Directional Change for Handling Tick-to-tick Data

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Time Series (TS) records transactions in a market at fixed intervals. Directional Change (DC) is an alternative way to record transactions: it only records transactions that represent significant price changes in the opposite direction in a trend, where "significance" is observer-defined. In this paper, we argue that DC is particularly suitable for recording and analysing tick-to-tick data. Firstly, significant data points and high activities between sampling points that may not be recorded in a TS will always be recorded in DC. Secondly, as transactions take place at irregular times, but TS records transactions at fixed intervals, adjustments are required in the recording process, which may distort the records; no adjustments are required in DC. Thirdly, as DC is data-driven: every new transaction could potentially provide us with new information on the pulse of the market. For these reasons, DC is more suitable than TS for tracking tick-to-tick data for signals.

Keywords: Finance, markets, time series, directional change

Introduction

Time and prices are recorded as transactions are made in a market. The record of all transactions is referred to as tick-to-tick data (TD). It is worth pointing out that transactions take place irregularly: while many transactions may take place in one second, no transaction may take place in the next. To facilitate analysis, these transactions are often summarized as time series (TS), where transactions are sampled at fixed intervals. For example, one could take the final transaction of every day to form the daily closing time series.

Olsen et al [7][9] introduced the concept of Directional Change (DC) as an alternative way to sample transactions: the idea is to let data determine when a transaction should be recorded. A transaction is only recorded in DC when a significant price change in the opposite direction of the current trend has taken place, where "significance" is observer-defined. This will be explained in detail later. DC has been applied to forecasting [2], market analysis [14][10], monitoring [6] and trading [8][3][4][16][1][6]. This paper aims to show that DC is more suitable than TS for handling TD.

An example set of Tick-to-tick Data (TD)

To support our discussion, an example set of TD is shown in Columns 2 and 3 of Table 1. These transactions are charted in Figure 1. It is worth reiterating that transactions take place irregularly in the market. In this example, nine transactions took place between 01:00 and 02:00. These transactions were (01:08, 98), (01:13, 105), …, (01:53, 100), as shown in rows 5 to 13 in columns 2 and 3 in Table 1. There were no transactions between 03:00 and 05:00, as shown in rows 15 and 16.

Table 1. Data used for explaining the relationship between TD, TS and DC

Tick-to-tick Data (TD)			**Time Series (TS)**		**Directional Change (DC) with a 5% threshold**				
Data Point	Time (mm:ss)	Price	Time (mm:ss)	Price	Time (mm:ss)	Extreme Point	Last Hi/Low	Change from last hi/low	DC Confirmation
1	00:00	100	00:00	100	00:00	100	100		
2	00:10	110			00:10	110	110		Up
3	00:40	106					110	-3.64%	
4	00:50	107					110	-2.73%	
5	01:08	98	01:00	107	01:08	98	98	-10.91%	Down
6	01:13	105			01:13	105	105	7.14%	Up
7	01:23	90					105	-14.29%	Down
8	01:28	92					105	-12.38%	
9	01:33	83			01:33	83	83	-20.95%	
10	01:38	98					83	18.07%	Up
11	01:43	95					83	14.46%	
12	01:48	104			01:48	104	104	25.30%	
13	01:53	100					104	-3.85%	
14	02:08	103	02:00	100			104	-0.96%	
15	02:38	101	03:00	101			104	-2.88%	
16	05:08	98	04:00	101	05:08	98	98	-5.77%	Down
17	05:38	100	05:00	101			98	2.04%	
18	06:08	104	06:00	100			98	6.12%	Up
19	06:28	106			06:28	106	106	8.16%	
20	06:43	102					106	-3.77%	
21	07:03	100	07:00	102			106	-5.66%	Down
22	07:33	95					106	-10.38%	
23	07:58	98					106	-7.55%	
24	08:28	90	08:00	98	08:28	90	90	-15.09%	
25	08:43	92					90	2.22%	
26	09:08	97	09:00	92			90	7.78%	Up
27	09:40	99					90	10.00%	
28	10:00	100	10:00	100	10:00	100	90	11.11%	

