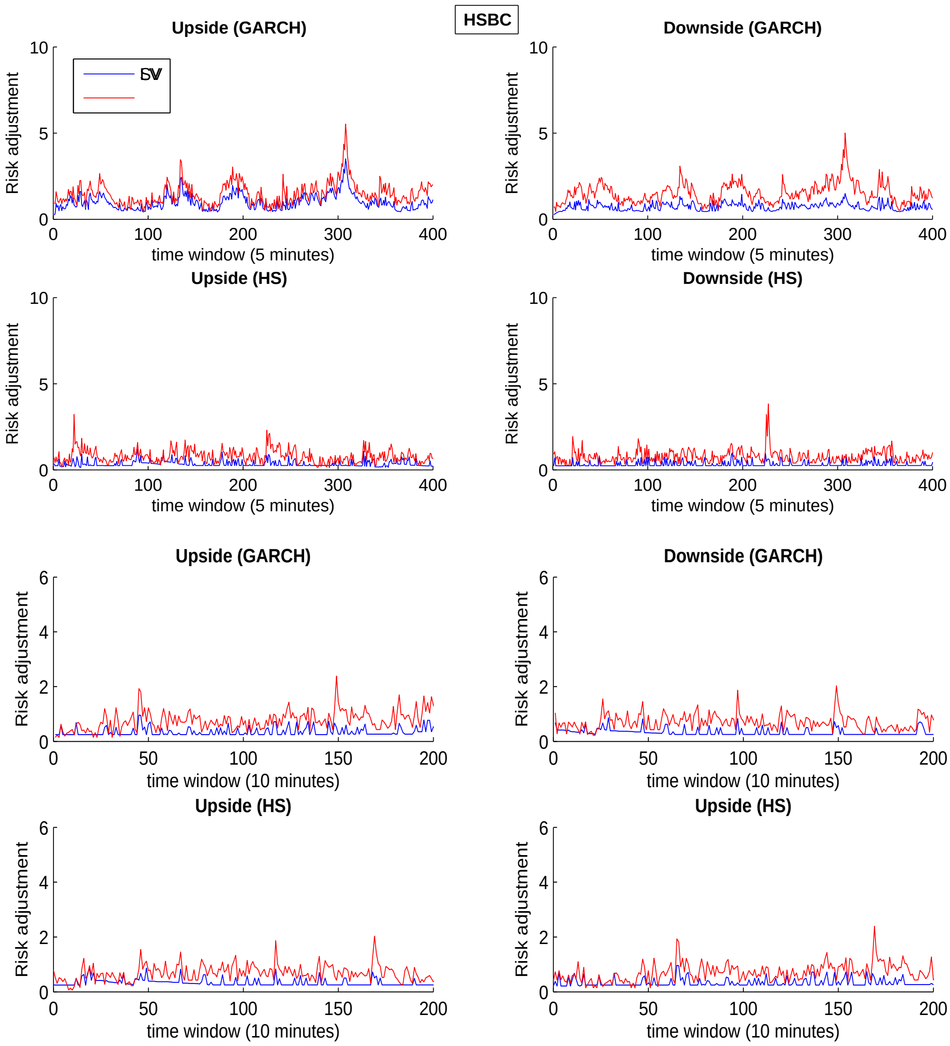


Figure B.8: Risk adjustment for HSBC with 5 and 10 minutes sampling frequency. The blue line displays risk adjustment for small volume and the red line is for large volume (SV=small volume; LV=large volume). The four subplots on the left are for upside risk and other four on the right are for downside risk.

B.6 Backtesting Results

Table B.2: Failure rates for conventional VaR, and two LAIVaR measures (with HS and GARCH models) at 5% quantile.

5 minutes	Backtesting Results (failure rates)					
Models	VaR		GARCH-LAIVaR		HS-LAIVaR	
NR	Ask	Bid	Ask	Bid	Ask	Bid
SV	0.1175	0.1225	0.0525	0.0450	0.0425	0.0475
LV	0.2000	0.2075	0.0475	0.0475	0.0575	0.0450
RBS	Ask	Bid	Ask	Bid	Ask	Bid
SV	0.1575	0.1775	0.0425	0.0475	0.0525	0.0575
LV	0.2950	0.2975	0.0475	0.0450	0.0500	0.0525
HSBC	Ask	Bid	Ask	Bid	Ask	Bid
SV	0.0850	0.8750	0.0525	0.0450	0.0425	0.0475
LV	0.1475	0.1525	0.0475	0.0475	0.0575	0.0450
10 minutes						
NR	Ask	Bid	Ask	Bid	Ask	Bid
SV	0.1450	0.1550	0.0450	0.0550	0.0450	0.0450
LV	0.1850	0.1950	0.0450	0.0500	0.0550	0.0600
RBS	Ask	Bid	Ask	Bid	Ask	Bid
SV	0.1300	0.1450	0.0500	0.0450	0.0450	0.0600
LV	0.1650	0.1800	0.0550	0.0550	0.0500	0.0550
HSBC	Ask	Bid	Ask	Bid	Ask	Bid
SV	0.0750	0.0900	0.0350	0.0450	0.0450	0.0350
LV	0.1150	0.1300	0.0450	0.0550	0.0500	0.0450

Failure rates for conventional VaR, and two LAIVaR measures (with HS and GARCH models). Failure rate is the proportion of VaR violations of the return which equals V/N, where V is the aggregated violation of stock and N is the size of sample. If the VaR model is accurate or has good forecasting performance, the failure rate should be equal to the present VaR level ($a = 5\%$).

B.7 Figures of Correlation

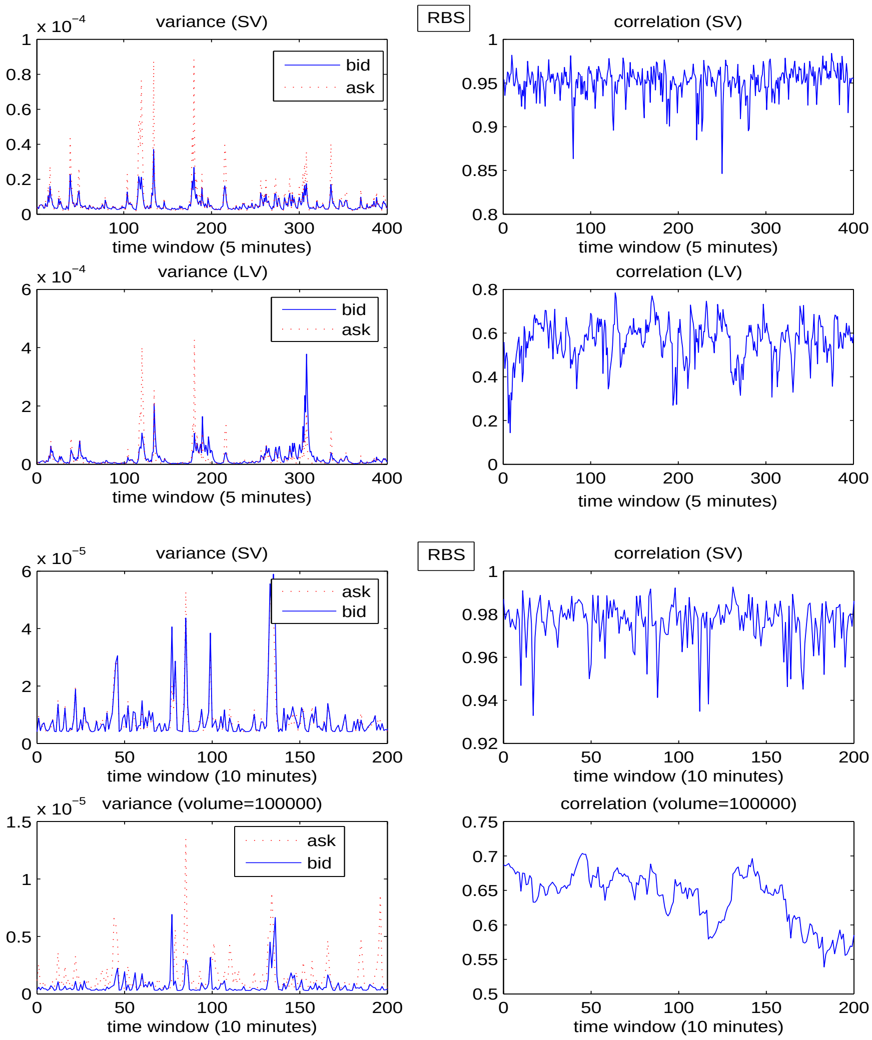


Figure B.9: Variance and correlation for RBS with different volume sizes (SV=small trading volume, LV=large trading volume)

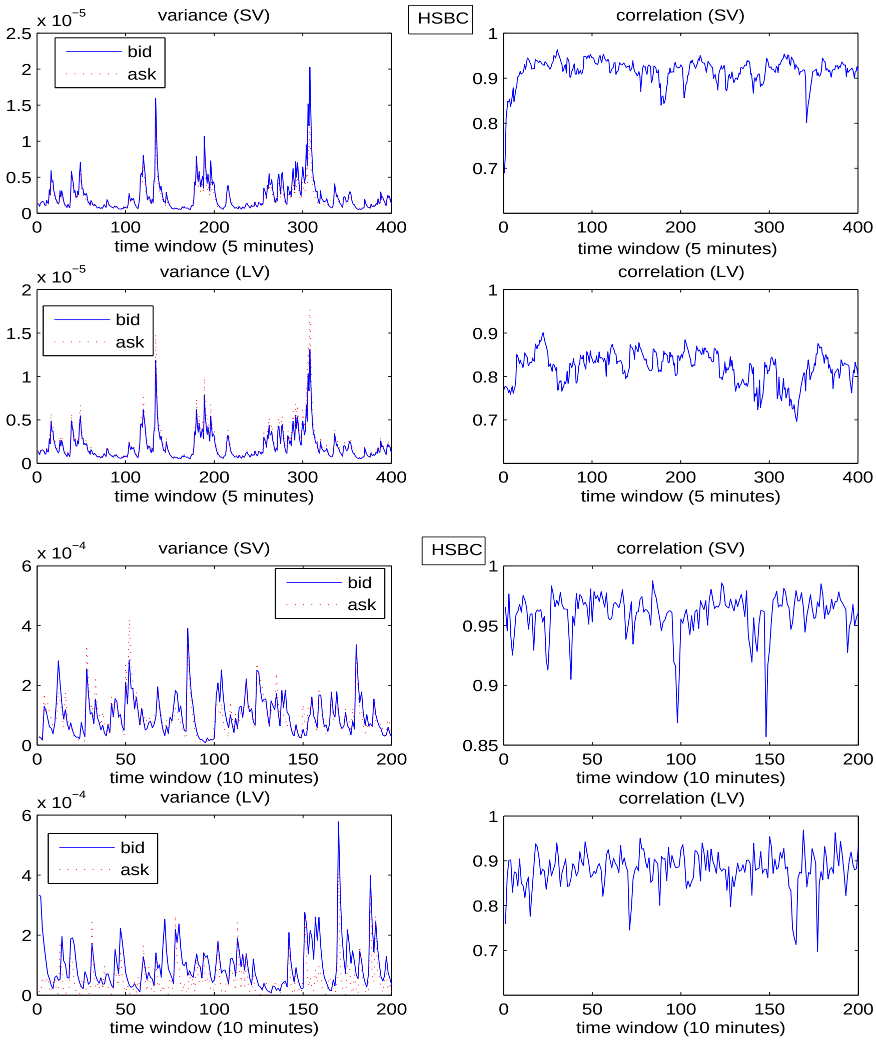


Figure B.10: Variance and correlation for HSBC with different volume sizes (SV=small trading volume, LV=large trading volume)

Appendix C

C.1 The Figure Outputs for MPC Scaling Law

C.2 Sample Moments of Forecast Errors

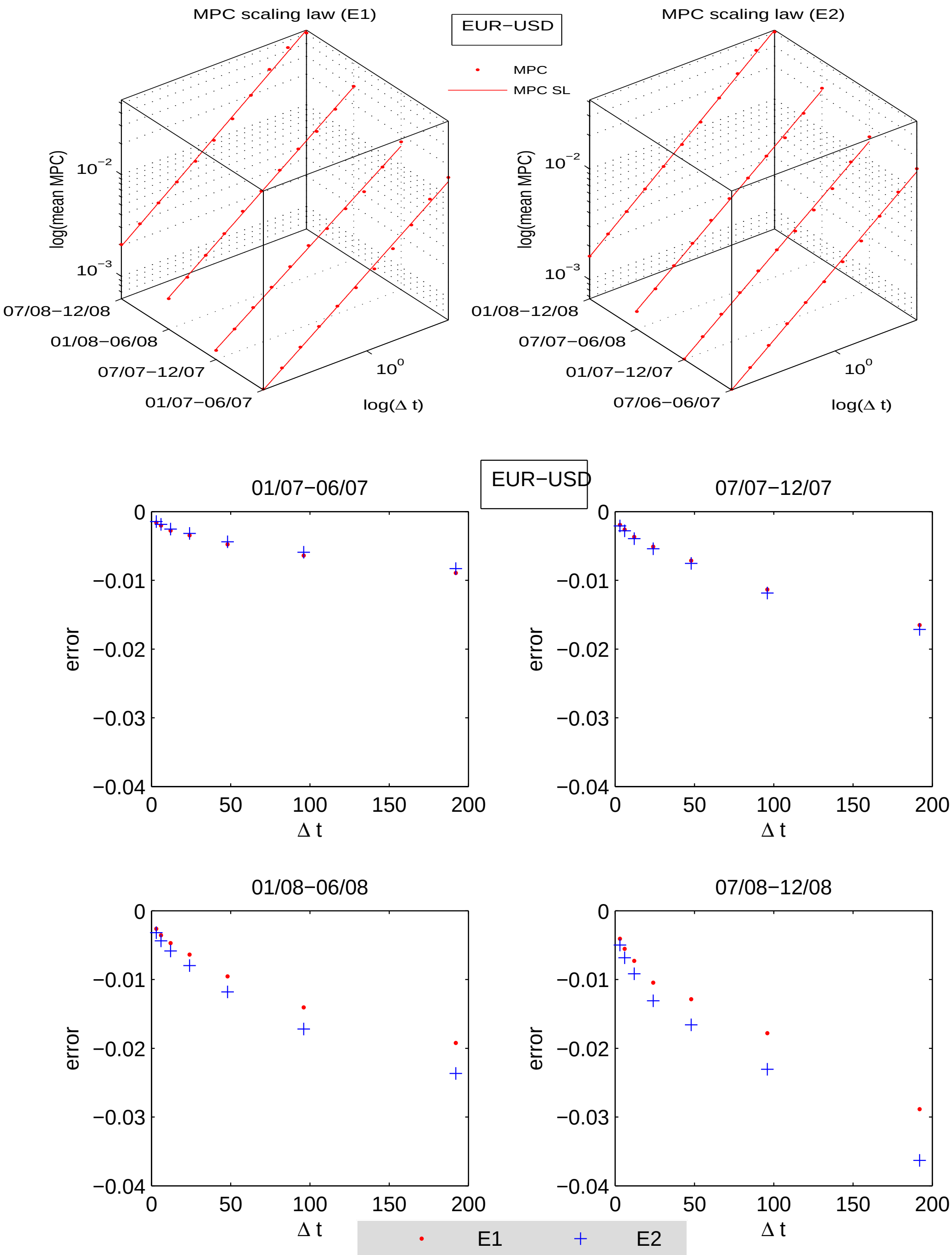


Figure C.1: The scaling law plot and error plot of out-of-sample data for EUR-USD. Note: E1: experiment 1 (6 month in-sample data); E2: experiment 2 (1 year in-sample data).