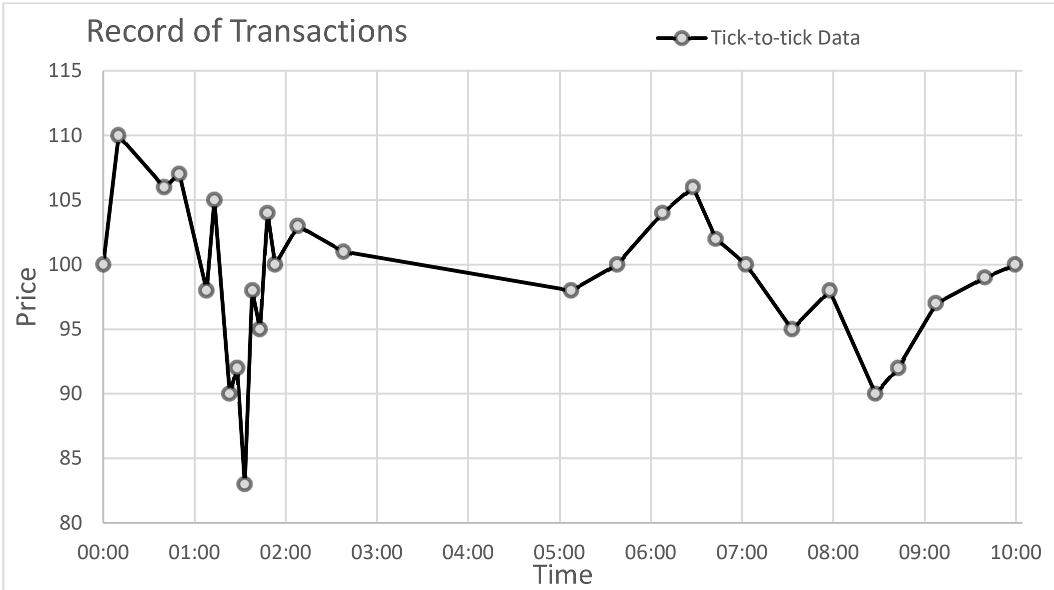

Figure 1. Tick-to-tick Data (TD) shown in Table 1

Recording transactions with TS

Given the TD shown in Columns 2 and 3 in Table 1, a time series is generated in Columns 4 and 5. This time series is summarised in Table 2 and plotted in Figure 2.

Table 2. Time Series generated from the TD in Table 1

	Time	Price	Returns
1	00:00	100	
2	01:00	107	7.00%
3	02:00	100	-6.54%
4	03:00	101	1.00%
5	04:00	101	0.00%
6	05:00	101	0.00%
7	06:00	100	-0.99%
8	07:00	102	2.00%
9	08:00	98	-3.92%
10	09:00	92	-6.12%
11	10:00	100	8.70%

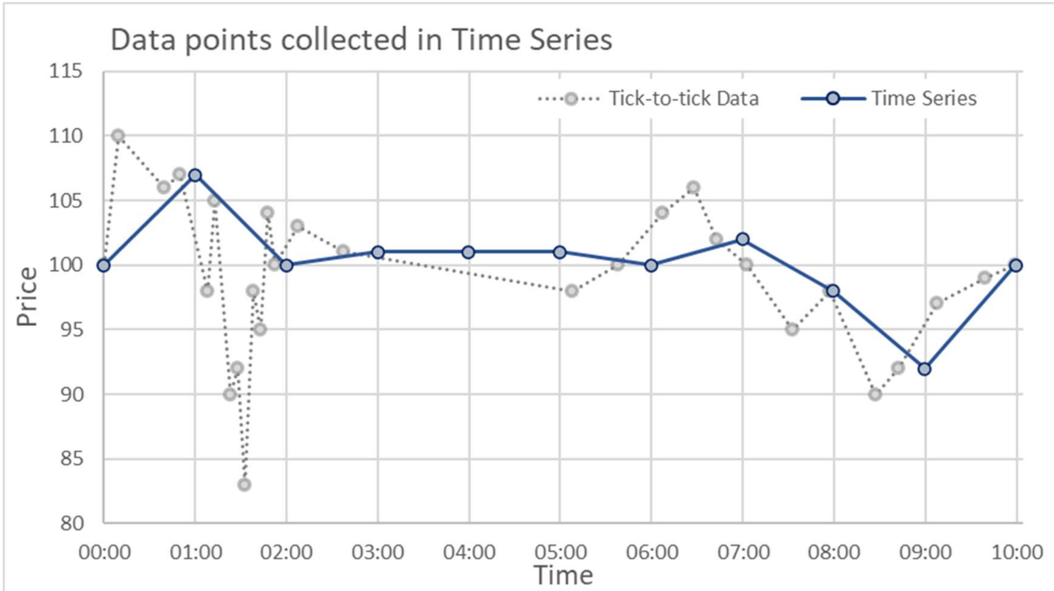

Figure 2. Data points collected Time Series (TS) for the TD shown in Table 1

Observation 1. In TS, a data point is an approximation of the nearest transaction times.

For example, TS records a price of 107 at time 01:00 (row 5, columns 4 and 5 in Table 1). This is a record of the price at 00:50 (row 4, column 2), the time of the latest transaction before 01:00. The price recorded for 03:00 by TS is 101 (row 15, columns 4 and 5), which is an approximation of the transaction (02:38, 101) (row 15, columns 2 and 3).

Time approximation may not a problem if there are many transactions within each sampling interval, in which case the error is potentially small. This would be the case for low-frequency data.

Observation 2. In TS, a data point must be created in the absence of transactions.

For example, as no transactions took place between 03:00 and 05:00, missing data points must be created in TS. One way to create a data point is to record the previous price. As the latest recorded price before 03:00 was 101 (row 15), this may be used as the price for 04:00 and 05:00.

What we have described above is just one way to reconstruct missing data points. If we allow hindsight when creating the missing data points, we may reconstruct the missing data by assuming a linear decrease between (02:38, 101) and (05:08, 98). This does not affect our observation here and therefore will not be elaborated.

In general, if we sample between large intervals, then the chance of missing data is low. The error of approximation is also proportionally reduced.[1] However, if we want to support high-frequency trading, we must sample frequently. The more frequently we sample data, the more chance that we may need to reconstruct missing data points.

Recording transactions with DC

First, we shall explain how DC records transactions in a market. Columns 6 to 10 of Table 1 show the DC summaries of the TD shown in columns 2 and 3. DC records an extreme point (Column 7) when price reverses by a significant amount from the current trend – in Table 1, we take 5% to be the threshold of a significant change. DC events are recorded in hindsight. At (01:08, 98) (row 5), the first DC event is confirmed. This event started at (00:10, 110) (row 2), which is called an *Extreme Point*. The price movement from (00:10, 110) to (01:08, 98) is called a *DC Event*. This DC event is

[1] For example, for a time series with daily closing prices, even if the data point is one minute after the actual transaction, the error is only 0.07% of the 24 hours period. However, in a minutely time series, an approximation by 1 second incurs an error of 1.7%.

confirmed because 98 is 10.91% (which is above the DC threshold of 5%) below 110. Here (01:08, 98) is called a *DC Confirmation* (DCC) point. The next transaction (01:13, 105) (row 6) confirms that the DCC point (01:08, 98) is itself an extreme point which ends the downtrend and starts the next uptrend. This is because 105 (row 6, column 3) is 7.14% above 98.

A *DC summary* records the extreme points (Columns 6 and 7 in Table 1) and DCC points (Columns 8 and 9). The key points recorded by DC are summarised in Table 3. The extreme points are plotted in Figure 3. It is worth clarifying that the first (00:00, 100) and final points (10:00, 100) in Table 3 and Figure 3 are not really extreme points in DC. They are plotted to show the directional changes in the first (00:10, 110) and the final (08:28, 90) extreme points in this summary.

Table 3. A 5% DC Summary generated from the TD in Table 1

Extreme Points		**DC Confirmation Points**		**Trends started**
Time	**Price**	**Time**	**Price**	
00:00	100	← For reference; this is not really an extreme point		
00:10	110	01:08	98	Down
01:08	98	01:13	105	Up
01:13	105	01:23	90	Down
01:33	83	01:38	98	Up
01:48	104	05:08	98	Down
05:08	98	06:08	104	Up
06:28	106	07:03	100	Down
08:28	90	09:08	97	Up
10:00	100	← For reference; this is not really an extreme point		