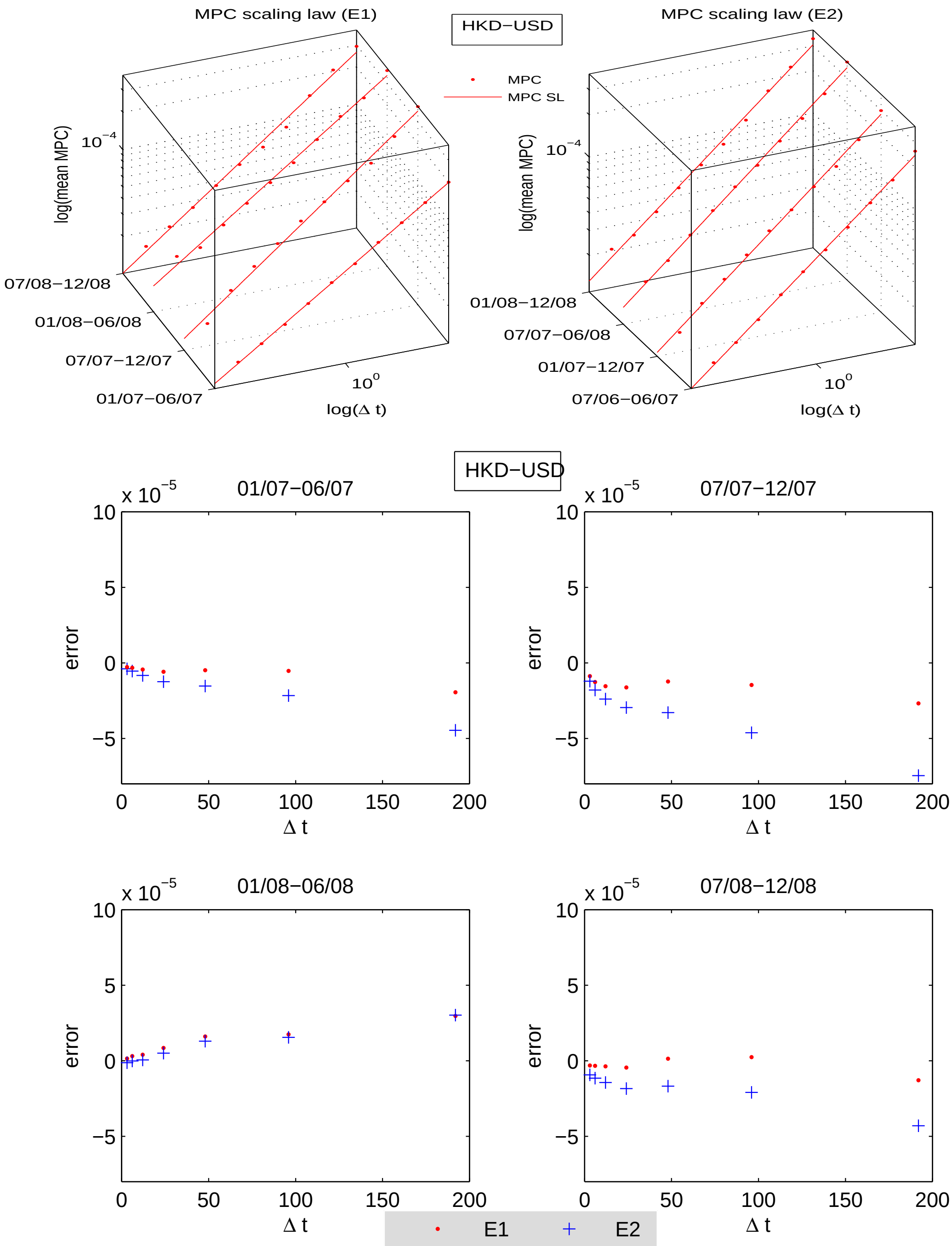


Figure C.2: The scaling law plot and prediction error plot of out-of-sample data for HKD-USD. Note: E1: experiment 1 (6 month in-sample data); E2: experiment 2 (1 year in-sample data).

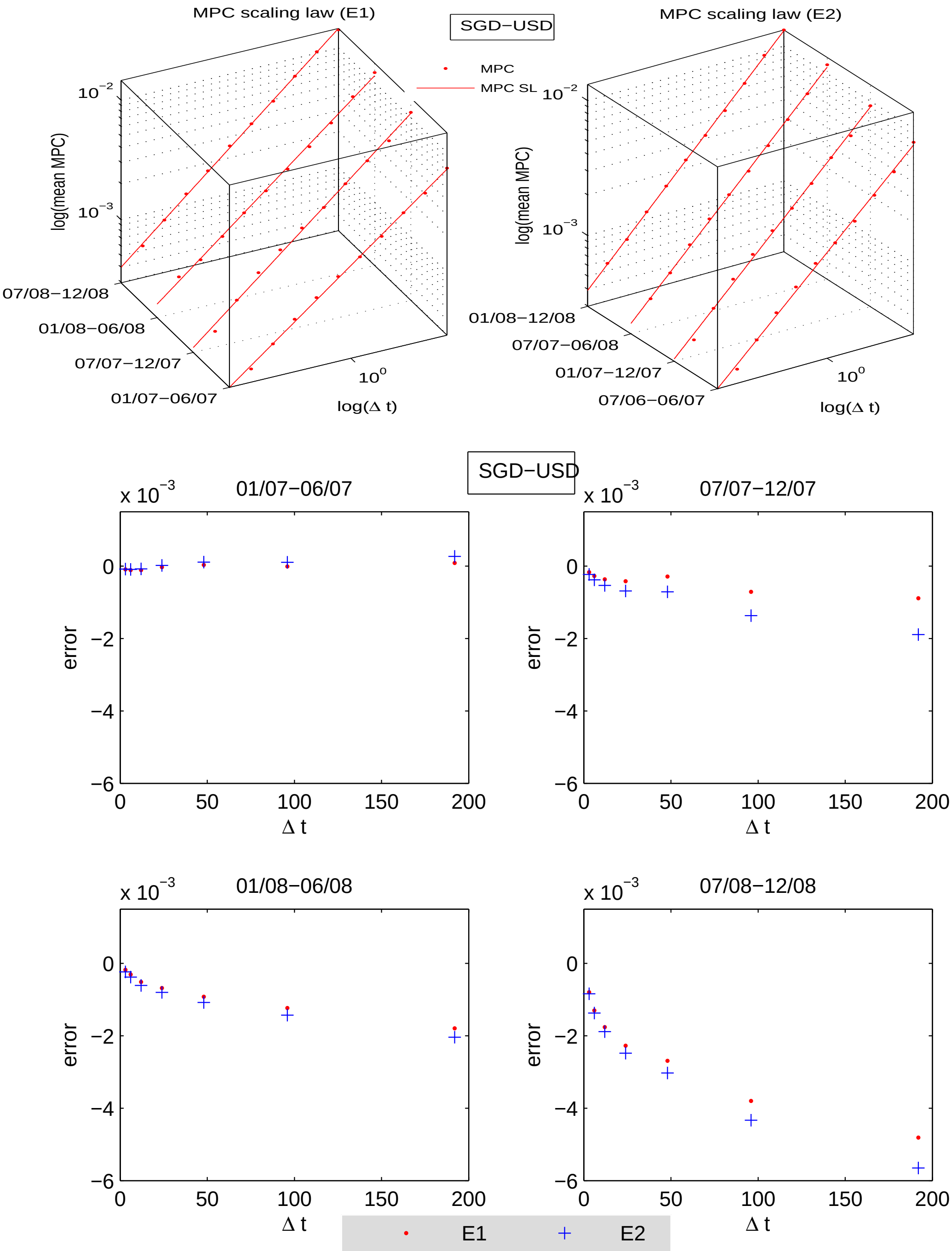


Figure C.3: The scaling law plot and prediction error plot of out-of-sample data for SGD-USD. Note: E1: experiment 1 (6 month in-sample data); E2: experiment 2 (1 year in-sample data).

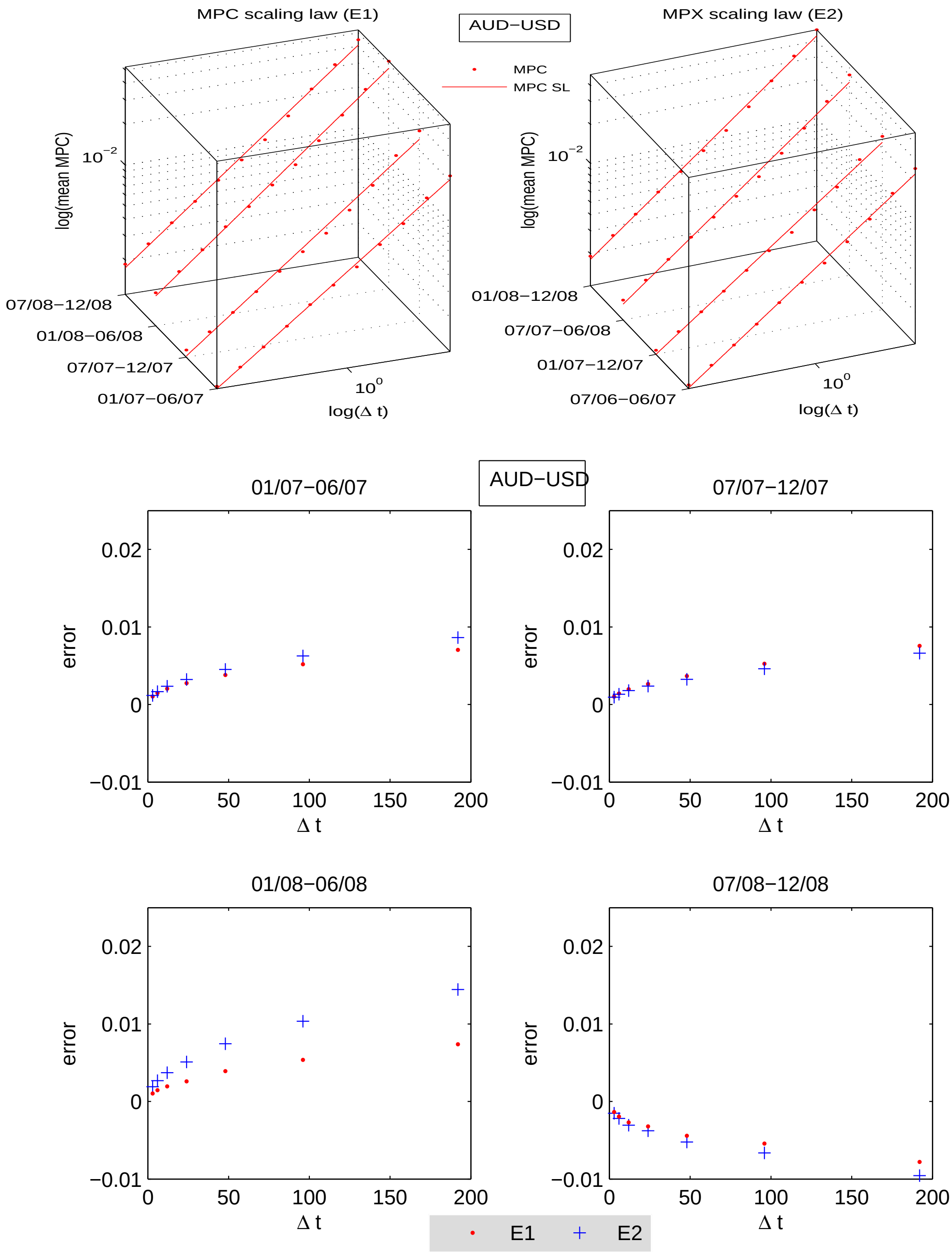


Figure C.4: The scaling law plot and prediction error plot of out-of-sample data for AUR-USD. Note: E1: experiment 1 (6 month in-sample data); E2: experiment 2 (1 year in-sample data).

Table C.1: Sample moments of the forecast errors for the VaR and scaling laws models A

	EUR-USD				AUD-EUR				SGD-USD			
Standard methods	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
V1: VaR (5%)	-0.0207	0.0227	1.2733	4.7533	-0.0081	0.0143	3.2581	16.2312	-0.0046	0.0051	0.9274	4.1676
V2: ETL (5%)	-0.0542	0.0080	0.4187	3.2199	-0.0282	0.0078	-1.2760	3.2583	-0.0139	0.0026	-0.8931	3.1808
V3: VaR (1%)	-0.0443	0.0235	1.2884	4.7530	-0.0184	0.0162	2.7086	14.8801	-0.0115	0.0059	0.4468	4.6530
V4: ETL (1%)	-0.0699	0.0099	-0.5934	4.4132	-0.0320	0.0114	-1.0552	2.6780	-0.0192	0.0040	-1.5624	4.4020
1 year	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	-0.0142	0.0228	1.1147	4.4674	-0.0075	0.0136	3.2921	16.7346	-0.0037	0.0053	1.1005	4.1893
L1: NMPC	-0.0048	0.0227	1.1469	4.4771	-0.0041	0.0135	3.2935	16.7776	-0.0010	0.0053	1.1157	4.2240
L2: EMAMPC	-0.0037	0.0227	1.3613	4.9643	-0.0046	0.0137	3.2974	16.8795	-0.0018	0.0052	0.8979	4.1867
L3a: ETLMPC (50%)	-0.0169	0.0234	0.7347	3.8628	-0.0109	0.0134	3.2588	16.8159	-0.0091	0.0056	-0.0443	3.6809
L3b: ETLMPC (10%)	-0.0381	0.0264	-0.1120	3.1835	- 0.0282	0.0147	2.1512	11.9876	-0.0145	0.0063	-0.4947	3.3870
6 month	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	-0.0298	0.0274	-0.2766	3.2112	-0.0082	0.0145	3.1792	16.2850	-0.0063	0.0054	0.9920	4.2401
L1: NMPC	-0.0213	0.0284	-0.4627	3.0587	-0.0049	0.0144	3.1473	16.1578	-0.0041	0.0054	0.8003	4.1982
L2: EMAMPC	-0.0251	0.0308	-0.6727	2.9690	-0.0055	0.0142	2.8491	15.1448	-0.0038	0.0055	0.6314	4.1088
L3a: ETLMPC (50%)	-0.0399	0.0358	-0.9024	2.9502	-0.0114	0.0151	2.7691	14.4296	-0.0129	0.0072	-0.1412	2.9724
L3b: ETLMPC (10%)	-0.0749	0.0558	-0.9183	2.4541	-0.0271	0.0205	0.7160	6.4843	-0.0182	0.0080	-0.2242	2.7876
3 month	Mean	Std	Sskewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	-0.0304	0.0261	0.4314	3.8657	-0.0103	0.0153	2.0094	11.5081	-0.0059	0.0058	0.4403	4.3028
L1: NMPC	-0.0217	0.0260	0.3041	3.6495	-0.0072	0.0147	2.0942	11.9389	-0.0034	0.0057	0.3274	4.0155
L2: EMAMPC	-0.0239	0.0266	0.1317	3.6062	-0.0075	0.0144	1.8461	11.2125	-0.0037	0.0055	0.4104	3.9639
L3a: ETLMPC (50%)	-0.0375	0.0292	-0.1707	3.4069	-0.0137	0.0164	1.0055	7.6502	-0.0120	0.0078	-0.6767	3.4803

cont

In-Sample Size	EUR-USD				AUD-EUR				SGD-USD			
VaR	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L3b: ETLMPC (10%)	-0.0640	0.0380	-0.7254	3.0580	-0.0249	0.0258	-1.2423	4.5572	-0.0148	0.0076	-0.4986	3.3035
2 month	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	-0.0277	0.0263	0.3043	3.2957	-0.0109	0.0157	1.3839	9.9275	-0.0063	0.0060	0.5948	4.3806
L1: NMPC	-0.0183	0.0253	0.2118	3.3467	-0.0075	0.0149	1.5454	10.3821	-0.0035	0.0059	0.4506	4.1099
L2: EMAMPC	-0.0186	0.0241	0.2632	3.3783	-0.0077	0.0142	1.2605	9.8513	-0.0033	0.0057	0.5272	4.0232
L3a: ETLMPC (50%)	-0.0355	0.0278	-0.1263	3.1494	-0.0163	0.0186	- 0.1509	5.6497	-0.0115	0.0083	-0.3274	3.0250
L3b: ETLMPC (10%)	-0.0581	0.0302	-0.7138	3.2662	-0.0249	0.0258	-1.2423	4.5572	-0.0176	0.0083	-0.2184	2.7686
1 month	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	-0.0293	0.0255	0.2449	3.3726	-0.0113	0.0159	0.3111	8.5056	-0.0070	0.0059	0.3418	4.6267
L1:NMPC	-0.0187	0.0244	0.3385	3.3058	-0.0079	0.0152	0.4049	8.7743	-0.0033	0.0058	0.3241	3.5710
L2:EMAMPC	-0.0185	0.0242	0.2618	3.4404	-0.0078	0.0148	0.2411	9.2701	-0.0029	0.0056	0.2087	3.5165
L3a:ETLMPC (50%)	-0.0262	0.0247	0.1092	3.1189	-0.0156	0.0192	-1.4774	8.1234	-0.0097	0.0066	-0.2153	3.0547
L3b:ETLMPC (10%)	-0.0395	0.0279	-0.1792	3.0089	-0.0328	0.0310	- 2.3354	8.7149	-0.0121	0.0070	-0.0928	2.6546

Table C.2: Sample moments of the absolute forecast errors for the VaR and scaling laws models B

In-Sample Size	HKD-USD				AUD-USD			
VaR	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
V1: VaR (5%)	-1.8E-4	5.7E-5	0.9578	4.5770	-0.0167	0.0274	2.4674	10.8171
V2: ETL (5%)	-3.3E-4	3.8E-5	0.8523	2.0780	-0.0584	0.0100	-0.8805	2.8885
V3: VaR (1%)	-4.5E-4	9.3E-5	0.7236	3.3802	-0.0671	0.0414	0.9581	5.8566
V4: ETL (1%)	-5.5E-4	7.7E-5	1.0144	2.1952	-0.1035	0.0304	-0.4971	2.3474
1 year	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	-1.6E-4	5.4E-5	1.3003	5.3284	-0.0119	0.0263	2.4118	10.7295
L1: NMPC	-1.1E-4	5.4E-5	1.2815	5.1670	-0.0045	0.0260	2.3978	10.7131
L2: EMAMPC	-1.0E-4	5.3E-5	1.1473	4.9790	-0.0033	0.0259	2.1557	10.0309
L3a: ETLMPC (50%)	-1.9E-4	5.4E-5	1.2513	5.1430	-0.0170	0.0262	2.2130	10.1745
L3b: ETLMPC (10%)	-3.3E-4	5.6E-5	0.9645	4.2592	-0.0484	0.0297	1.1522	7.1199
6 month	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	-1.6E-4	6.2E-5	0.5818	4.3775	-0.0150	0.0283	1.9631	9.0626
L1: NMPC	-1.1E-4	5.7E-5	0.9453	4.8996	-0.0081	0.0279	1.9552	9.1493
L2: EMAMPC	-1.4E-4	6.5E-5	0.5222	3.9902	-0.087	0.0274	1.6007	8.3464
L3a: ETLMPC (50%)	-1.8E-4	6.5E-5	0.4924	4.2881	-0.0216	0.0289	1.5063	7.8157
L3b: ETLMPC (10%)	-2.0E-4	8.2E-5	0.2028	3.6194	-0.0150	0.0283	1.9631	9.0626
3 month	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	-1.5E-4	6.5E-5	0.4401	3.0560	-0.0194	0.0296	0.9182	6.4987
L1: NMPC	-1.0E-4	6.0E-5	0.4708	4.1157	-0.0129	0.0287	0.9316	6.5939
L2: EMAMPC	-0.9E-4	6.0E-5	0.4154	3.4231	-0.0139	0.0268	0.8897	6.9165
L3a: ETLMPC (50%)	-1.6E-4	6.6E-5	0.2454	3.6464	-0.0262	0.0316	0.1765	4.9851

cont

In-Sample Size	HKD-USD				AUD-USD			
VaR	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L3b: ETLMPC (q=0.1)	-2.7E-4	9.9E-5	-0.1039	2.5175	-0.0544	0.0449	-0.8561	3.5662
2 month	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	-1.4E-4	8.2E-5	-0.1268	2.3636	-0.0208	0.0297	0.7559	6.3380
L1: NMPC	-0.9E-4	6.6E-5	0.6587	3.3827	-0.0141	0.0286	0.8002	6.4433
L2: EMAMPC	-0.9E-4	6.5E-5	0.4970	3.2705	-0.0143	0.0269	0.7301	6.8436
L3a: ETLMPC (50%)	-1.5E-4	7.4E-5	0.2433	2.5218	-0.0269	0.0313	-0.1113	4.8667
L3b: ETLMPC (10%)	-2.1E-4	1.0E-4	-0.1953	2.2214	-0.0493	0.0427	-0.9127	4.0071
1 month	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	-1.3E-4	8.6E-5	-0.2141	2.3079	-0.0217	0.0278	0.3826	6.0926
L1: NMPC	-0.8E-4	6.1E-5	0.2410	2.7950	-0.0145	0.0276	0.4260	6.5838
L2: EMAMPC	-0.7E-4	6.2E-5	-0.0295	2.8746	-0.0144	0.0264	0.2605	7.2373
L3a: ETLMPC (50%)	-1.3E-4	7.6E-5	-0.0046	2.2577	-0.0259	0.0283	-0.7851	5.8344
L3b: ETLMPC (10%)	-1.9E-4	1.0E-4	-0.4600	2.3211	-0.0413	0.0357	-1.4869	5.8317

Table C.3: Sample moments of the relative forecast errors for the VaR and scaling law models A

In-Sample Size	EUR-USD				AUD-EUR				SGD-USD			
VaR	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
V1: VaR (5%)	0.0194	0.0347	-2.3269	10.5824	0.0137	0.0255	-3.2909	16.4266	0.0066	0.0074	-0.7730	4.0225
V2: ETL (5%)	0.0458	0.0397	-1.4544	8.1986	0.0389	0.0293	-2.2981	13.2113	0.0125	0.0081	-0.5070	4.2467
V3: VaR (1%)	0.0835	0.0625	0.1108	4.3058	0.0705	0.0485	0.2032	4.7153	0.0165	0.0088	-0.1327	4.5833
V4: ETL (1%)	0.1022	0.0647	-0.0840	4.2778	0.0929	0.0535	0.3013	4.1521	0.0201	0.0095	0.1785	4.5176
1 year	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	0.0138	0.0337	-2.2078	10.2510	0.0128	0.0244	-3.2623	16.6805	0.0052	0.0076	-1.0131	4.1100
L1: NMPC	0.0052	0.0333	-2.2566	10.3735	0.0069	0.0243	-3.2660	16.6779	0.0014	0.0075	-1.0463	4.1309
L2: EMAMPC	0.0092	0.0338	-1.8076	9.2759	0.0078	0.0245	-3.2532	16.7185	0.0026	0.0075	-0.7822	4.0734
L3a: ETLMPC (50%)	0.0204	0.0343	-1.7551	9.1410	0.0188	0.0242	-3.1202	16.3628	0.0130	0.0083	0.2598	3.6875
L3b: ETLMPC (10%)	0.0595	0.0439	0.0132	5.2838	0.0495	0.0283	-1.3649	9.5974	0.0207	0.0095	0.6794	3.4456
6 month	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	0.0184	0.0373	-1.5060	8.2222	0.0141	0.0260	-3.0816	15.9628	0.0069	0.0078	-0.8615	4.1141
L1: NMPC	0.0103	0.0365	-1.5736	8.4440	0.0085	0.0259	-3.0677	15.8918	0.0031	0.0079	-0.6303	4.1093
L2: EMAMPC	0.0142	0.0369	-1.0226	7.3398	0.0096	0.0261	-2.7690	14.7246	0.0041	0.0080	-0.4263	4.0245
L3a: ETLMPC (50%)	0.0271	0.0395	-0.7689	6.5730	0.0201	0.0276	-2.4778	13.4589	0.0106	0.0088	0.1815	4.0096
L3b: ETLMPC (10%)	0.0643	0.0564	0.7545	4.0107	0.0484	0.0400	-0.1761	5.5375	0.0154	0.0090	0.1823	3.8379
3 month	Mean	Std	Sskewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	0.0253	0.0414	-0.1805	5.5741	0.0183	0.0283	-1.6495	10.6041	0.0085	0.0085	-0.2438	4.2050
L1: NMPC	0.0176	0.0396	-0.2774	5.8094	0.0129	0.0266	-1.6665	10.7422	0.0049	0.0082	-0.1474	3.9414

cont

In-Sample Size	EUR-USD				AUD-EUR				SGD-USD			
VaR	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L2: EMAMPC	0.0189	0.0373	-0.1765	6.0734	0.0136	0.0262	-1.3209	9.8899	0.0054	0.0080	-0.2273	3.8593
L3a: ETLMPC (50%)	0.0345	0.0462	0.5198	4.4442	0.0245	0.0309	-0.6006	6.9424	0.0172	0.0117	0.8312	3.5634
L3b: ETLMPC (10%)	0.0713	0.0708	1.2270	3.5866	0.0540	0.0539	1.0773	3.4401	0.0212	0.0115	0.6728	3.3691
2 month	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	0.0274	0.0423	0.0220	5.4816	0.0197	0.0295	-0.9419	8.9387	0.0091	0.0087	-0.3810	4.2170
L1: NMPC	0.0193	0.0400	-0.1015	5.7601	0.0137	0.0272	-1.1529	9.5178	0.0055	0.0085	-0.2668	3.9774
L2: EMAMPC	0.0195	0.0376	0.0073	6.2774	0.0140	0.0258	-0.7728	9.0506	0.0056	0.0082	-0.3457	3.8921
L3a: ETLMPC (50%)	0.0357	0.0466	0.7813	4.5775	0.0296	0.0361	0.5109	5.3802	0.0166	0.0124	0.4979	3.0744
L3b: ETLMPC (10%)	0.0648	0.0672	1.3180	4.1729	0.0456	0.0513	1.4163	4.7060	0.0252	0.0125	0.3769	2.8039
1 month	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	0.0283	0.0397	0.4472	6.9828	0.0206	0.0303	0.1615	8.2887	0.0100	0.0086	-0.1382	4.5083
L1: NMPC	0.0186	0.0383	0.3480	7.6640	0.0146	0.0288	0.0344	8.6037	0.0059	0.0084	-0.1675	3.5912
L2: EMAMPC	0.0183	0.0367	0.5168	8.3293	0.0146	0.0281	0.2336	9.2395	0.0059	0.0081	-0.0654	3.5426
L3a: ETLMPC (50%)	0.0340	0.0422	1.4572	7.4915	0.0286	0.0377	1.8018	8.6864	0.0139	0.0097	0.3729	3.1726
L3b: ETLMPC (10%)	0.0537	0.0553	1.9205	7.7158	0.0597	0.0621	2.4385	9.0879	0.0173	0.0104	0.2230	2.7562

Table C.4: Sample moments of the relative forecast errors for the VaR and scaling law models B

In-Sample Size	EUR-USD				HKD-USD			
1 year	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
V1: VaR (5%)	0.0143	0.0161	-1.1129	4.6982	0.0014	0.0005	-1.5033	8.4492
V2: ETL (5%)	0.0233	0.0165	-0.9931	4.5047	0.0023	0.0005	-0.9563	5.0519
V3: VaR (1%)	0.0303	0.0169	-0.9813	4.4604	0.0035	0.0007	-0.7085	3.3173
V4: ETL (1%)	0.0341	0.0174	-0.5740	4.1552	0.0040	0.0006	-0.7182	3.4563
1 year	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	0.0099	0.0163	-0.9333	4.4012	0.0013	0.0004	-1.2990	5.3295
L1: NMPC	0.0023	0.0158	-1.3541	5.1383	0.0009	0.0004	-1.2808	5.1663
L2: EMAMPC	0.0031	0.0162	-1.0708	4.5914	0.0008	0.0004	-1.1426	4.9674
L3a: ETLMPC (50%)	0.0119	0.0169	-0.4609	3.7779	0.0015	0.0004	-1.2501	5.1457
L3b: ETLMPC (10%)	0.0267	0.0202	0.5054	3.2615	0.0026	0.0004	-0.9630	4.2548
6 months	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	0.0212	0.0208	0.5953	3.2724	0.0013	0.0005	-0.5730	4.3692
L1: NMPC	0.0170	0.0237	0.8903	3.2993	0.0009	0.0005	-0.9375	4.8785
L2: EMAMPC	0.0143	0.0215	0.6912	3.2933	0.0012	0.0005	-0.5141	3.9770
L3a: ETLMPC (50%)	0.0287	0.0279	1.0825	3.1089	0.0014	0.0005	-0.4828	4.2706
L3b: ETLMPC (10%)	0.0536	0.0441	1.0316	2.6168	0.0023	0.0006	-0.1920	3.6163
3 months	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	0.0213	0.0193	-0.1255	3.6098	0.0012	0.0005	-0.4438	3.0683

cont

In-Sample Size	EUR-USD				HKD-USD			
1 year	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L1: NMPC	0.0164	0.0184	0.3906	3.0348	0.0008	0.0005	-0.4634	4.1147
L2: EMAMPC	0.0146	0.0178	0.1823	3.0321	0.0007	0.0005	-0.4113	3.4212
L3a: ETLMPC (50%)	0.0263	0.0217	0.3440	3.1084	0.0013	0.0005	-0.2379	3.6618
L3b: ETLMPC (10%)	0.0446	0.0284	0.7364	2.6685	0.0021	0.0008	0.1077	2.5439
2 months	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	0.0195	0.0196	0.1003	3.2767	0.0011	0.0006	0.1166	2.3519
L1: NMPC	0.0132	0.0187	0.1579	3.3755	0.0007	0.0005	-0.6580	3.3779
L2: EMAMPC	0.0133	0.0178	0.1179	3.4142	0.0007	0.0005	-0.4963	3.2629
L3a: ETLMPC (50%)	0.0251	0.0213	0.5386	3.2639	0.0012	0.0006	-0.2480	2.5208
L3b: ETLMPC (10%)	0.0407	0.0241	1.0172	3.4160	0.0017	0.0008	0.1848	2.2104
1 month	Mean	Std	Skewness	Kurtosis	Mean	Std	Skewness	Kurtosis
L: MPC	0.0206	0.0188	0.1008	3.2746	0.0010	0.0007	0.2135	2.3028
L1: NMPC	0.0133	0.0178	-0.0183	3.3666	0.0006	0.0005	-0.2395	2.7859
L2: EMAMPC	0.0132	0.0176	0.0553	3.5388	0.0006	0.0005	0.0314	2.8670
L3a: ETLMPC (50%)	0.0239	0.0198	0.4269	3.3844	0.0010	0.0006	0.0052	2.2534
L3b: ETLMPC (10%)	0.0421	0.0247	1.2439	4.6305	0.0015	0.0008	0.4587	2.3198

C.3 The Estimated MPC Scaling Law Parameters

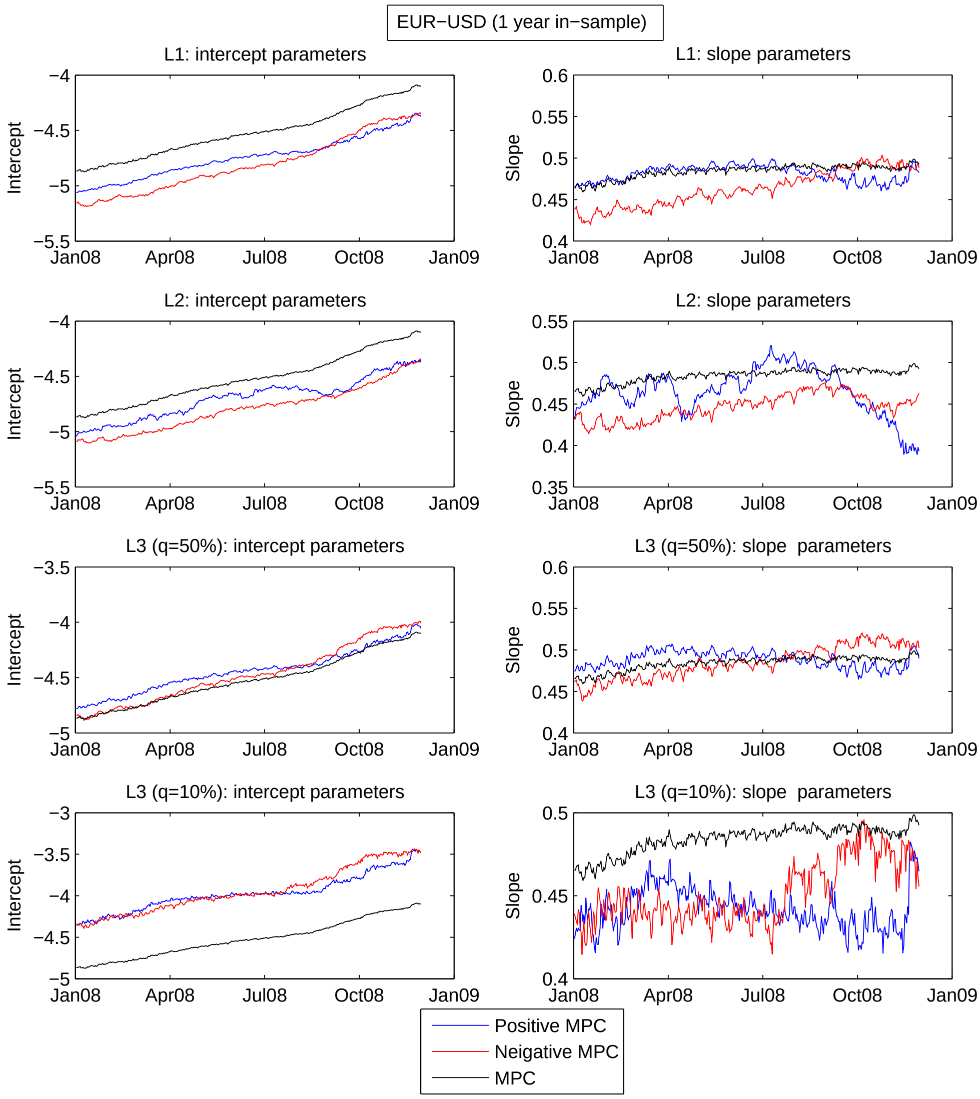


Figure C.5: The estimated parameters of scaling laws for EUR-USD with 10 day time intervals with 1 year in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