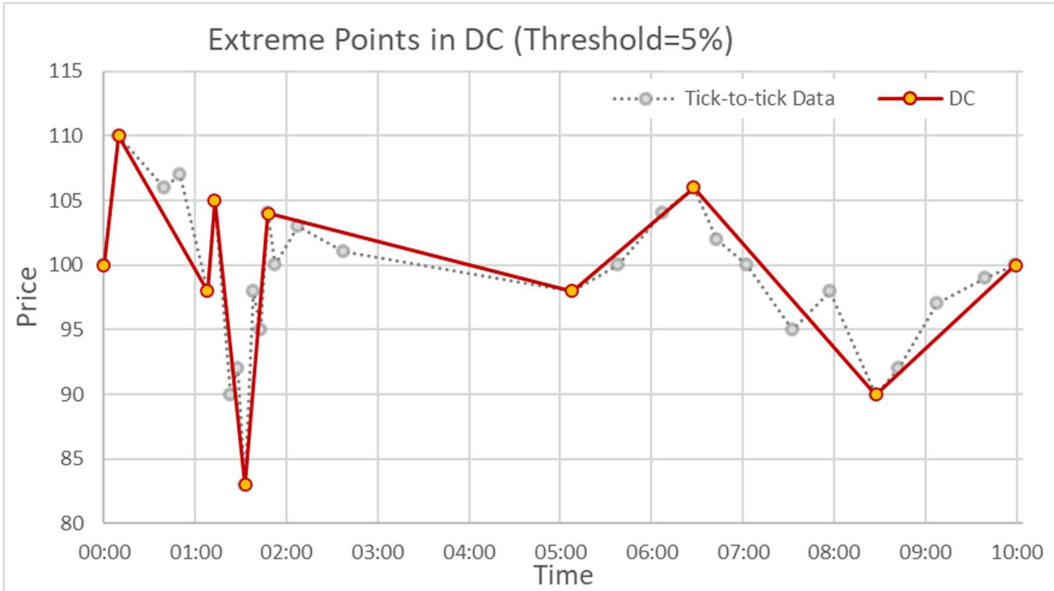

Figure 3. DC extreme points collected from the TD in Table 1

The extreme points shown in Figure 3 correspond to the "zig-zag indicator" in technical analysis [13]. The formal definition of DC is presented in Appendix A of [6][2].

Observation 3. DC records transactions as they are. No approximation is required. There is no missing data to be reconstructed.

By definition, every extreme point and DC confirmation point recorded in a DC summary is a transaction in the TD.

Contrasting TS and DC

Figure 4 highlights the difference between the TS and DC summaries above. There are 11 data points in the TS and only 8 extreme points in DC.

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[2] The formal definition of DC is mutually recursive. For simplicity, this will not be elaborated in this paper. Interested readers may refer to [6], Appendix A.

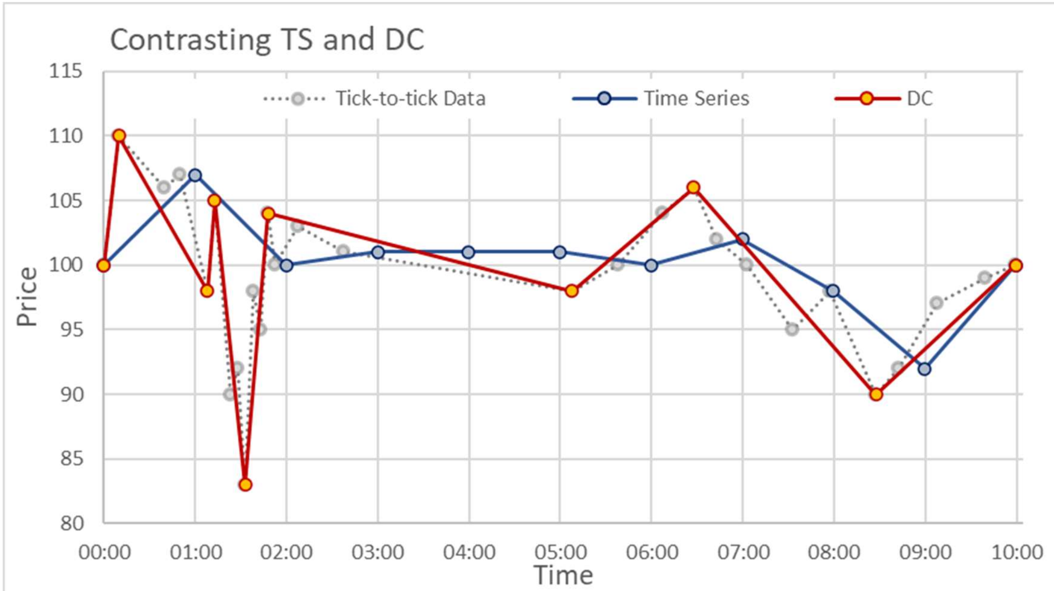

Figure 4. Contrasting TS and DC in recording the same TD in Table 1

Observation 4. DC records all the extreme points in TD. They could be missed by TS.

By nature, a DC summary records all the extreme points. In the TS (Table 2), the lowest price in the whole period (01:33, 83) (row 9 in Table 1) is not recorded because it is not the final point before a sampling time.

Observation 5. TS may miss high activities in the market, which will be captured by DC.

By recording at fixed intervals, TS may not be able to record high activities in the market. For example, the market fluctuated between 01:00 and 02:00 (rows 5 to 13 in Table 1). As DC is data-driven, it records all significant changes (as defined by the DC threshold) that took place. Between 01:00 and 02:00, DC records four extreme points.

Remarks on sampling frequency under TS:

By sampling more frequently under TS, one could increase the chance of capturing extreme points. However, there is a price to pay: by increasing the sampling frequency under TS, potentially more missing data will need to be recreated (Observation 2). Besides, due to time approximation (Observation 1), a high-frequency TS may present a distorted picture. As an example, suppose transactions were sampled half-minutely. A TS data point that follows (02:00, 100) (row 14 in Table 1) will be (02:30, 103) (which takes the transaction (02:08, 103) in row 14, columns 2 and 3). The time approximation is (02:30 – 02:08 =) 12 seconds; the error is therefore (12 ÷ 30 =) 73%. The calculated return from 02:00 to 02:30 in the TS will be ((103–100) ÷ 100 =) 3% over 30 seconds. This distorts the actual picture that (i) price rose by 3% over 15 seconds (from (01:53, 100) to (02:08, 103), see rows 13 to 14 in Table 1), not 30 seconds as calculated in TS; and (ii) contrary to what TS showed, the price was actually in decline at 02:30. Sampling at higher frequency risks adding distortions to TS.