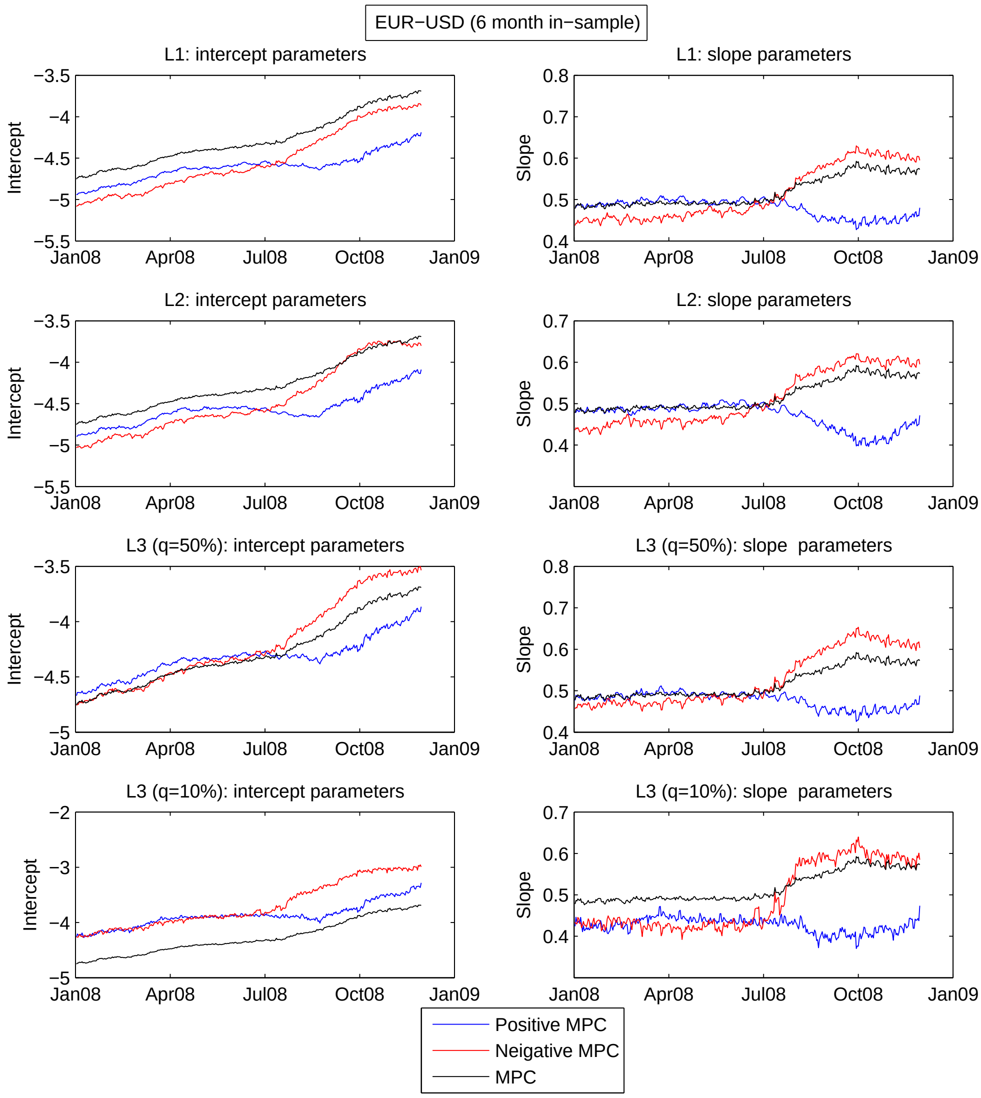


Figure C.6: The estimated parameters of scaling laws for EUR-USD with 10 day time intervals with 6 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

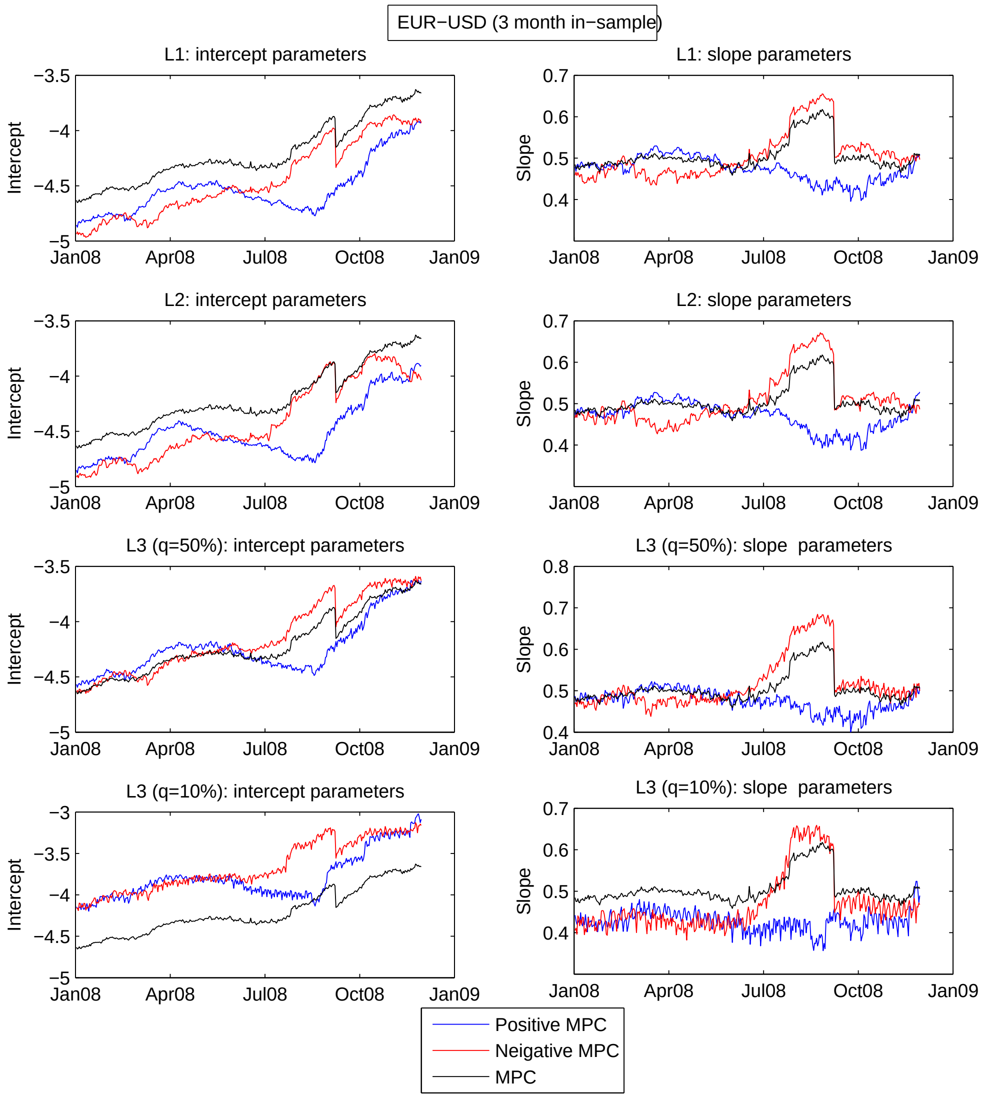


Figure C.7: The estimated parameters of scaling laws for EUR-USD with 10 day time intervals with 3 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

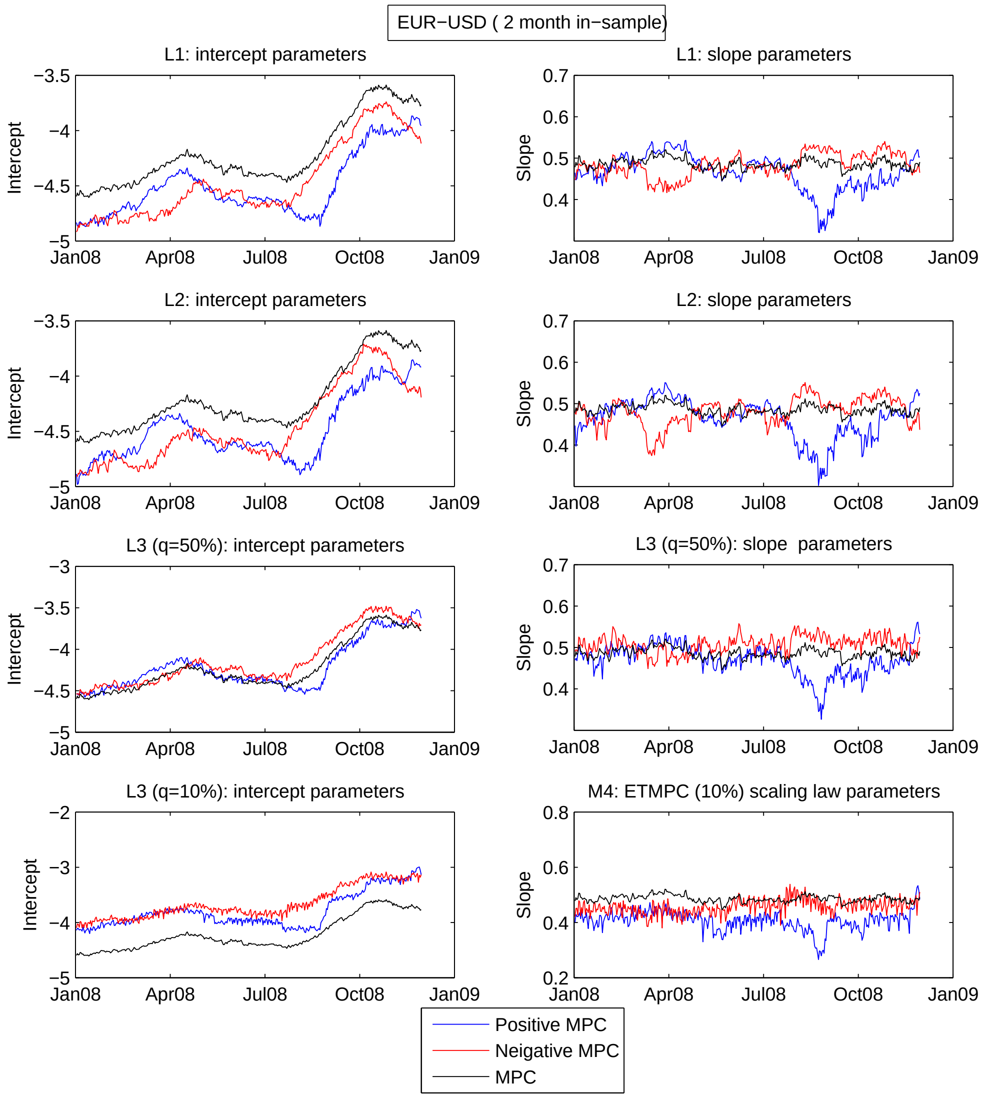


Figure C.8: The estimated parameters of scaling laws for EUR-USD with 10 day time intervals with 2 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

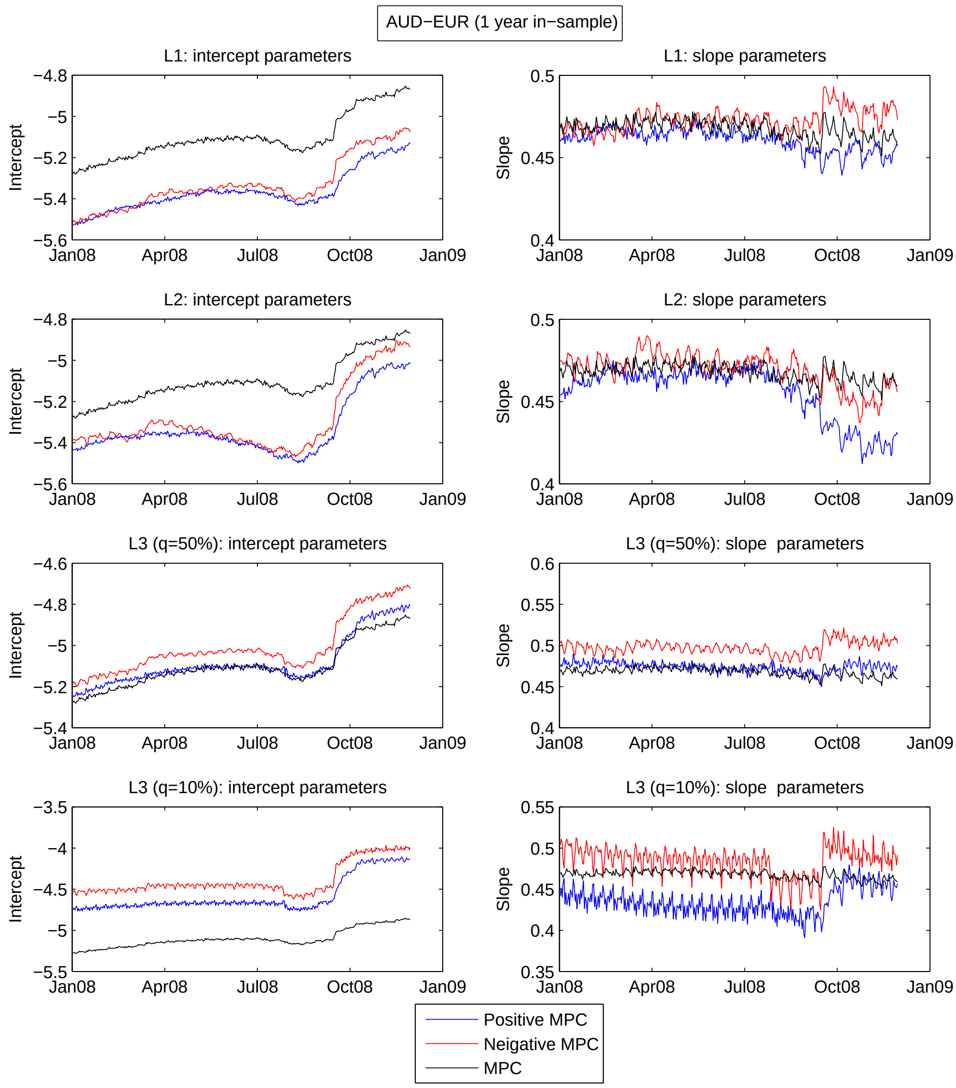


Figure C.9: The estimated parameters of scaling laws for AUD-EUR with 10 day time intervals with 1 year in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

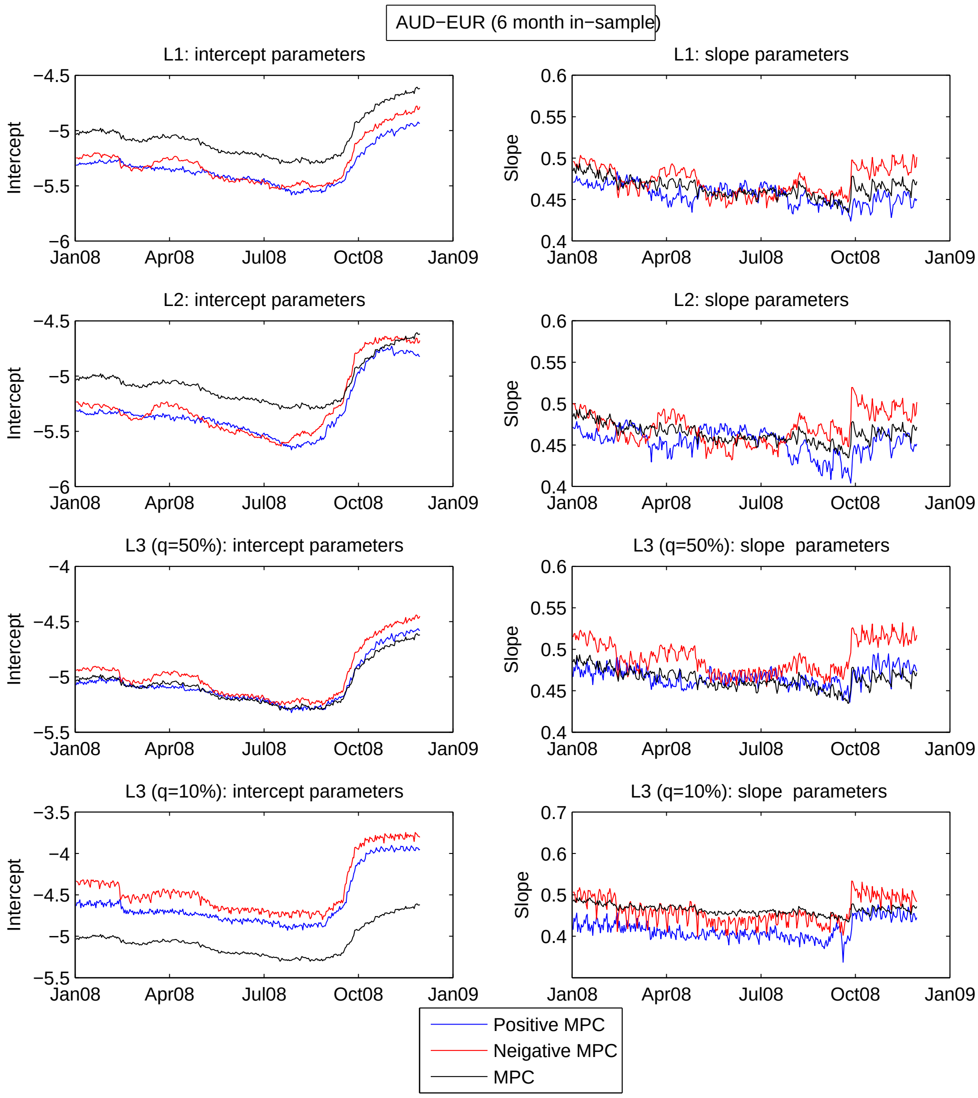


Figure C.10: The estimated parameters of scaling laws for AUD-EUR with 10 day time intervals with 6 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

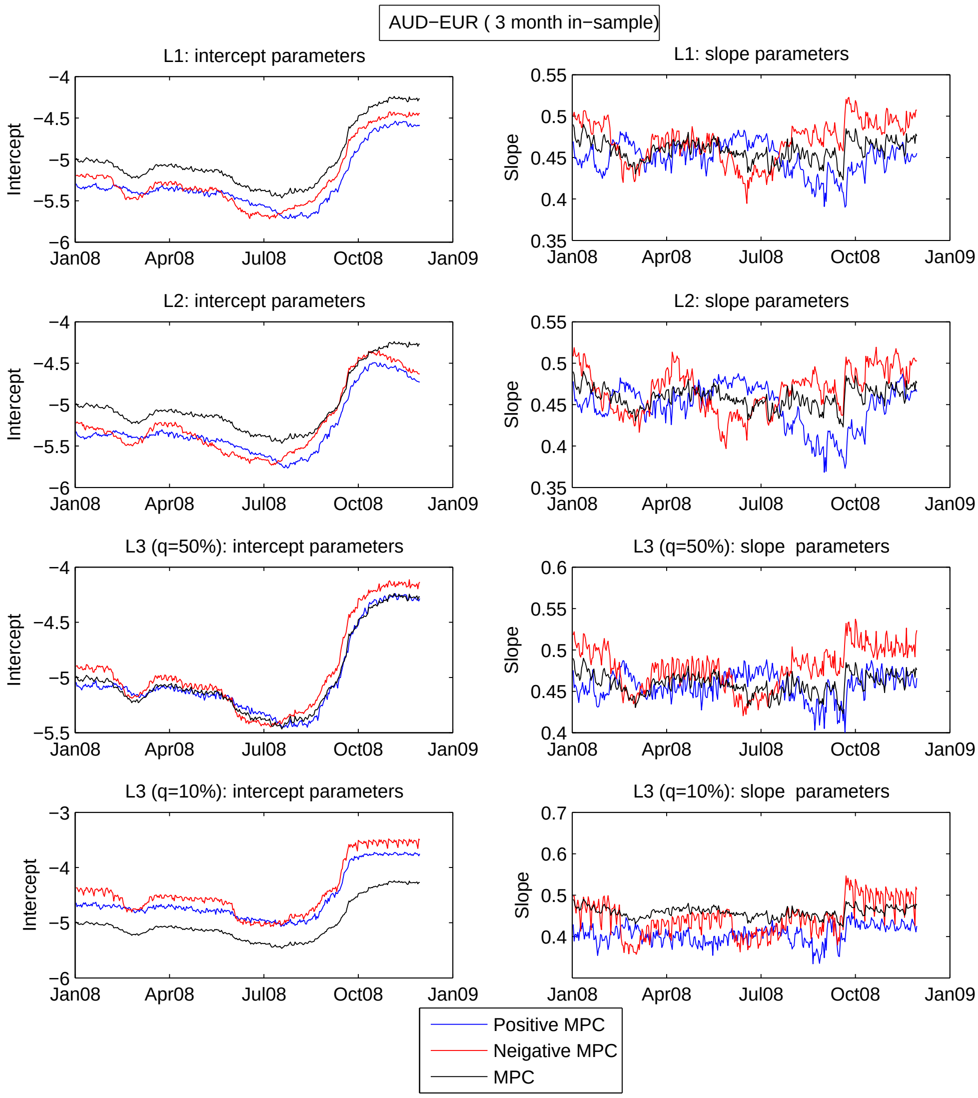


Figure C.11: The estimated parameters of scaling laws for AUD-EUR with 10 day time intervals with 3 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

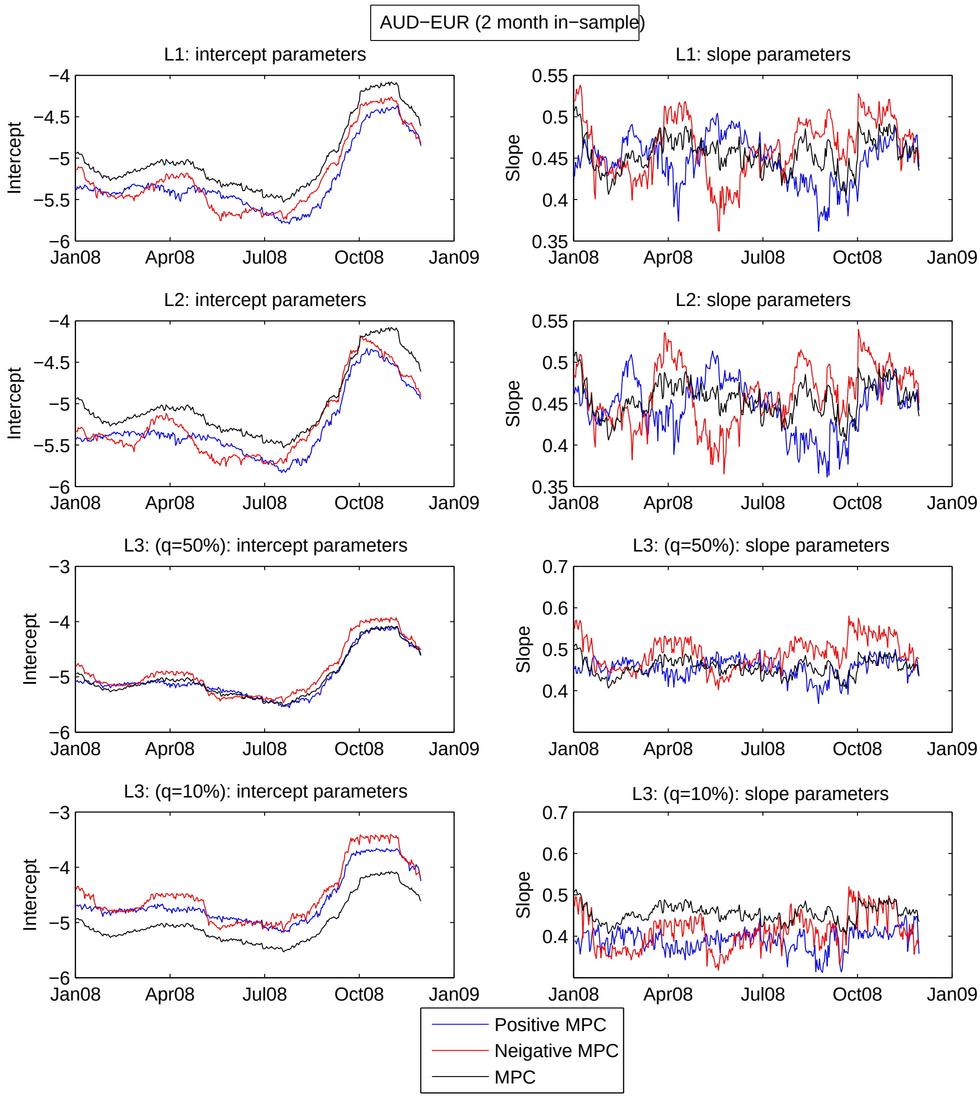


Figure C.12: The estimated parameters of scaling laws for AUD-EUR with 10 day time intervals with 2 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

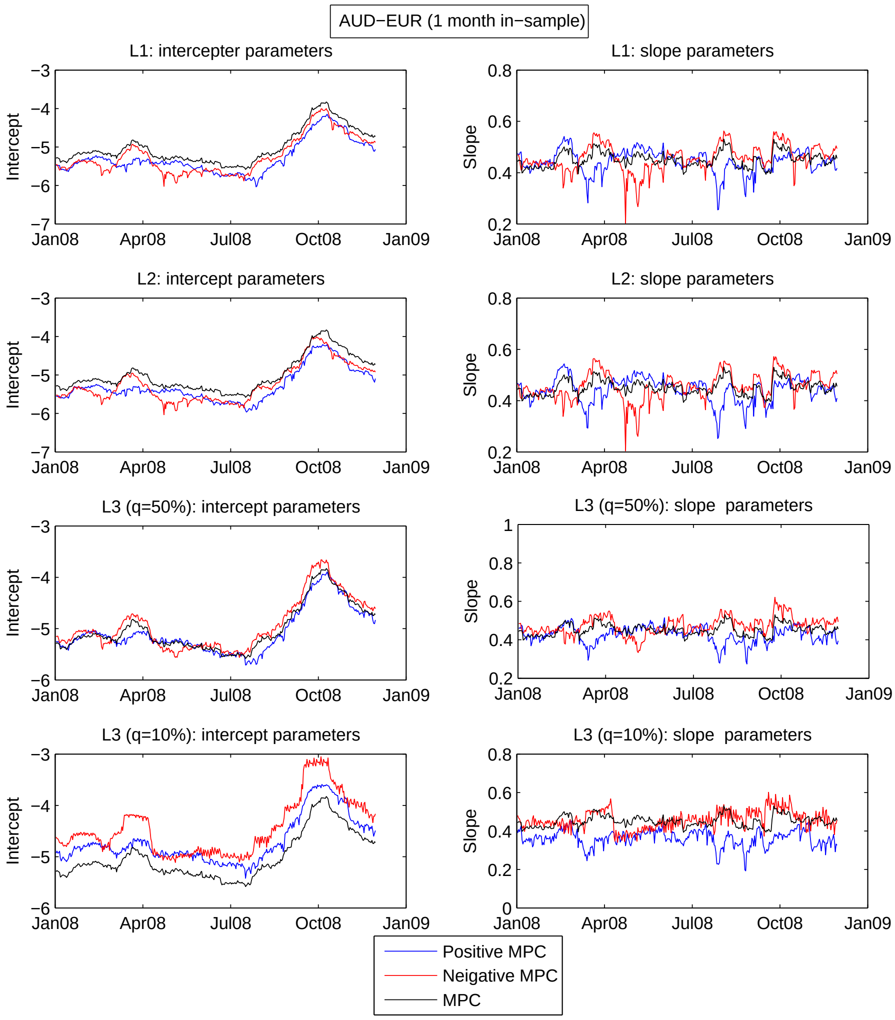


Figure C.13: The estimated parameters of scaling laws for AUD-EUR with 10 day time intervals with 1 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

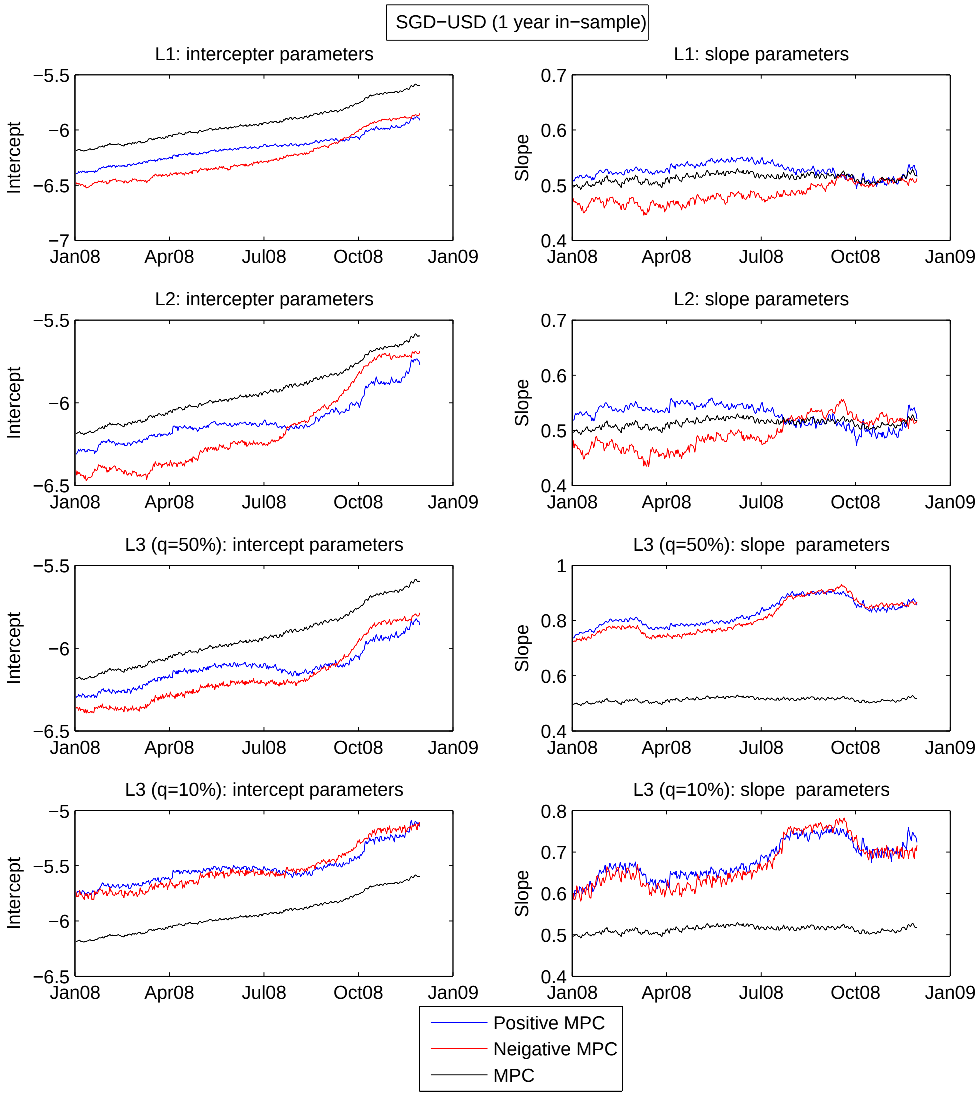


Figure C.14: The estimated parameters of scaling laws for SGD-USD with 10 day time intervals with 1 year in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