DC does not have the approximation problem (Observation 3). Does DC have its own problems? For example, what DC threshold should one use? By using different thresholds, one sees the market in different levels of details, which may produce different observations. However, as demonstrated by Glattfelder et al [9], the market exhibits similar statistical properties under different DC thresholds. The decision of what threshold to use is actually a freedom (to reflect the needs of the observer) rather than a hindrance.

DC can be used to track the market tick by tick

Volatility in TS is often measured by the standard deviation of returns over a period of time. In the above example, the final three data points in TS were (08:00, 98), (09:00, 92), (10:00, 100) (see Table 2, rows 9 to 11). While the two returns (-6.1% and +8.7%, as shown in Table 2, column 4) are big, taking the standard deviation of just two numbers is not a very meaningful measure of the volatility of the market. Volatility in TS is normally measured with a reasonably large number of returns. When taking the standard deviation of a large number of returns, these two big changes may not look significant.

We shall explain below that volatility in DC is measured more directly. Tsang et al [14] introduced several measures of volatility in DC. Two of them are adapted here for our discussion:

Definition 1. Absolute Total Movement (**aTMV**) in DC:

$$aTMV = (|P_c - P_{EP}| \div P_{EP}) \div \text{Threshold} \quad (1)$$

where P_c is the current price, P_{EP} is the price of the preceding extreme point, Threshold is the threshold used to determine 'significance' in the DC summary. ■

By normalisation with the Threshold, aTMV values obtained from different DC summaries can be compared, even if they were obtained using different DC thresholds.

In the above example, the final extreme point in the DC summary in Table 3 is (08:28, 90). At (09:08, 97) (data point 26 in Table 1), the aTMV is ((|97 – 90| ÷ 90) ÷ 5% =) 1.556. At the next transaction (09:40, 99) (data point 27 in Table 1), the aTMV is ((|99 – 90| ÷ 90) ÷ 5% =) 2. At (10:00, 100), the aTMV is 2.222.

Observation 6. The aTMV can be calculated for each transaction. This gives us a tick-by-tick measure of the volatility in the market under DC, which is not available under TS.

For example, the extreme point (01:13, 105) is immediately followed by the transaction (01:23, 90) (rows 6 and 7, columns 2 and 3 in Table 1). The aTMV value at (01:23, 90) is (|90 – 105| ÷ 105) ÷ 5% =) 2.8571. Olsen et al [5][9] observed that historically, on average, price reverse when the price reaches twice the threshold; this means, on average, DC events take place at aTMV equals to 2. The aTMV at (01:33, 83) is 4.1905, which is significantly above 2. Historically, it indicates a reasonably high chance of imminent price reversion.

We also know from historical data that aTMV values roughly follow a power-law distribution [15]: the chance of a DC event happening increases exponentially as aTMV increases. Being able to monitor every transaction allows one to compare the current aTMV with the historical aTMV distribution. A transaction-by-transaction measure of aTMV allows us to estimate the chance of a directional change after the current transaction, according to historical data.

On the other hand, under TS, the high activities between 01:00 and 02:00 are not recorded (Observation 5). As the extreme points are not recorded, TS would not be able to calculate the equivalence of the aTMV values even if one were to track the transactions tick-by-tick.

Definition 2. Absolute Return **aR** in DC:

$$aR = (|P_c - P_{EP}| \div P_{EP}) \div (T_c - T_{EP}) \tag{2}$$

where T_c and P_c are the time and price of the current transaction, T_{EP} and P_{EP} are the time and price of the preceding extreme point. aR is the return from the preceding extreme point to the current transaction, normalized by time. ■

For example, the final extreme point was (08:28, 90) (row 24 in Table 1). At the DCC point (09:08, 97) (row 26), aR is (|97 – 90| ÷ (09:08 – 08:28) =) 11.67% per minute[3]. At the next transaction point (09:40, 99) (row 27 in Table 1), aR is dropped to (|99 – 90| ÷ (09:40 – 08:28) =) 8.33%.

Observation 7. In DC, the absolute return aR can be calculated after each transaction. This gives us a transaction-by-transaction measure of the return since the preceding extreme point. This measure is not available under TS.