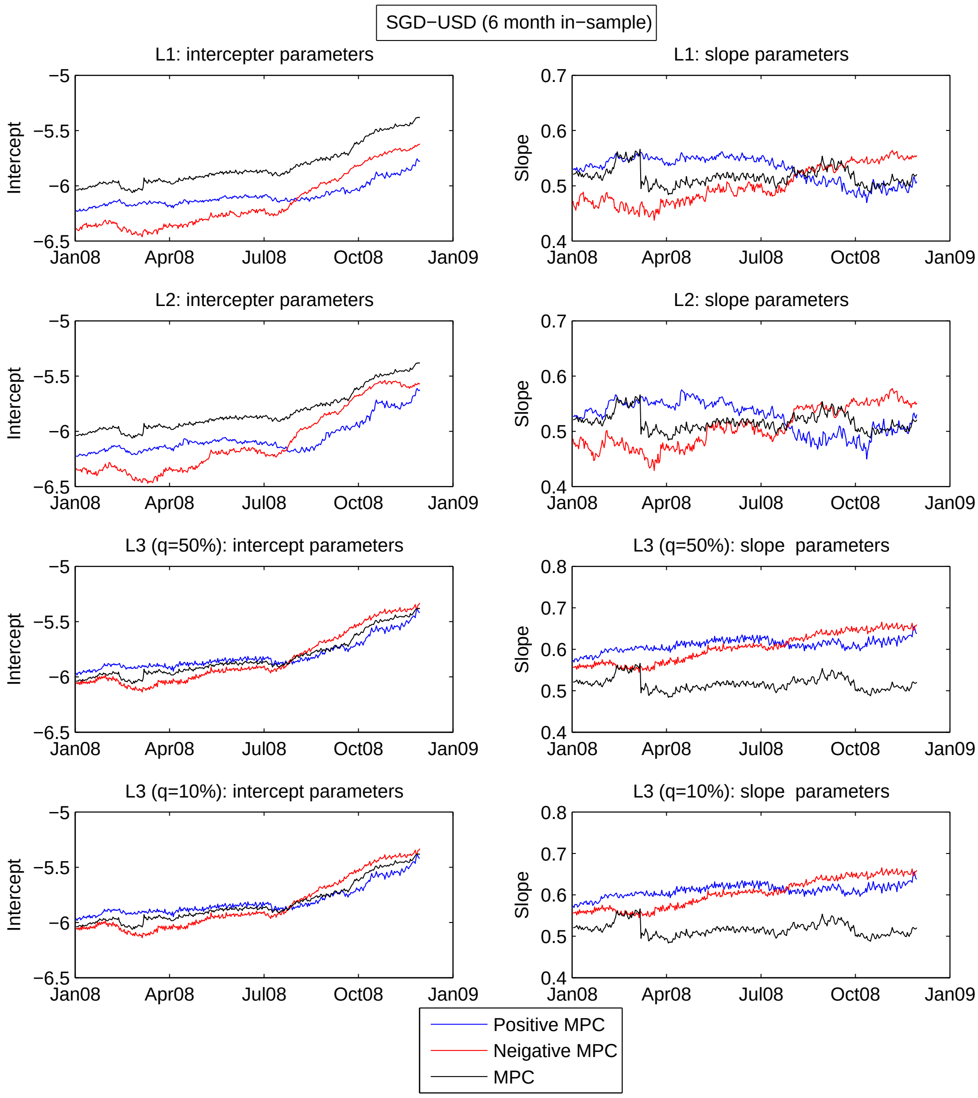


Figure C.15: The estimated parameters of scaling laws for SGD-USD with 10 day time intervals with 6 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

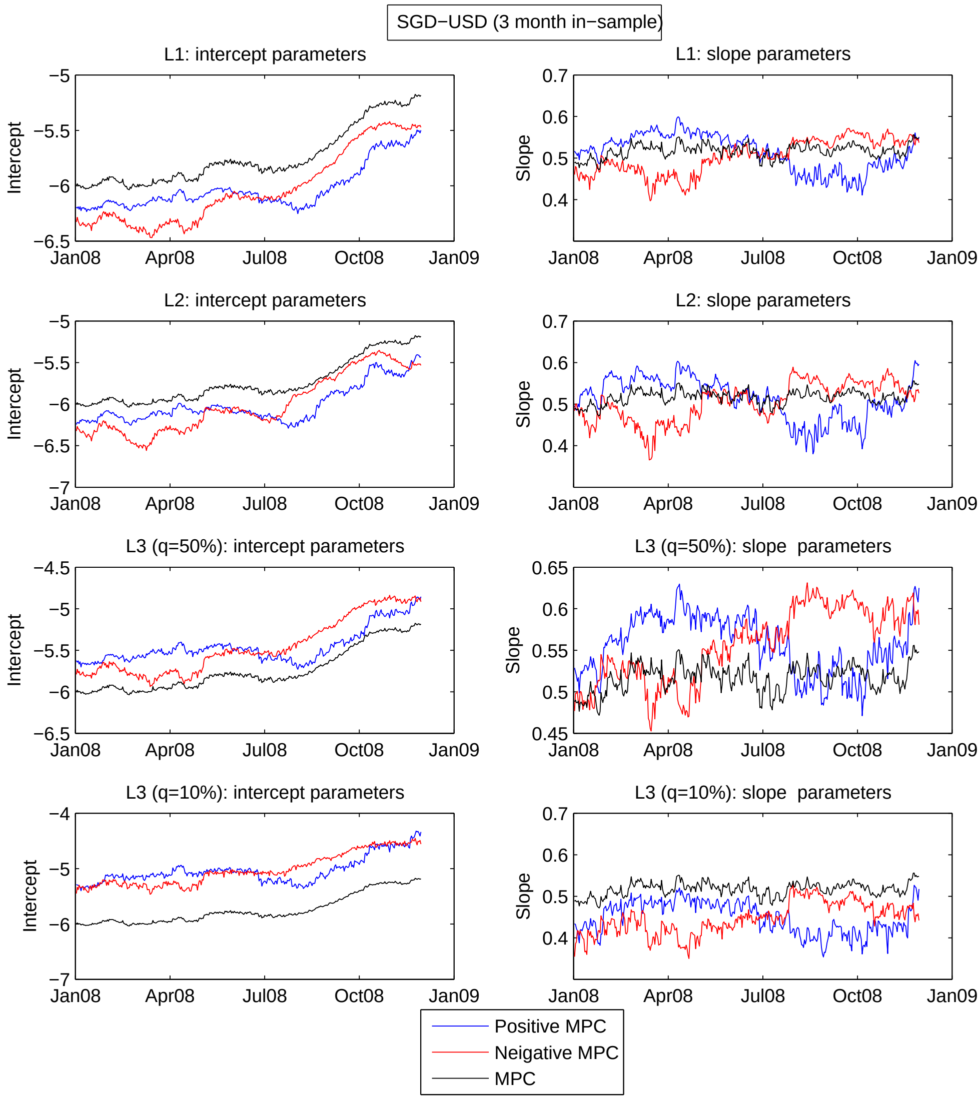


Figure C.16: The estimated parameters of scaling laws for SGD-USD with 10 day time intervals with 3 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

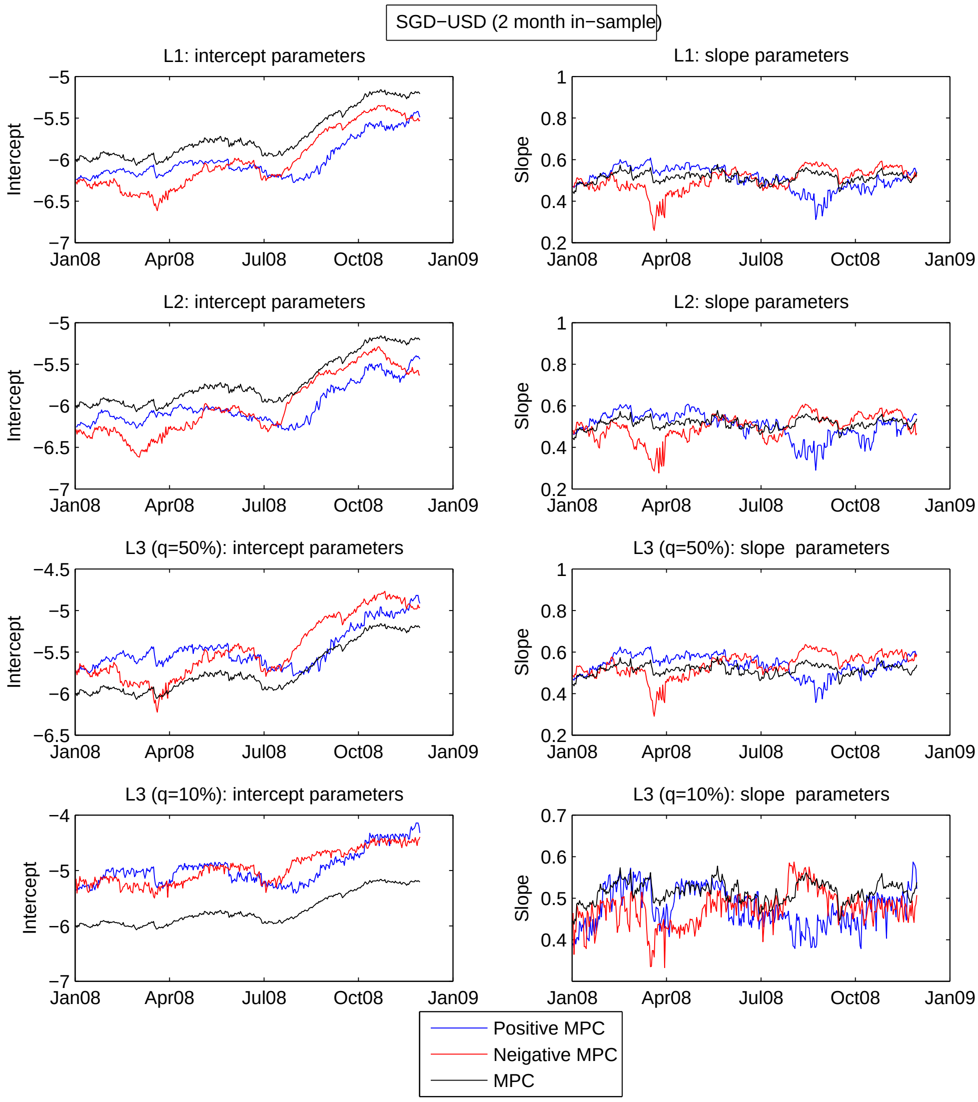


Figure C.17: The estimated parameters of scaling laws for SGD-USD with 10 day time intervals with 2 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

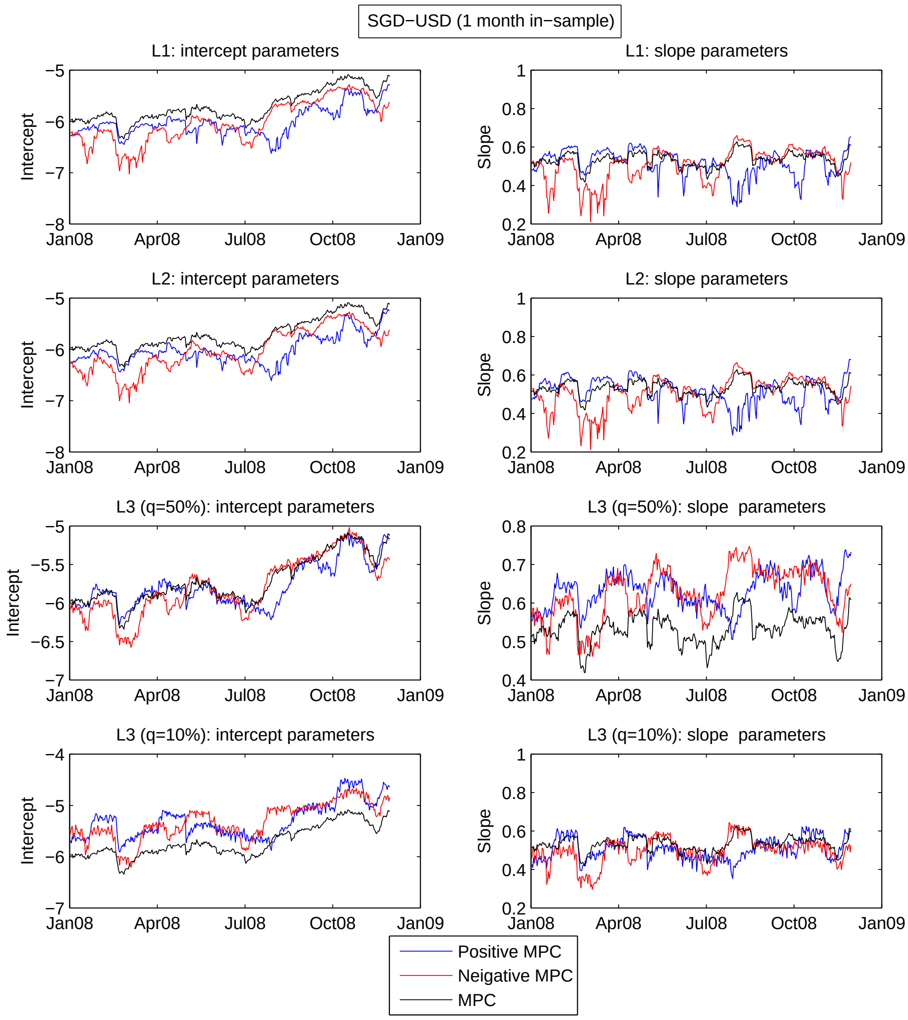


Figure C.18: The estimated parameters of scaling laws for SGD-USD with 10 day time intervals with 1 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

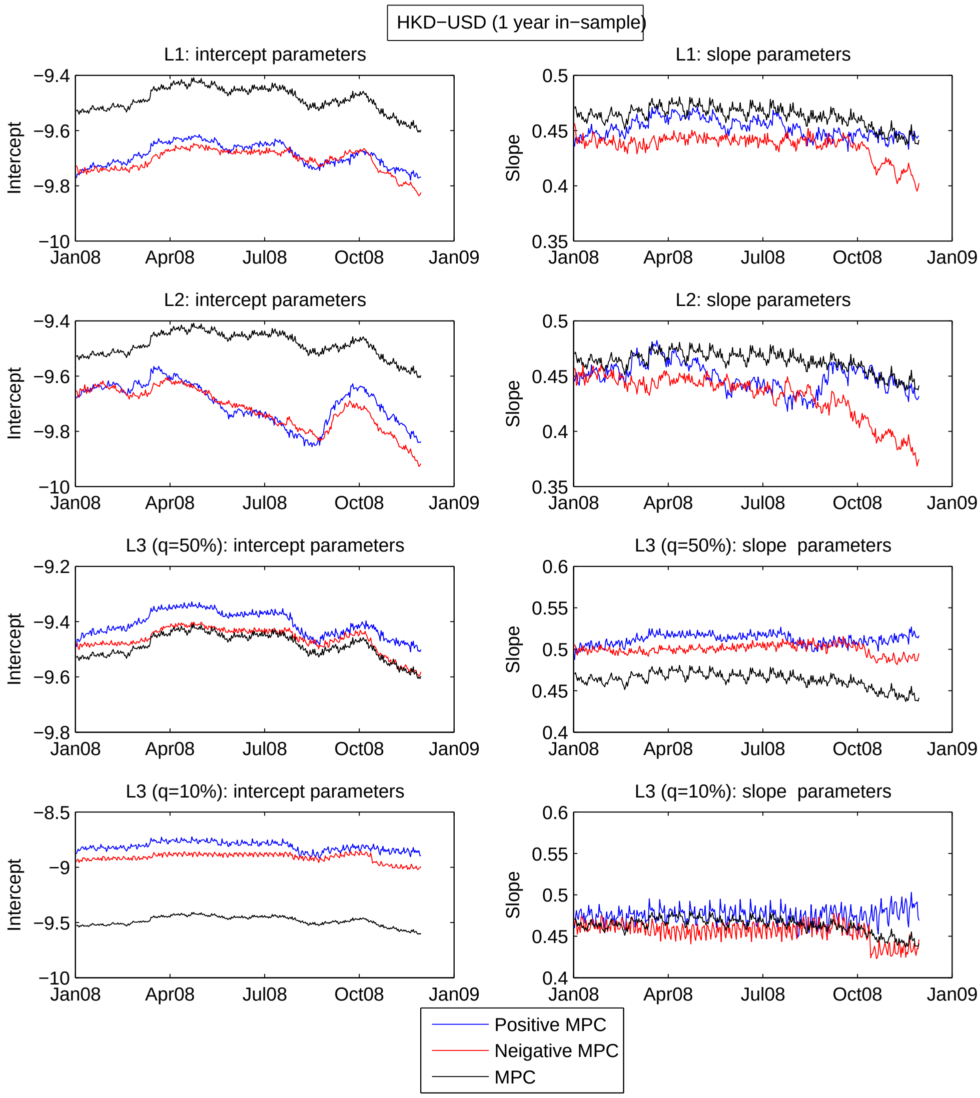


Figure C.19: The estimated parameters of scaling laws for HKD-USD with 10 day time intervals with 1 year in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