If we look at the transaction (01:23, 90) (row 7 in Table 1), we can see that it represents a sharp drop from the preceding extreme point (01:13, 105) (row 6). The aR value is equivalent to (|90 – 105| ÷ (01:23 – 01:13) =) 85.71% per minute. Unlike aTMV, aR takes time into consideration. While the aTMV (2.8571) mentioned above may not be large enough to cause any concern[4], this aR value is alarmingly large. When such a big drop is found in the market, traders and regulators may benefit from being alerted.

Can we monitor the price change and return at each transaction in TS? Yes, we can monitor the price change and return from the last sampled point. However, the last sampled point may not (and most likely will not) be the lowest or highest point in the current trend. So, these measures under TS do not bear the same significance as aTMV and aR in DC. For reference, Chen & Tsang [6] demonstrated how regime changes in the market can be tracked under DC.

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[3] The returns in this artificial data set are large; this is designed for illustration purpose.

[4] This value (2.8571) is not much bigger than the average aTMV at extreme points (which is 2, according to [5][9]).

Real-world high-frequency data

Above we have used artificial data to illustrate why DC could be useful for handling high-frequency data. In this section, we shall support our observations with real-world data. For this purpose, we shall use the tick-to-tick GBP-USD foreign exchange transactions on 23rd June 2016, from 17:00 to 23:55. That was the market on the evening of the Brexit referendum in the UK.

Figure 5 shows a TS with 5-minutely closing prices and a DC Summary under the threshold 0.4% for the above period. The TS has 84 data points; the DC summary has 80 points[5]. Before 19:12, TS has sampled 27 data points whereas DC has only detected 5 directional changes (hence only sampled 5 data points). After 19:12, the market has become much more volatile. From 19:12 to 21:36 (inclusive), during the 29 5-minutely intervals in TS, 34 directional changes took place[6]. From 21:36 to 23:59, during the 28 5-minutely intervals in TS, 41 directional changes took place.

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[5] Readers are reminded that DC is data-driven in sampling data points.

[6] It is difficult to show all the points in Figure 5, as some of the extreme points were close to each other. In fact, two directional changes took place within on second, at 21:36.

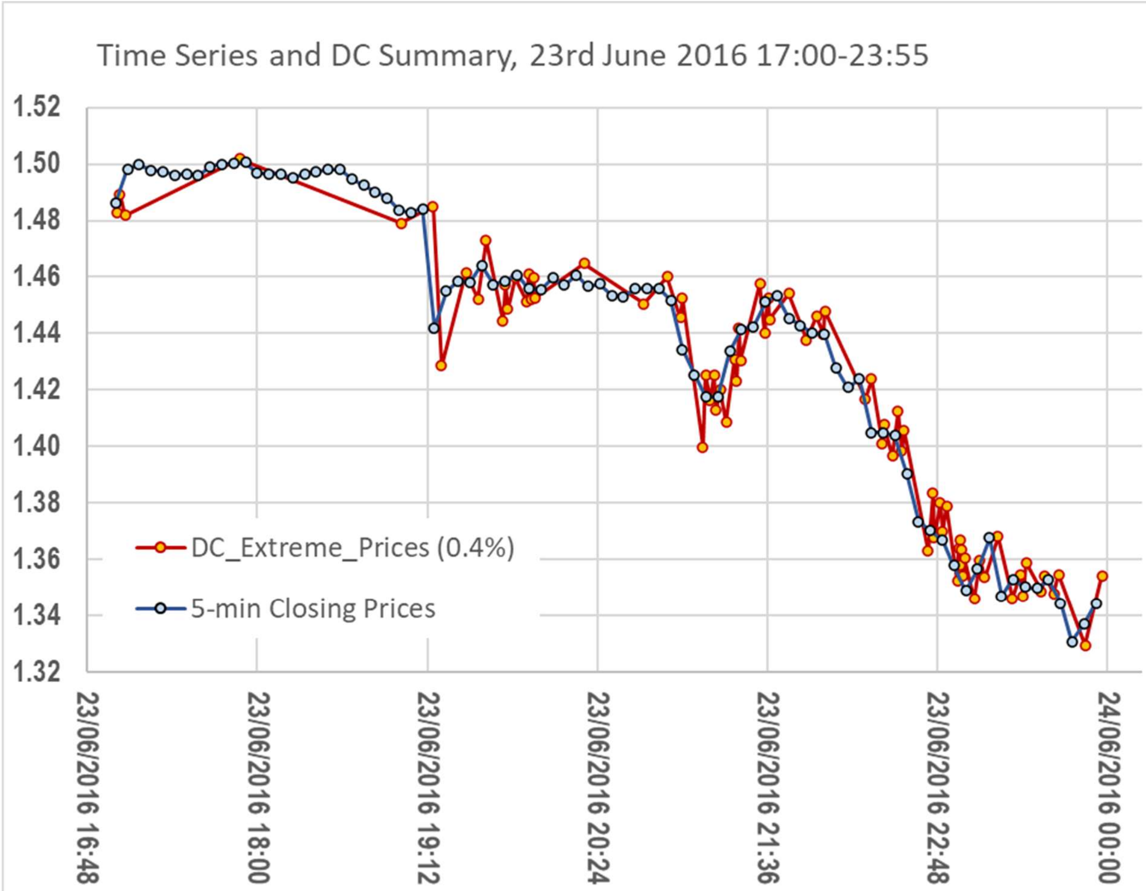

Figure 5. 5-minutely Time Series and DC summary (threshold 0.4%) in 23rd June 2016 17:00-23:55

The number of directional changes (NDC) within a period reflects the volatility of the market. The above NDC shows that volatility increased after 19:12, and increased further after 21:36. This volatility information in TS shows a different picture. Using the standard deviation of log returns, the volatility measured in TS are 0.00200, 0.00696 and 0.00558 in the three periods 17:00-19:12, 19:12-21:36 and 21:36-23:59, respectively. That means, according to the standard deviation of log returns, TS measures higher volatility in 19:12-21:36 than 21:36-23:59. This is different from our observation under NDC, which suggests higher volatility in the latter period than the former. This difference does not suggest any contradiction between TS and DC.

Standard deviation of log-return in TS and NDC just two different ways of measuring volatility. As argued in [14], they are useful for different purposes and should be used in parallel. However, as explained above, log-return standard deviation can only be calculated statistically over a period of time. Therefore, price changes within a few data points do not normally raise the alarm in TS. On the other hand, a few transactions could dramatically increase the aTMV and aR values. Therefore, as mentioned above (Observation 6 and Observation 7), volatility could be monitored tick-by-tick in DC.

A closeup of the market from 21:00 to 21:59 is shown in Figure 6. Within this hour, 12 closing prices were sampled in TS and 19 directional changes were detected. Between 21:10 and 21:15 in TS (the third and fourth data points in blue in Figure 6), the price changed from 1.41763 to 1.41754, or 0.006%. This does not suggest high volatility. In reality, this period is rather volatile, as three DCs took place within these five minutes (the third to fifth data points in red in Figure 6). Similarly, between 21:20 and 21:25 (the fifth and sixth data points in blue in Figure 6), four DCs took place. These support Observation 5 above, that TS may miss high activities in the market. Here, NDC shows high volatility that was not observable in a 5-minutely TS.