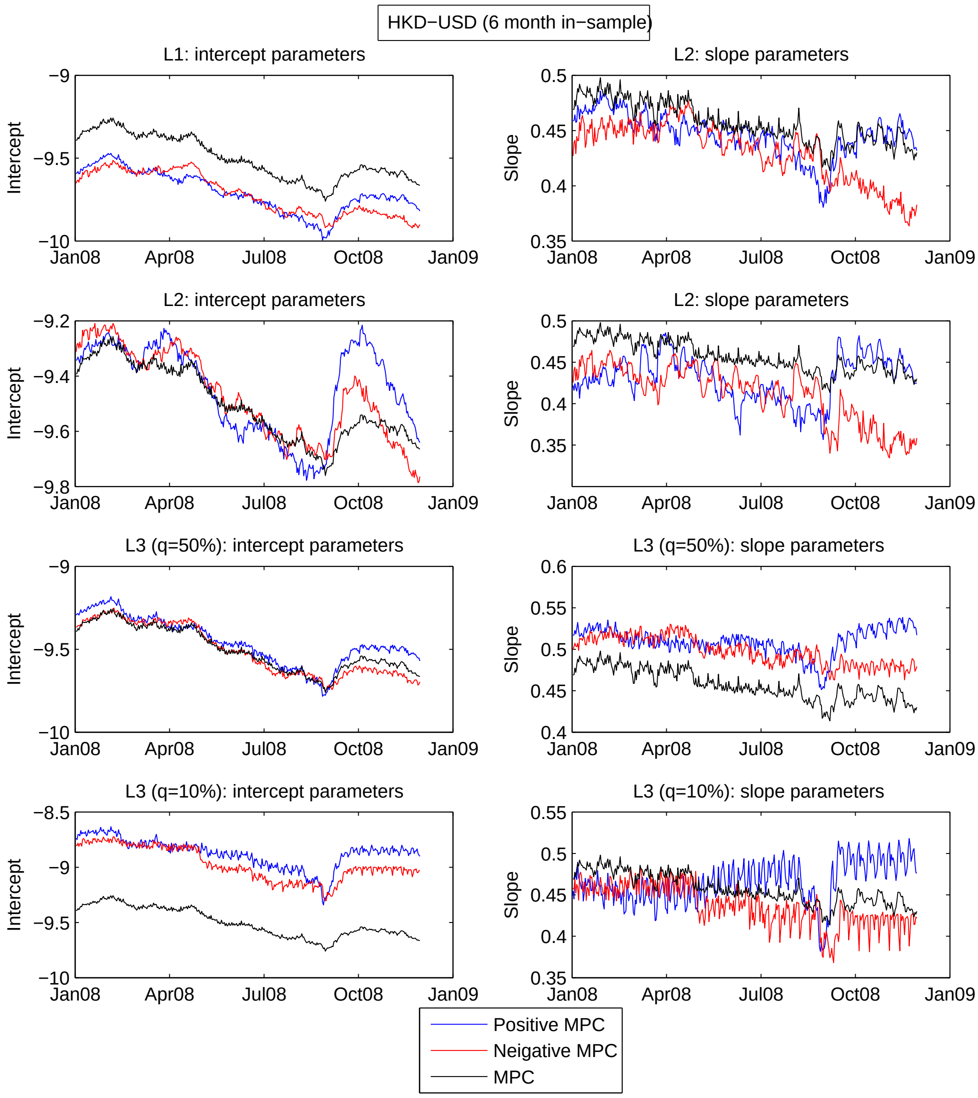


Figure C.20: The estimated parameters of scaling laws for HKD-USD with 10 day time intervals with 6 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

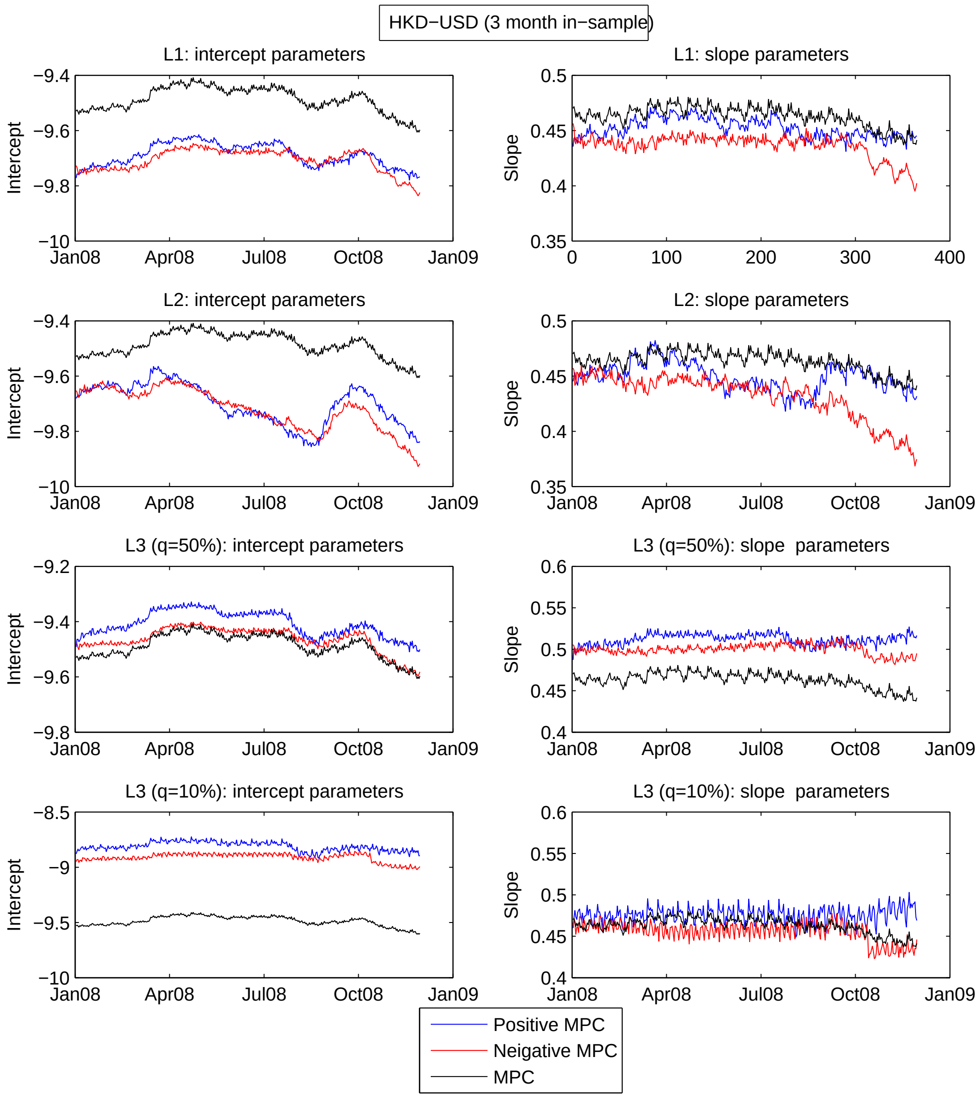


Figure C.21: The estimated parameters of scaling laws for HKD-USD with 10 day time intervals with 3 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

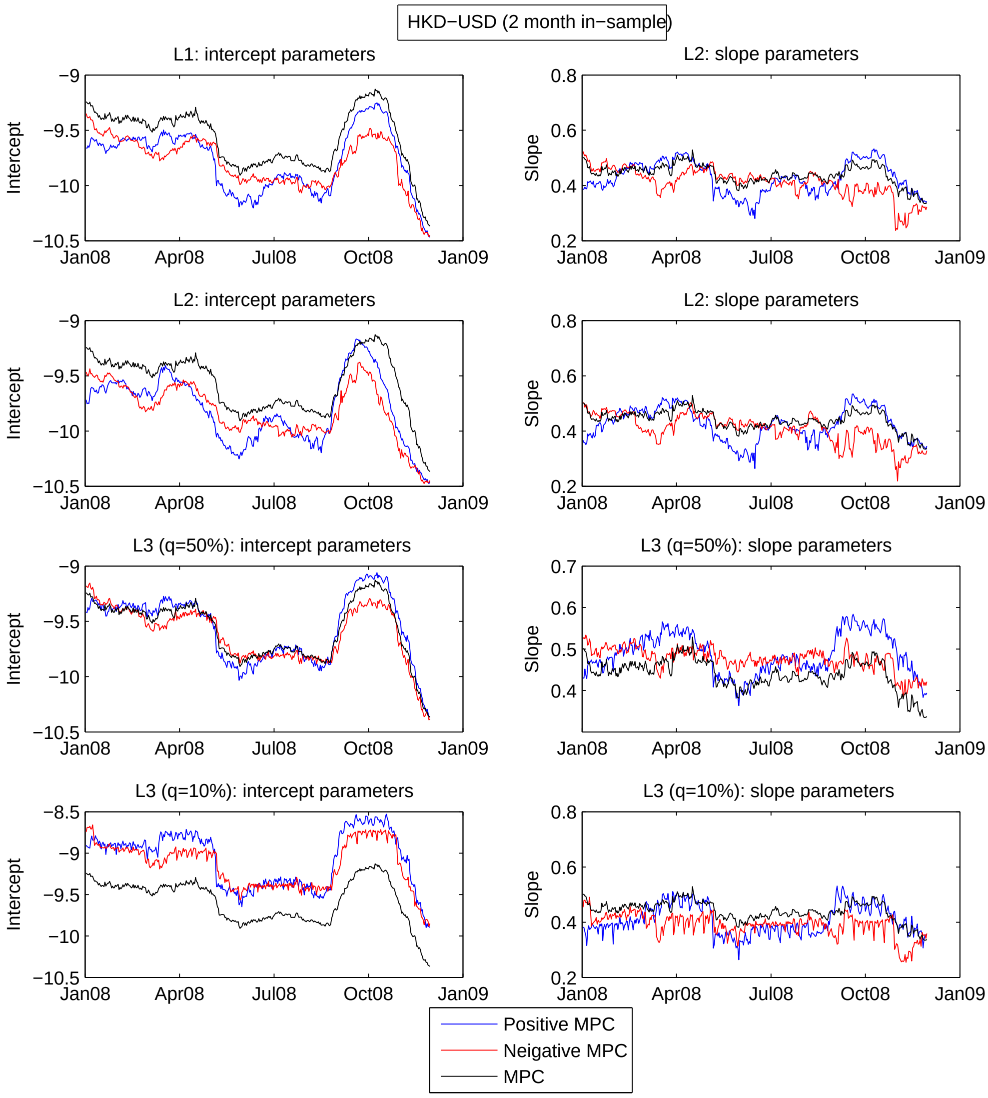


Figure C.22: The estimated parameters of scaling laws for HKD-USD with 10 day time intervals with 2 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

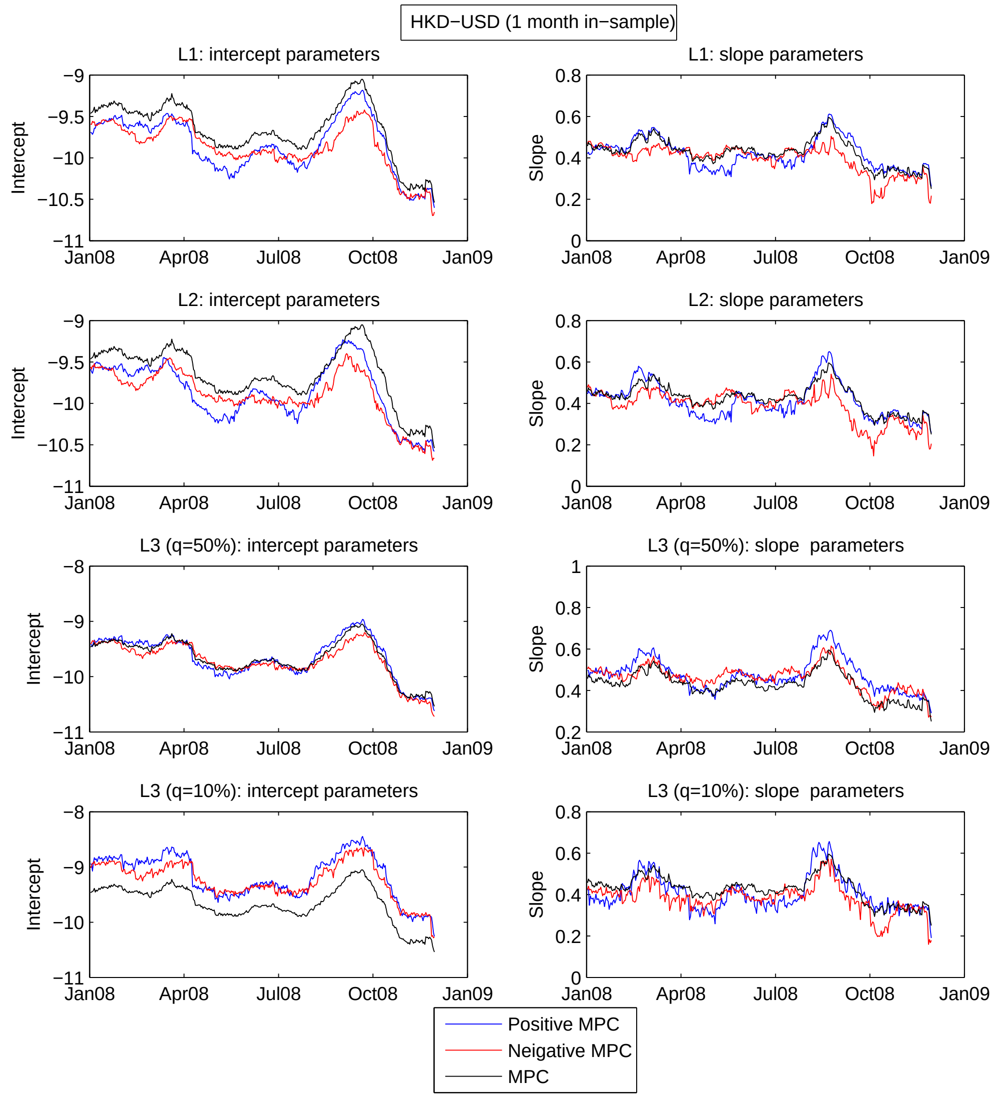


Figure C.23: The estimated parameters of scaling laws for HKD-USD with 10 day time intervals with 1 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

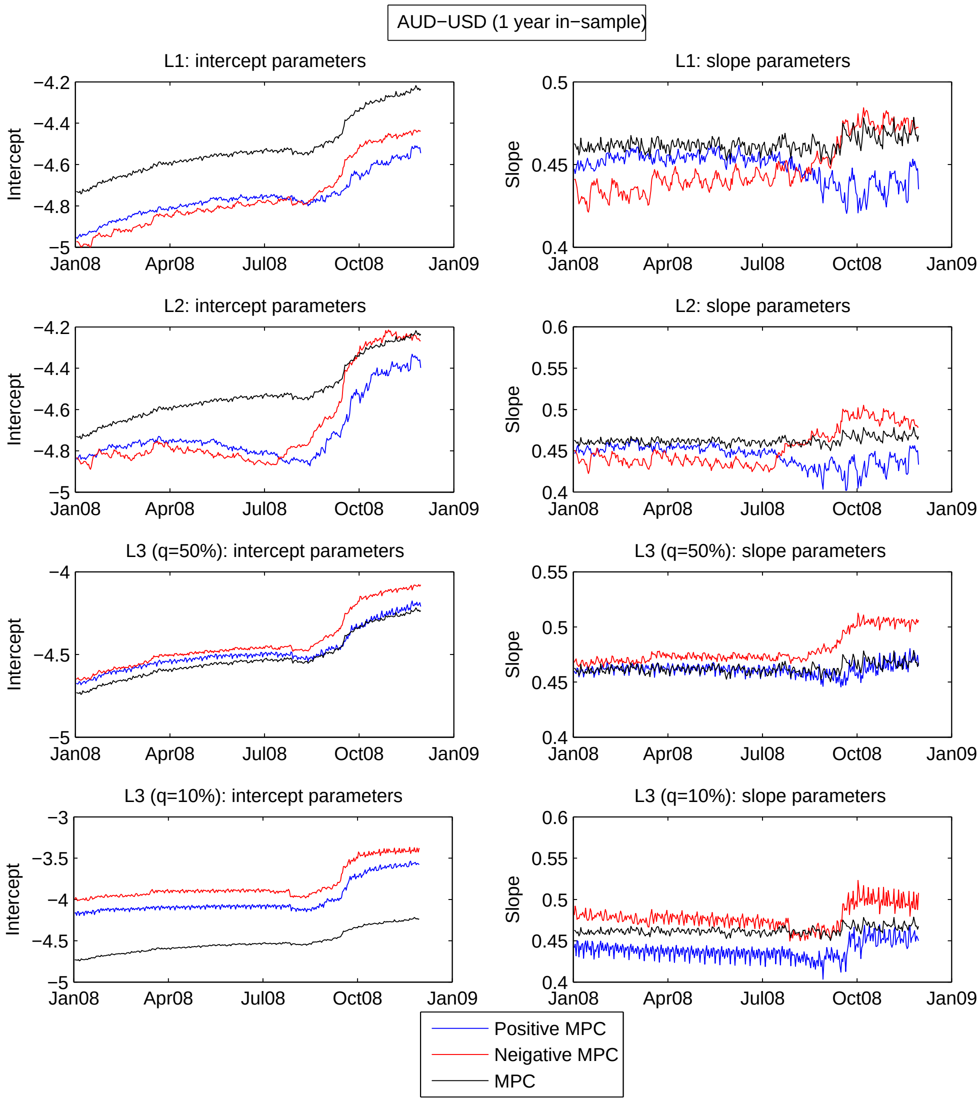


Figure C.24: The estimated parameters of scaling laws for AUD-USD with 10 day time intervals with 1 year in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

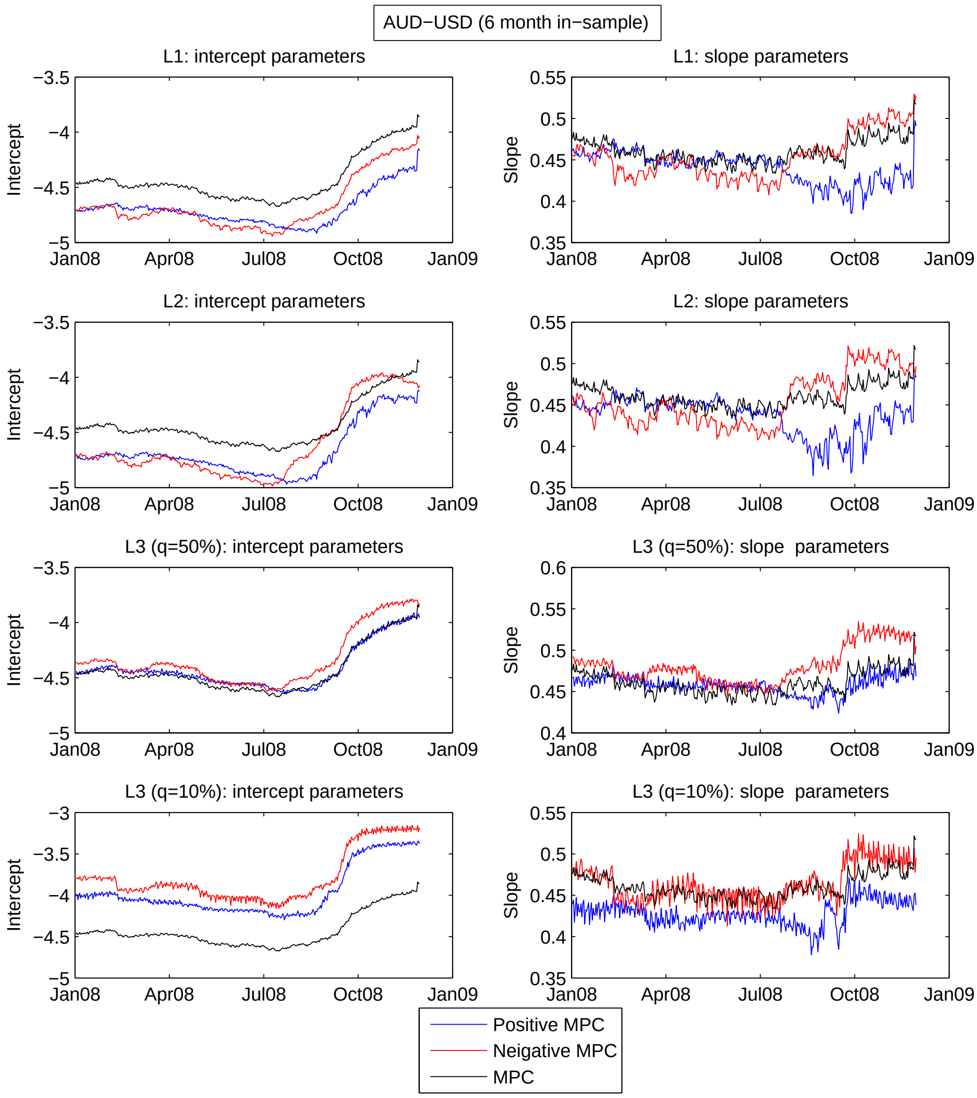


Figure C.25: The estimated parameters of scaling laws for AUD-USD with 10 day time intervals with 6 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

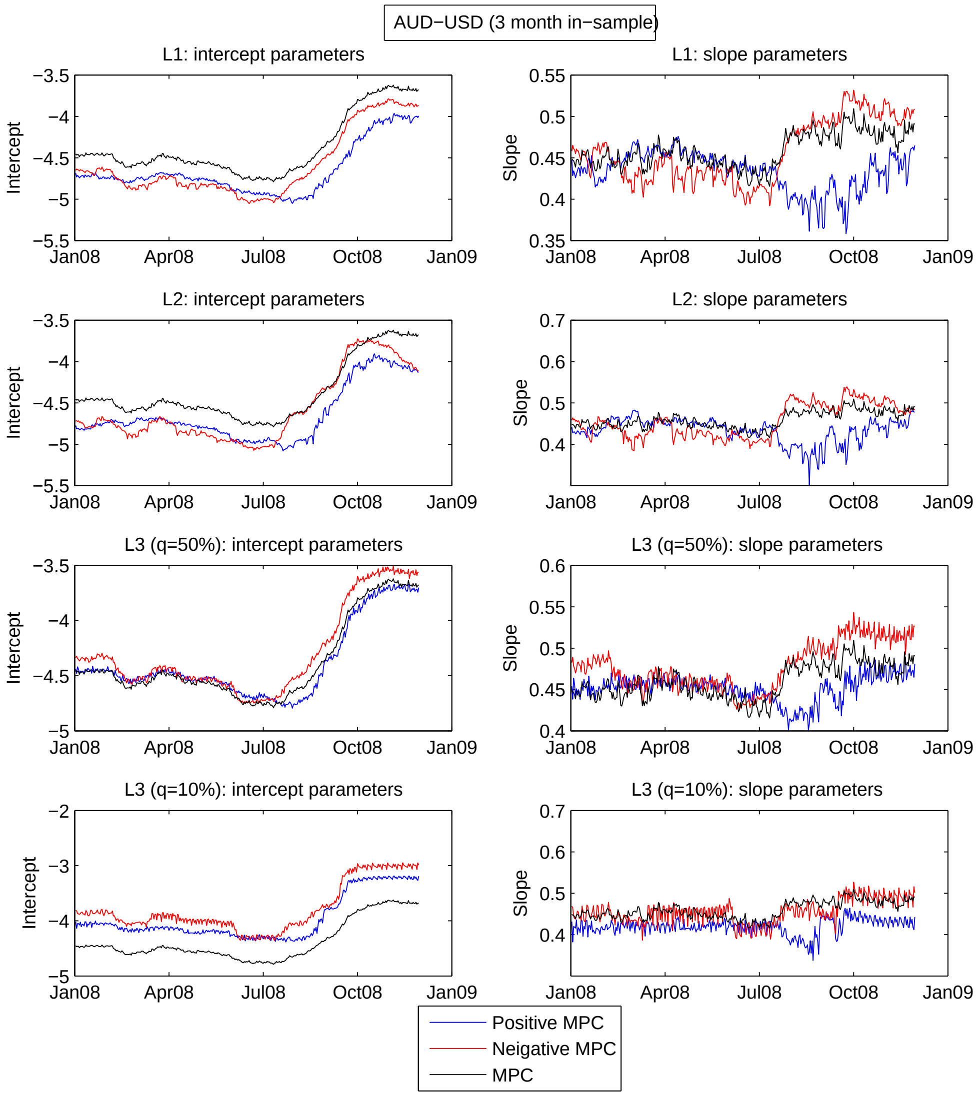


Figure C.26: The estimated parameters of scaling laws for AUD-USD with 10 day time intervals with 3 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

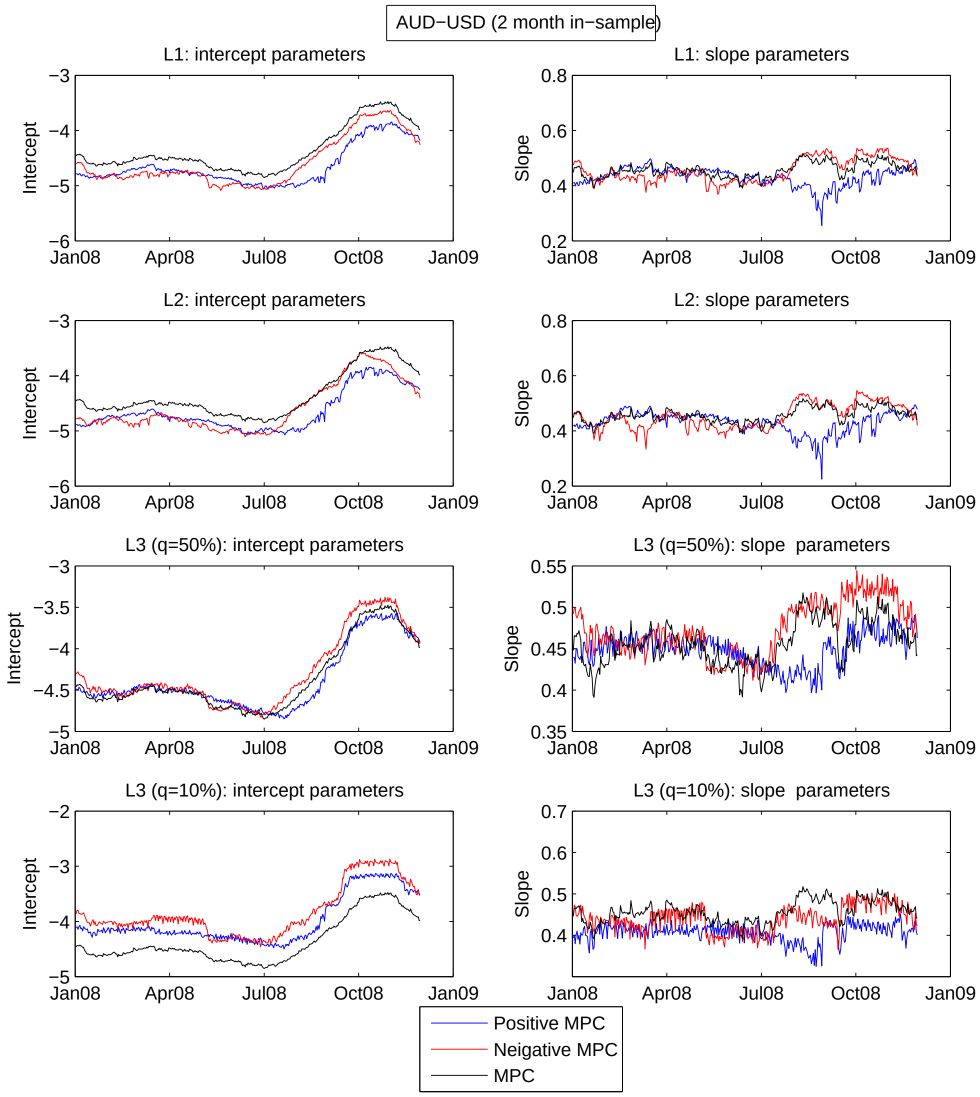


Figure C.27: The estimated parameters of scaling laws for AUD-USD with 10 day time intervals with 2 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).

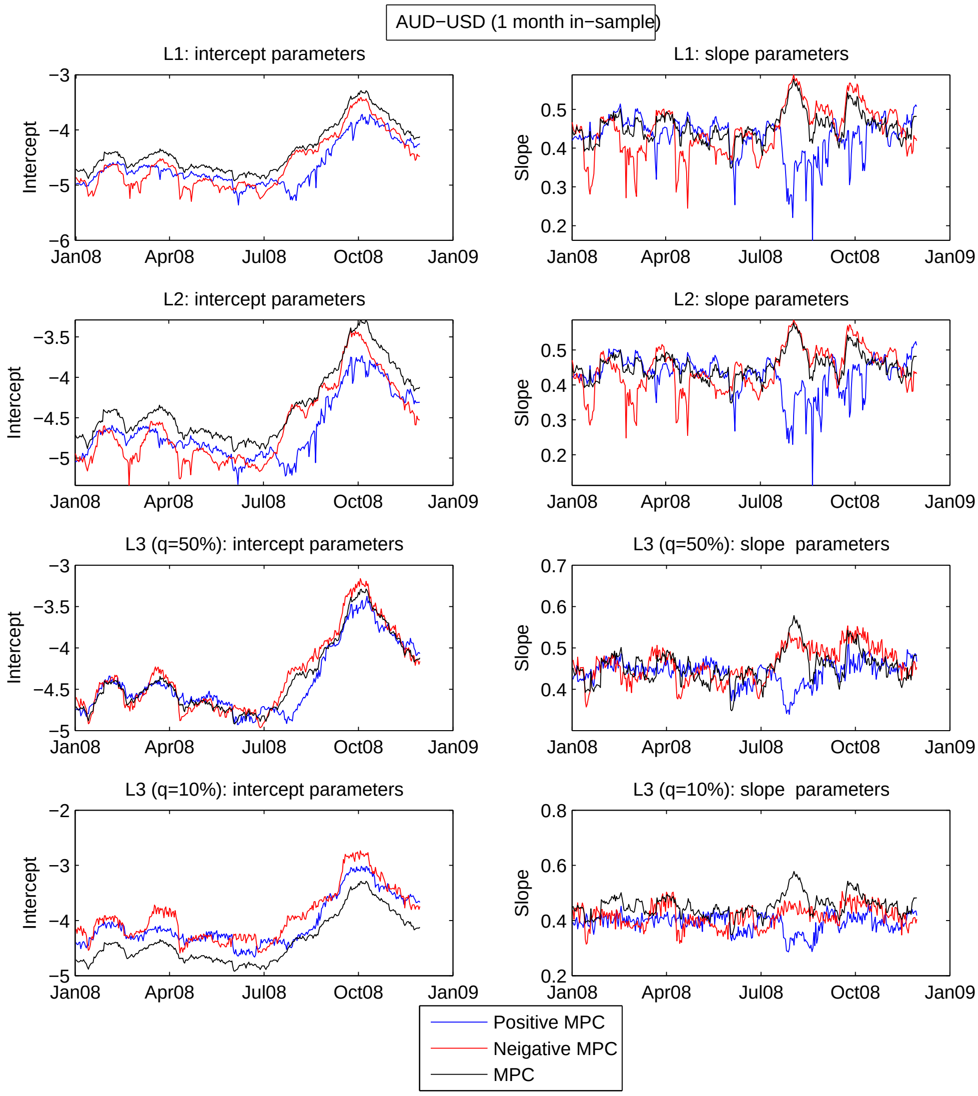


Figure C.28: The estimated parameters of scaling laws for AUD-USD with 10 day time intervals with 1 month in-sample data (Out-of-sample period is from Jan 1, 2008 to Dec 31, 2008).