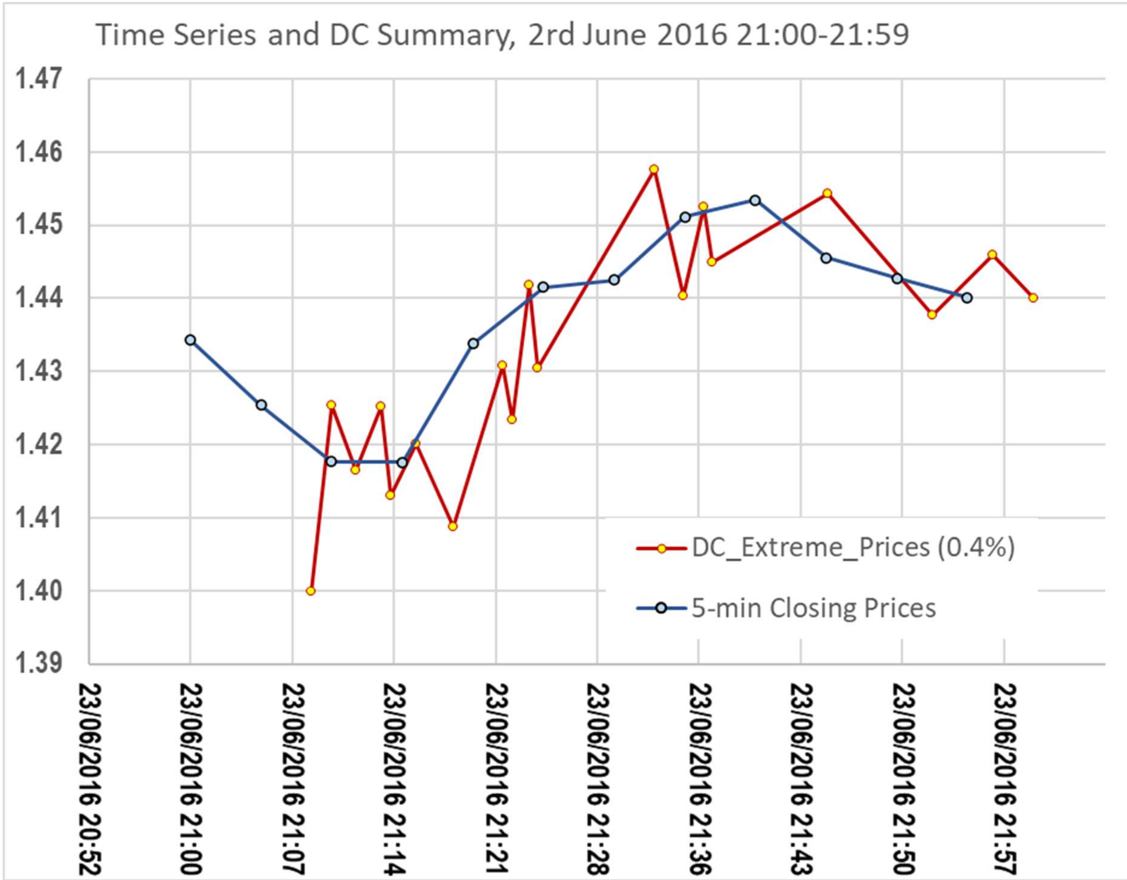

Figure 6. 5-minutely Time Series and DC summary (threshold 0.4%) in 23rd June 2016 21:00-21:59

It is useful to introduce the concept of coastlines in the market [12]. The coastline of the market is the summation of price changes (in absolute value) between consecutive prices in a TS (Definition 3) or a DC summary (Definition 4).

Definition 3. Length of the coastline in a time series $\mathbb{TS}$:

$$\mathrm{LC}(\mathbb{TS}) = \sum_{i=1}^{n-1} \left| \frac{P_{i+1}^{TS} - P_i^{TS}}{P_i^{TS}} \right| \tag{4}$$

Where $P_1^{TS}, P_2^{TS}, \dots, P_n^{TS}$ are prices collected at fixed intervals in $\mathbb{TS}$. ■

Definition 4. Length of the coastline in a DC summary $\mathbb{DC}$:

$$\mathrm{LC}(\mathbb{DC}) = \sum_{i=1}^{n-1} \left| \frac{P_{i+1}^{EXT} - P_i^{EXT}}{P_i^{EXT}} \right| \tag{3}$$

Where $P_1^{EXT}, P_2^{EXT}, \ldots, P_n^{EXT}$ are prices at extreme points in $\mathbb{DC}$. ■

The coastline of the TS in Figure 5 is 29.7%. The coastline of the corresponding DC extreme series is 79.6%. The two series use a similar number of data points (84 and 80, respectively) to capture the same period of the market (23rd June 2016 17:00-23:55). The difference in the lengths of the coastlines suggests that a DC-based trading strategy with perfect foresight could potentially gain more than a TS-based strategy. That means there is potentially more profit to be gained by using DC summaries for research.

It is reasonable to conjecture that given a TS and a DC summary that use the same number of data points to cover the same period, the length of the DC coastline is at least as long as that of the TS. This is because, by definition, peaks and troughs are captured by DC (under a given threshold) but not necessarily TS (see Observation 4 above). A formal proof of this conjecture will be left for future work.

Coastline Conjecture: Given a time series $\mathbb{TS}$ and a DC summary $\mathbb{DC}$ that cover the same period with the same number of data points, the coastline of $\mathbb{DC}$ is at least as long as the coastline of $\mathbb{TS}$.

$$\mathrm{LC}(\mathbb{TS}) \leq \mathrm{LC}(\mathbb{DC})$$ ■

Concluding Summary

We have used an artificial data set (Table 1) to compare and contrast how transactions can be summarised under TS and DC. We have used a real market scenario (Figure 5) to support our findings. We recapitulate our observations below:

1. Data points in a TS record approximations of actual transaction times (Observation 1), whereas DC records transactions as they are (Observation 3).
2. As transactions take place at irregular times in tick-to-tick data (TD), reconstruction of missing data may be needed in TS (Observation 2).
3. All extreme points are recorded in DC summaries; they may be missed by TS sampling (Observation 4).
4. High activities in the market captured by DC may be missed in TS (Observation 5).
5. To increase the chance of capturing extreme points and high activities in TS, one could choose to increase the sampling frequency. But doing so may increase the error in approximations (Observation 1) and the need to fill in missing data (Observation 2).
6. Volatility measures in DC are more suitable for market monitoring: a single transaction may provide us with valuable signals about the market. With aTMV (Definition 1) and aR (Definition 2), one could monitor the price movement transaction-by-transaction. The same cannot be done in TS, as it does not guarantee recording extreme points (Observation 6 and Observation 7).

For these reasons, we argue that DC is more suitable than TS for recording and analysing TD.

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